Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Factsheet: Matrices

Size or **Order** of a matrix is given by **number of rows × number of columns** so an $n \times m$ matrix has n rows and m columns. **Square matrices** have the same number of rows and columns so $n = m$ and a square matrix is an $n \times n$ matrix.

An **element** of a matrix A is an entry into that matrix, usually denoted by a_{ij} where i is the row the element is in and j is the column the element is in.

The **transpose** of an matrix A, written A^{T} swaps the rows and columns in A so a_{ij} becomes a_{ji} .

The **main diagonal** of a square matrix goes from top left to bottom right in the matrix. The sum of the elements of the main diagonal is called the **trace** of the matrix.

Operation	Symbol	Size of Input	Size of Output	2×2	General Element
Addition	$A+B$	$n \times m$ and $n \times m$	$n \times m$	$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$	$a_{ij} + b_{ij}$
Subtraction	$A-B$	$n \times m$ and $n \times m$	$n \times m$	$\begin{pmatrix} a & b \\ c & d \end{pmatrix} - \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a-e & b-f \\ c-g & d-h \end{pmatrix}$	$a_{ij} - b_{ij}$
Scalar Multiplication	λA	Scalar and $n \times m$	$n \times m$	$\lambda \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \lambda a & \lambda b \\ \lambda c & \lambda d \end{pmatrix}$	λa_{ij}
Matrix Multiplication	AB	$n \times m$ and $m \times p$	$n \times p$	$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{pmatrix}$	$\sum_k a_{ik} b_{kj}$

In general: $AB \neq BA$ (matrix multiplication **does not commute**)

The **identity matrix** I is a square matrix where each element in the main diagonal is 1 and all the other elements are 0.

Multiplication by the identity matrix: $AI = IA = A$

Property	Symbol	2×2	3×3
General Matrix	A	$A=\begin{pmatrix} a & b \\ c & d \end{pmatrix}$	$A=\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$
Identity	I	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$
Determinant	$\det A = \|A\|$	$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$	$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$ $= a(ei-fh)-b(di-fg)+c(dh-ge)$ **Rule of Sarrus**
Inverse* (if $\det A \neq 0$)	A^{-1}	$\frac{1}{\det A}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	$\frac{1}{\det A}\begin{pmatrix} (ei-fh) & (fg-di) & (dh-eg) \\ (ch-bi) & (ai-cg) & (bg-ah) \\ (bf-ce) & (cd-af) & (ae-bd) \end{pmatrix}$

*If a matrix has a determinant of 0 then it has no inverse and is **singular**.

Multiplication by an inverse: $AA^{-1} = A^{-1}A = I$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761
Ask: ask.let@uea.ac.uk
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