

## Factsheet: Logic and Propositions

Here propositions are denoted by  $p$ ,  $q$  and so on,  $T$  denotes True and  $F$  denotes False.

### Basic Connectives and Statements

<p style="text-align: center;"><b>Conjunction</b> <math>p \wedge q</math> "p and q"</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>p</math></th> <th><math>q</math></th> <th><math>p \wedge q</math></th> </tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>F</td></tr> <tr><td>F</td><td>T</td><td>F</td></tr> <tr><td>F</td><td>F</td><td>F</td></tr> </tbody> </table>	$p$	$q$	$p \wedge q$	T	T	T	T	F	F	F	T	F	F	F	F	<p style="text-align: center;"><b>Disjunction</b> <math>p \vee q</math> "p or q"</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>p</math></th> <th><math>q</math></th> <th><math>p \vee q</math></th> </tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>T</td></tr> <tr><td>F</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>F</td></tr> </tbody> </table>	$p$	$q$	$p \vee q$	T	T	T	T	F	T	F	T	T	F	F	F	<p style="text-align: center;"><b>Negation</b> <math>\bar{p}, \neg p</math> or <math>\sim p</math> "not p"</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>p</math></th> <th><math>\bar{p}</math></th> </tr> </thead> <tbody> <tr><td>T</td><td>F</td></tr> <tr><td>F</td><td>T</td></tr> </tbody> </table>	$p$	$\bar{p}$	T	F	F	T
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<p style="text-align: center;"><b>Conditional</b> <math>p \rightarrow q, p \Rightarrow q</math> or <math>p \supset q</math> "If p then q" or "p implies q"</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>p</math></th> <th><math>q</math></th> <th><math>p \rightarrow q</math></th> </tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>F</td></tr> <tr><td>F</td><td>T</td><td>T</td></tr> <tr><td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	$p$	$q$	$p \rightarrow q$	T	T	T	T	F	F	F	T	T	F	F	T	<p style="text-align: center;"><b>Biconditional</b> <math>p \leftrightarrow q, p \Leftrightarrow q</math> or <math>p \equiv q</math> "p if and only if q" "p iff q" "p implies q and q implies p"</p> <table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <thead> <tr> <th><math>p</math></th> <th><math>q</math></th> <th><math>p \leftrightarrow q</math></th> </tr> </thead> <tbody> <tr><td>T</td><td>T</td><td>T</td></tr> <tr><td>T</td><td>F</td><td>F</td></tr> <tr><td>F</td><td>T</td><td>F</td></tr> <tr><td>F</td><td>F</td><td>T</td></tr> </tbody> </table>	$p$	$q$	$p \leftrightarrow q$	T	T	T	T	F	F	F	T	F	F	F	T							
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### Tautology

$\vDash p$  "p is always true"

### Contradiction

$\not\vdash p$  "p is always false"

### Algebraic Equivalence

$p \equiv q$  "p is algebraically equivalent to q"

### Two Useful Logical Equivalences

$$p \rightarrow q \equiv \bar{p} \vee q$$

$$p \leftrightarrow q \equiv (\bar{p} \vee q) \wedge (p \vee \bar{q})$$

## Algebraic Laws of Propositions

1.	Idempotent Laws	(i)	$p \vee p \equiv p$	(ii)	$p \wedge p \equiv p$
2.	Associative Laws	(i)	$(p \vee q) \vee r \equiv p \vee (q \vee r)$		
		(ii)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$		
3.	Commutative Laws	(i)	$p \vee q \equiv q \vee p$	(ii)	$p \wedge q \equiv q \wedge p$
4.	Distributive Laws	(i)	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$		
		(ii)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$		
5.	Identity Laws	(i)	$p \vee F \equiv p$	(ii)	$p \vee T \equiv T$
		(iii)	$p \wedge T \equiv p$	(iv)	$p \wedge F \equiv F$
6.	Complement Laws	(i)	$p \vee \bar{p} \equiv T$	(ii)	$p \wedge \bar{p} \equiv F$
		(iii)	$\bar{\bar{T}} \equiv F$	(iv)	$\bar{\bar{F}} \equiv T$
7.	Involution Law		$\bar{\bar{p}} \equiv p$		
8.	De Morgan's Laws	(i)	$\overline{p \vee q} \equiv \bar{p} \wedge \bar{q}$	(ii)	$\overline{p \wedge q} \equiv \bar{p} \vee \bar{q}$
9.	Modus Ponens		$(p \wedge (p \rightarrow q)) \rightarrow q$		
10.	Modus Tollens		$(\bar{p} \wedge (q \rightarrow p)) \rightarrow \bar{q}$		
11.	Disjunctive Syllogism		$(\bar{p} \wedge (p \vee q)) \rightarrow q$		
12.	Hypothetical Syllogism		$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$		

## Want to know more?

If you have any further questions about this topic you can make an appointment to see a [Learning Enhancement Tutor](#) in the [Student Support Service](#), as well as speaking to your lecturer or adviser.

- ☎ Call: 01603 592761
- ✉ Ask: [ask.let@uea.ac.uk](mailto:ask.let@uea.ac.uk)
- 🖱 Click: <https://portal.uea.ac.uk/student-support-service/learning-enhancement>

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