Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Factsheet: Vectors

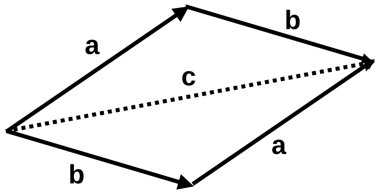
Definitions

In this factsheet vectors are depicted in bold type (e.g. **a**) and scalars in italics (e.g. *m*).

A vector it has both magnitude *and* direction whereas a scalar only has magnitude

Parallelogram law of vector addition

If $\mathbf{a}+\mathbf{b}=\mathbf{c}$ then $\mathbf{a}=\mathbf{c}-\mathbf{b}$ and $\mathbf{b}=\mathbf{c}-\mathbf{a}$.



Vector properties and special vectors

Magnitude of a vector **a**, written as $|\mathbf{a}|$ or *a*, represents the vector length and is a scalar.

Unit vectors have a magnitude of 1. A unit vector having the same direction as **a** is written $\hat{\mathbf{a}}$ with the hat above the **a** indicating the unit vector.

Specifically $\hat{\mathbf{a}}=\dfrac{\mathbf{a}}{a}$.

Rectangular unit vectors represent unit steps in the positive *x*, *y* and *z*-directions respectively.

Unit step in the *x*-direction is represented as $\hat{\mathbf{i}}$
Unit step in the *y*-direction is represented as $\hat{\mathbf{j}}$
Unit step in the *z*-direction is represented as $\hat{\mathbf{k}}$

Any vector can be described in terms of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$.

For example the vector **a** can be written as $\mathbf{a}=a_1\hat{\mathbf{i}}+a_2\hat{\mathbf{j}}+a_3\hat{\mathbf{k}}$ where a_1, a_2 and a_3 are the **components** of **a**.

The magnitude of **a** is from Pythagoras' theorem: $a=|\mathbf{a}|=\sqrt[2]{a_1^2+a_2^2+a_3^2}$.

For three vectors **a**, **b** and **c** and scalars *m* and *n*:

$$\mathbf{a}+\mathbf{b}=(a_1+b_1)\hat{\mathbf{i}}+(a_2+b_2)\hat{\mathbf{j}}+(a_3+b_3)\hat{\mathbf{k}}$$
$$m\mathbf{a}=ma_1\hat{\mathbf{i}}+ma_2\hat{\mathbf{j}}+ma_3\hat{\mathbf{k}}$$

A **Position Vector r** describes a journey from the origin to a particular coordinate. The coordinate (x,y,z) has a corresponding position vector $\mathbf{r}=x\hat{\mathbf{i}}+y\hat{\mathbf{j}}+z\hat{\mathbf{k}}$.

Dot or scalar product

The dot or scalar product of vectors $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$ is defined as:

$$\mathbf{a}\cdot\mathbf{b} = ab\cos\theta = a_1b_1 + a_2b_2 + a_3b_3$$
$$\mathbf{a}\cdot\mathbf{a} = a^2 = a_1^2 + a_2^2 + a_3^2$$

where θ is the angle between **a** and **b** if they share the same start point. The result of a dot product is a scalar.

Specifically

$$\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$$
$$\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{c}$$
$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$$
$$\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = 0$$

If $\mathbf{a}\cdot\mathbf{b} = 0$ then **a** and **b** are perpendicular to each other.

Cross or vector product

The cross or vector product of vectors $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$ is defined as:

$$\mathbf{a}\times\mathbf{b} = ab\sin\theta\,\hat{\mathbf{n}} = (a_2b_3 - a_3b_2)\hat{\mathbf{i}} - (a_1b_3 - a_3b_1)\hat{\mathbf{j}} + (a_1b_2 - a_2b_1)\hat{\mathbf{k}}$$

Specifically

$$\mathbf{a}\times\mathbf{b} = -\mathbf{b}\times\mathbf{a}$$
$$\mathbf{a}\times\mathbf{a} = 0$$
$$\mathbf{a}\times(\mathbf{b}+\mathbf{c}) = \mathbf{a}\times\mathbf{b} + \mathbf{a}\times\mathbf{c}$$
$$m(\mathbf{a}\times\mathbf{b}) = (m\mathbf{a})\times\mathbf{b} = \mathbf{a}\times(m\mathbf{b}) = (\mathbf{a}\times\mathbf{b})m$$
$$\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}}\times\hat{\mathbf{k}} = 0$$
$$\hat{\mathbf{i}}\times\hat{\mathbf{j}} = \hat{\mathbf{k}} \qquad \hat{\mathbf{j}}\times\hat{\mathbf{k}} = \hat{\mathbf{i}} \qquad \hat{\mathbf{k}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}$$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761

Ask: ask.let@uea.ac.uk

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