Factsheet: Differential Equations

First-order differential equations

(1) Simple integrable $\frac{dy}{dx} = p(x)$

Integrate both sides to find *y*.

(2) Separable $\frac{dy}{dx} = \frac{p(x)}{q(y)}$

Separate the variables by rearranging (all *x* on one side and all *y* on the other) then integrate both sides. It may be possible to rearrange your answer for *y*.

(3) Homogeneous $\frac{dy}{dx} = p(x,y)$

If $p(x,y) = p(ax,ay)$ then you can use the substitutions:

$y = ux$ and $\frac{dy}{dx} = u + x\frac{du}{dx}$ (by the product rule)

to make the differential equation separable and then solve using method (2).

(4) Linear $\frac{dy}{dx} + p(x)y = q(x)$

If you multiply through by the integrating factor $I(x) = \exp\left[\int p(x)dx\right]$ you get:

$$I(x)\frac{dy}{dx} + I(x)p(x)y = I(x)q(x)$$

The product rule shows that $\frac{d}{dx}(Iy) = I(x)\frac{dy}{dx} + I(x)p(x)y$ and so:

$$\frac{d}{dx}(I(x)y) = I(x)q(x)$$

which is type (1).

(5) Bernoulli Type $\frac{dy}{dx} + p(x)y = q(x)y^n$

Divide through by y^n and choose $u = \frac{1}{y^{n-1}}$, the differential equation should now be linear in *u* and solvable by method (4).

Second-order differential equations

If they are of the form: $$\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=f(x)$$

(1) Known as **homogeneous** if $f(x)=0$, so $y''+ay'+by=0$. Choose $y=e^{kx}$ thus $k^2+ak+b=0$ (**Auxiliary Equation**) with roots k_1 and k_2.

(a) When roots are real and different $y=Ae^{k_1x}+Be^{k_2x}$

(b) When roots are real and equal $y=(Ax+B)e^{kx}$

(c) When roots are complex $k_1=\alpha+\mathrm{i}\beta$, $k_2=\alpha-\mathrm{i}\beta$, $y=e^{\alpha x}(A\cos\beta x+B\sin\beta x)$

(2) Known as **non-homogeneous** if $f(x)\neq 0$. **General solution** (GS) is sum of a **complimentary function** (CF) and a **particular integral** (PI) so $\mathrm{GS}=\mathrm{CF}+\mathrm{PI}$. For CF set $f(x)=0$ and solve the result (see above). For PI:

(a) If $f(x)$ is a polynomial try a general polynomial of the same degree.

(b) If $f(x)$ is an exponential function try a general exponential function $y=Ae^{kx}$ if it not already part of the CF. If part of the CF try $y=(Ax+B)e^{kx}$.

(c) If $f(x)$ is a trigonometric function try $y=A\sin nx+B\cos nx$

Want to know more?

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