

## Worksheet: The Cross Product

This worksheet has questions on the cross product between two vectors. Before attempting the questions below, you could read the study guide: [The Cross Product](#).

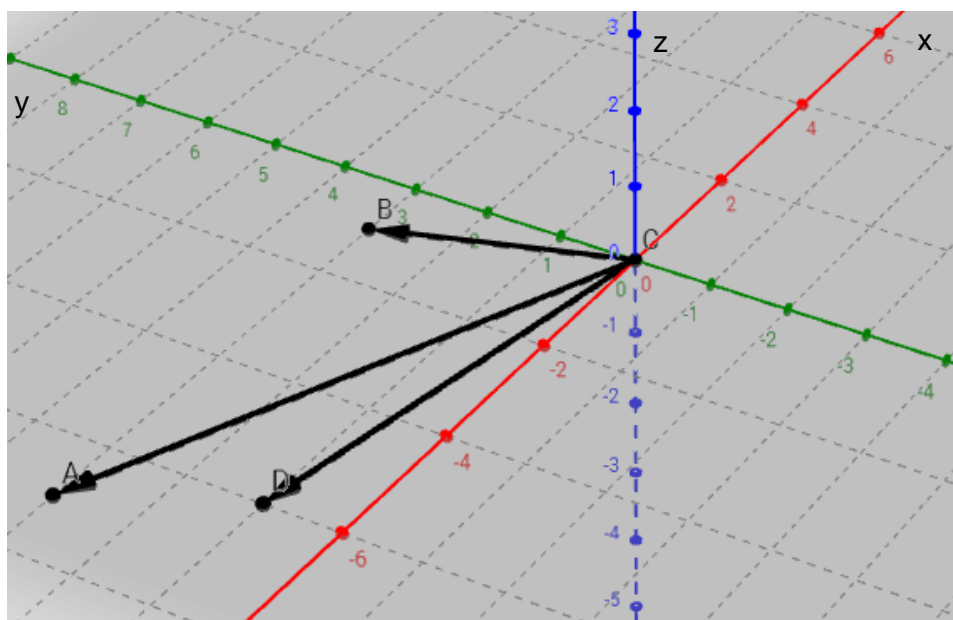
The Cross Product  
study guide



Model Answers  
to this sheet



1. Look at the vectors in the following image.



- a. The vectors  $\overrightarrow{CD}$ ,  $\overrightarrow{CA}$  and  $\overrightarrow{CB}$  are in the  $xy$ -plane (their  $z$ -coordinate is zero). Express the vectors in terms of the rectangular unit vectors  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$ .
- b. Calculate the following cross products and think about what they will look like:
- |  |   |
|--|---|
| i. $\overrightarrow{CD} \times \overrightarrow{CA}$  | iii. $\overrightarrow{CD} \times \overrightarrow{CB}$ |
| ii. $\overrightarrow{CA} \times \overrightarrow{CD}$ | iv. $\overrightarrow{CB} \times \overrightarrow{CD}$  |
- c. What do you notice about your answers to parts (i) and (ii) above? What do you notice about your answers to parts (iii) and (iv) above? Write a piece of maths that describes your observations.

2. What is the cross product of the vectors **a** and **b** when:
- $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$  and  $\mathbf{b} = -2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ ?
  - $\mathbf{a} = \frac{2}{3}\mathbf{i} + \frac{1}{2}\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = -\mathbf{i} + \frac{1}{3}\mathbf{j} - \frac{1}{2}\mathbf{k}$ ?
  - $\mathbf{a} = -\mathbf{i} + \sqrt{2}\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = \sqrt{2}\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ ?
3. You are given the following vectors  $\mathbf{a} = -2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$  and  $\mathbf{b} = 4\mathbf{i} - 1\mathbf{j} + 3\mathbf{k}$ .
- Find a vector that is normal to both vectors and call it **c**.
  - Verify that your vector **c** is normal to both **a** and **b**, using the dot product.
  - Calculate the magnitude of **c**.
  - Given that the cross product is  $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin \theta \hat{\mathbf{n}}$ , find the angle  $\theta$  between the vectors **a** and **b**.
4. Given the two vectors  $\mathbf{a} = \frac{1}{4}\mathbf{i} + 2\mathbf{j} - \mathbf{k}$  and  $\mathbf{b} = \mathbf{i} + 8\mathbf{j} - 4\mathbf{k}$ .
- Calculate their cross product. What does the cross product tell you about the direction and the angle  $\theta$  between the vectors **a** and **b**?
  - Find the angle  $\phi$  between the vectors **a** and  $\mathbf{c} = -\mathbf{i} - 8\mathbf{j} + 4\mathbf{k}$ . What do you notice?
5. You are given the following vectors  $\mathbf{a} = -\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$  and  $\mathbf{b} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$ .
- Calculate the cross products  $\mathbf{a} \times \mathbf{b}$  and  $\mathbf{b} \times \mathbf{a}$ . Do your answers confirm what you wrote for question 1c?
  - Given the vector  $\mathbf{c} = 2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$  calculate  $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$  and  $\mathbf{b} \cdot (\mathbf{c} \times \mathbf{a})$ . What do you notice? Can you write a piece of mathematics that describes your observation?



This worksheet is one of a series on mathematics produced by the Learning Enhancement Team with funding from the UEA Alumni Fund. Scan the QR-code with a smartphone app for [more resources](#).



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