University of East Anglia
STUDENT SUPPORT SERVICE

Learning Enhancement Team

Model Answers: Implicit Differentiation

These are the model answers for the worksheet that has questions on implicit differentiation.

Implicit Differentiation study guide

The implicit functions below contain two variables. Remember that you assume that *y* is a function of *x*. You need to differentiate them with respect to *x.*

a. $x^2 + y^2 + y = 0$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(x^2\right) + \frac{d}{dx}\left(y^2\right) + \frac{d}{dx}(y) = \frac{d}{dx}(0)$$

The first term on the left is a basic function of *x* (to remind yourself how to differentiate basic functions of *x* you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}\left(x^2\right) = 2x$$

The second term on the left requires the chain rule form of the differential operator as y^2 is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(y^2\right) = 2y\frac{dy}{dx}$$

The third term is the derivative of *y* with respect to *x*:

$$\frac{d}{dx}(y) = \frac{dy}{dx}$$

The right hand side is the derivative of 0 with respect to x (which is zero). Combining these results together gives the derivative of $x^2 + y^2 + y = 0$ as:

$$2x + 2y\frac{dy}{dx} + \frac{dy}{dx} = 0$$

This result is fine as it is written but you can rearrange it for dy/dx. Firstly, subtract $2x$ from each side to give:

$$2y\frac{dy}{dx} + \frac{dy}{dx} = -2x$$

Now each term on the left hand side is multiplied by dy/dx and so you can factorise:

$$(2y+1)\frac{dy}{dx} = -2x$$

Finally, dividing by $2y+1$ gives:

$$\frac{dy}{dx} = \frac{-2x}{2y+1}$$

Note that the derivative is a function of both y and x, this is very common when you use implicit differentiation.

b. $x^4 + y^4 - 4 = 0$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(x^4\right) + \frac{d}{dx}\left(y^4\right) + \frac{d}{dx}(-4) = \frac{d}{dx}(0)$$

The first term on the left is a basic function of x (to remind yourself how to differentiate basic functions of x you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}\left(x^4\right) = 4x^3$$

The second term on the left requires the chain rule form of the differential operator as y^4 is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form: ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(y^4\right) = 4y^3 \frac{dy}{dx}$$

The third term is the derivative of -4 with respect to *x* (which is zero) and so:

$$\frac{d}{dx}(-4) = 0$$

The right hand side is the derivative of 0 with respect to *x* (which is zero). Combining these results together gives the derivative of $x^2 + y^2 + y = 0$ as:

$$4x^3 + 4y^3 \frac{dy}{dx} + 0 = 0$$

This result is fine as it is written but you can rearrange it for dy/dx. Firstly, subtract $4x^3$ from each side to give:

$$4y^3 \frac{dy}{dx} = -4x^3$$

Finally, dividing by $4y^3$ gives:

$$\frac{dy}{dx} = \frac{-4x^3}{4y^3}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

c. $x^3y^4 + x^4y^3 - y = 0$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(x^3y^4\right) + \frac{d}{dx}\left(x^4y^3\right) + \frac{d}{dx}(-y) = \frac{d}{dx}(0)$$

The first term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) and so:

$$\frac{d}{dx}\left(x^3y^4\right) = x^3\frac{d}{dx}\left(y^4\right) + y^4\frac{d}{dx}\left(x^3\right)$$

As y^4 is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(x^3y^4\right)=x^3 4y^3\frac{dy}{dx}+y^4\left(3x^2\right)=4x^3y^3\frac{dy}{dx}+3x^2y^4$$

The second term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) and so:

$$\frac{d}{dx}\left(x^4y^3\right)=x^4\frac{d}{dx}\left(y^3\right)+y^3\frac{d}{dx}\left(x^4\right)$$

As y^4 is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(x^4y^3\right)=x^3 3y^2\frac{dy}{dx}+y^3\left(4x^3\right)=3x^3y^2\frac{dy}{dx}+4x^3y^3$$

The third term is the derivative of $-y$ with respect to *x*:

$$\frac{d}{dx}\left(-y\right)=-\frac{dy}{dx}$$

The right hand side is the derivative of 0 with respect to *x* (which is zero). Combining these results together gives the derivative of $x^3y^4+x^4y^3-y=0$ as:

$$4x^3y^3\frac{dy}{dx}+3x^2y^4+3x^3y^2\frac{dy}{dx}+4x^3y^3-\frac{dy}{dx}=0$$

You can rearrange for dy/dx by subtracting $\left(3x^2y^4+4x^3y^3\right)$ from each side to give:

$$4x^3y^3\frac{dy}{dx}+3x^3y^2\frac{dy}{dx}-\frac{dy}{dx}=-3x^2y^4-4x^3y^3$$

Finally, dividing by $4x^3y^3+3x^3y^2-1$ gives:

$$\frac{dy}{dx}=\frac{-3x^2y^4-4x^3y^3}{4x^3y^3+3x^3y^2-1}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

d. $-x^2+7xy+6y^3-6=0$

As you cannot use basic methods to rearrange this function to get $y=f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(-x^2\right)+\frac{d}{dx}(7xy)+\frac{d}{dx}\left(6y^3\right)+\frac{d}{dx}(-6)=\frac{d}{dx}(0)$$

The first term on the left is a basic function of x (to remind yourself how to differentiate basic functions of x you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}\left(-x^2\right)=-2x$$

The second term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) and so:

$$\frac{d}{dx}(7xy)=7x\frac{d}{dx}(y)+y\frac{d}{dx}(7x)$$

As y is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}(7xy)=7x\frac{dy}{dx}+y(7)=7x\frac{dy}{dx}+7y$$

The third term on the left requires the chain rule form of the differential operator as $6y^3$ is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(6y^3\right)=18y^2\frac{dy}{dx}$$

The fourth term is the derivative of -6 with respect to x (which is zero) and so:

$$\frac{d}{dx}(-6)=0$$

The right hand side is the derivative of 0 with respect to x (which is zero). Combining these results together gives the derivative of $-x^2+7xy+6y^3-6=0$ as:

$$-2x+7x\frac{dy}{dx}+7y+18y^2+0=0$$

You can rearrange for dy/dx by subtracting $\left(-2x+7y+18y^2\right)$ from each side to give:

$$7x\frac{dy}{dx} = 2x - 7y - 18y^2$$

Finally, dividing by $7x$ gives:

$$\frac{dy}{dx} = \frac{2x - 7y - 18y^2}{7x}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

e. $$\frac{x^2 - y^3}{y + x} - x + y - 2 = 0$$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(\frac{x^2 - y^3}{y + x}\right) + \frac{d}{dx}(-x) + \frac{d}{dx}(y) + \frac{d}{dx}(-2) = \frac{d}{dx}(0)$$

The first term on the left is a quotient of two functions and so you use the quotient rule (to remind yourself how to use the quotient rule you can check the study guide: *The Quotient Rule*) and so:

$$\frac{d}{dx}\left(\frac{x^2 - y^3}{y + x}\right) = \frac{(y + x)\frac{d}{dx}(x^2 - y^3) - (x^2 - y^3)\frac{d}{dx}(y + x)}{(y + x)^2}$$

$$= \frac{(y + x)\left(2x - 3y^2\frac{dy}{dx}\right) - (x^2 - y^3)\left(\frac{dy}{dx} + 1\right)}{(y + x)^2}$$

The second term is a basic function of *x* (to remind yourself how to differentiate basic functions of *x* you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}(-x) = -1$$

The third term is the derivative of *y* with respect to *x*:

$$\frac{d}{dx}(y) = \frac{dy}{dx}$$

The fourth term is the derivative of -2 with respect to x (which is zero) and so:

$$\frac{d}{dx}(-2) = 0$$

The right hand side is the derivative of 0 with respect to x (which is zero). Combining these results together gives the derivative of $\frac{x^2 - y^3}{y + x} - x + y - 2 = 0$ as:

$$\frac{(y+x)\left(2x - 3y^2\frac{dy}{dx}\right) - (x^2 - y^3)\left(\frac{dy}{dx} + 1\right)}{(y+x)^2} - 1 + \frac{dy}{dx} + 0 = 0$$

You can rearrange for dy/dx by subtracting $\left(\frac{2xy + x^2 + y^3}{(y+x)^2} - 1\right)$ from each side to give:

$$\left(\frac{-2y^3 - 3xy^2 - x^2}{(y+x)^2} + 1\right)\frac{dy}{dx} = -\frac{2xy + x^2 + y^3}{(y+x)^2} + 1$$

Finally, dividing by $\left(\frac{-2y^3 - 3xy^2 - x^2}{(y+x)^2} + 1\right)$ gives:

$$\frac{dy}{dx} = \frac{-\frac{2xy + x^2 + y^3}{(y+x)^2} + 1}{\frac{-2y^3 - 3xy^2 - x^2}{(y+x)^2} + 1}$$

$$= \frac{y(y - y^2)}{y(-2y^2 - 3xy + y + 2x)}$$

$$= \frac{y - y^2}{-2y^2 - 3xy + y + 2x}$$

Note that the derivative is a function of both y and x, this is very common when you use implicit differentiation.

f. $\frac{x^4 + 3y^3}{2y^4 + x} - x^2y + y^{-4} - 2 = 0$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(\frac{x^4 + 3y^3}{2y^4 + x}\right) + \frac{d}{dx}\left(-x^2y\right) + \frac{d}{dx}\left(y^{-4}\right) + \frac{d}{dx}(-2) = \frac{d}{dx}(0)$$

The first term on the left is a quotient of two functions and so you use the quotient rule (to remind yourself how to use the quotient rule you can check the study guide:

The Quotient Rule) and so:

$$\frac{d}{dx}\left(\frac{x^4+3y^3}{2y^4+x}\right)=\frac{\left(2y^4+x\right)\frac{d}{dx}\left(x^4+3y^3\right)-\left(x^4+3y^3\right)\frac{d}{dx}\left(2y^4+x\right)}{\left(2y^4+x\right)^2}$$

$$=\frac{\left(2y^4+x\right)\left(4x^3+3y^2\frac{dy}{dx}\right)-\left(x^4+3y^3\right)\left(8y^3\frac{dy}{dx}+1\right)}{\left(2y^4+x\right)^2}$$

The second term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: The Product Rule) and so:

$$\frac{d}{dx}\left(-x^2y\right)=-x^2\frac{d}{dx}\left(y\right)+y\frac{d}{dx}\left(-x^2\right)$$

As y is a composite function of x. The rules on the first page of the study guide: Implicit Differentiation tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(-x^2y\right)=-x^2\frac{dy}{dx}+y\left(-2x\right)=-x^2\frac{dy}{dx}-2xy$$

The third term on the left requires the chain rule form of the differential operator as y^{-4} is a composite function of x. The rules on the first page of the study guide: Implicit Differentiation tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(y^{-4}\right)=-4y^{-5}\frac{dy}{dx}$$

The fourth term is the derivative of -2 with respect to x (which is zero) and so:

$$\frac{d}{dx}\left(-2\right)=0$$

The right hand side is the derivative of 0 with respect to x (which is zero). Combining these results together gives the derivative of $\frac{x^4+3y^3}{2y^4+x}-x^2y+y^{-4}-2=0$ as:

$$\frac{\left(2y^4+x\right)\left(4x^3+3y^2\frac{dy}{dx}\right)-\left(x^4+3y^3\right)\left(8y^3\frac{dy}{dx}+1\right)}{\left(2y^4+x\right)^2}-x^2\frac{dy}{dx}-2xy-4y^{-5}\frac{dy}{dx}+0=0$$

You can rearrange for dy/dx by subtracting $\left(\frac{8x^3y^4+3x^4-24y^6-3y^3}{(2y^4+x)^2}-2xy\right)$ from each side to give:

$$\left(\frac{6y^6+3xy^2-8x^4y^3}{(2y^4+x)^2}-x^2-4y^{-5}\right)\frac{dy}{dx}=-\frac{8x^3y^4+3x^4-24y^6-3y^3}{(2y^4+x)^2}+2xy$$

Finally, dividing by $\left(\frac{6y^6+3xy^2-8x^4y^3}{(2y^4+x)^2}-x^2-4y^{-5}\right)$ gives:

$$\frac{dy}{dx}=\frac{-\frac{8x^3y^4+3x^4-24y^6-3y^3}{(2y^4+x)^2}+2xy}{\left(\frac{6y^6+3xy^2-8x^4y^3}{(2y^4+x)^2}-x^2-4y^{-5}\right)}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

2.

a. $-xy+e^y=0$

As you cannot use basic methods to rearrange this function to get $y=f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}(-xy)+\frac{d}{dx}(e^y)=\frac{d}{dx}(0)$$

The first term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) and so:

$$\frac{d}{dx}(-xy)=-x\frac{d}{dx}(y)+y\frac{d}{dx}(-x)$$

As *y* is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}(-xy)=-x\frac{dy}{dx}+y(-1)=-x\frac{dy}{dx}-y$$

The second term involves differentiating e^y which is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ae^{ky} the derivative is given by $ake^{ky}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(e^y\right)=e^y\frac{dy}{dx}$$

The right hand side is the derivative of 0 with respect to *x* (which is zero). Combining these results together gives the derivative of $-xy+e^y=0$ as:

$$-x\frac{dy}{dx}-y+e^y\frac{dy}{dx}=0$$

You can rearrange for dy/dx by adding *y* to each side to give:

$$-x\frac{dy}{dx}+e^y\frac{dy}{dx}=y$$

Finally, dividing by $\left(-x+e^y\right)$ gives:

$$\frac{dy}{dx}=\frac{y}{\left(-x+e^y\right)}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

b. $-x\left(y^2+xe^y+\cos x\right)=0$

Opening the brackets you can see that this function can be rewritten as:

$$-xy^2-x^2e^y-x\cos x=0$$

As you cannot use basic methods to rearrange this function to get $y=f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(-xy^2\right)+\frac{d}{dx}\left(-x^2e^y\right)+\frac{d}{dx}\left(-x\cos x\right)=\frac{d}{dx}(0)$$

The first term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) and so:

$$\frac{d}{dx}\left(-xy^2\right)=-x\frac{d}{dx}\left(y^2\right)+y^2\frac{d}{dx}(-x)$$

As y is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(-xy^2\right)=-x2y\frac{dy}{dx}+y^2(-x)=-2xy\frac{dy}{dx}-2xy^2$$

The second term is a product of two functions and so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) and so:

$$\frac{d}{dx}\left(-x^2e^y\right)=-x^2\frac{d}{dx}\left(e^y\right)+e^y\frac{d}{dx}\left(-x^2\right)$$

As y is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form: ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So,

$$\frac{d}{dx}\left(-x^2e^y\right)=-x^2e^y\frac{dy}{dx}+e^y(-2x)=-x^2e^y\frac{dy}{dx}-2xe^y$$

The third term is a product of basic function of x (to remind yourself how to differentiate basic functions of x you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}(-x\cos x)=-x\frac{d}{dx}(\cos x)+(\cos x)\frac{d}{dx}(-1)=x\sin x-\cos x$$

The right hand side is the derivative of 0 with respect to x (which is zero). Combining these results together gives the derivative of $-x\left(y^2+xe^y+\cos x\right)=0$ as:

$$-2xy\frac{dy}{dx}-2xy^2-x^2e^y\frac{dy}{dx}-2xe^y+x\sin x-\cos x=0$$

You can rearrange for dy/dx by subtracting $\left(-2xy^2-2xe^y+x\sin x-\cos x\right)$ from each side to give:

$$\left(-2xy-x^2e^y\right)\frac{dy}{dx}=2xy^2+2xe^y-x\sin x+\cos x$$

Finally, dividing by $\left(-2xy-x^2e^y\right)$ gives:

$$\frac{dy}{dx}=\frac{2xy^2+2xe^y-x\sin x+\cos x}{-2xy-x^2e^y}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

c. $e^{xy}+e^{6y}-e^{2x}=0$

As you cannot use basic methods to rearrange this function to get $y=f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}\left(e^{xy}\right)+\frac{d}{dx}\left(e^{6y}\right)+\frac{d}{dx}\left(-e^{2x}\right)=\frac{d}{dx}(0)$$

The first term involves differentiating e^{xy} which is a composition of functions and so you use the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) and so:

$$\frac{d}{dx}\left(e^{xy}\right)=e^{xy}\frac{d}{dx}(xy)$$

$$=e^{xy}\left(x\frac{d}{dx}(y)+y\frac{d}{dx}(x)\right)$$

$$=e^{xy}\left(x\frac{d}{dx}(y)+y\right)$$

As y is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(e^{xy}\right)=xe^{xy}\frac{dy}{dx}+ye^{xy}$$

The second term involves differentiating e^{6y} which is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form: ae^{ky} the derivative is given by $ake^{ky}\frac{dy}{dx}$. So,

$$\frac{d}{dx}\left(e^{6y}\right)=6e^{6y}\frac{dy}{dx}$$

The third term is a basic function of *x* (to remind yourself how to differentiate basic functions of *x* you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}\left(-e^{2x}\right) = -2e^{2x}$$

The right hand side is the derivative of 0 with respect to *x* (which is zero). Combining these results together gives the derivative of $e^{xy} + e^{6y} - e^{2x} = 0$ as:

$$xe^{xy}\frac{dy}{dx} + ye^{xy} + 6e^{6y}\frac{dy}{dx} - 2e^{2x} = 0$$

You can rearrange for dy/dx by subtracting $ye^{xy} - 2e^{2x}$ from each side to give:

$$xe^{xy}\frac{dy}{dx} + 6e^{6y}\frac{dy}{dx} = 2e^{2x} - ye^{xy}$$

Finally, dividing by $\left(xe^{xy} + 6e^{6y}\right)$ gives:

$$\frac{dy}{dx} = \frac{\left(2e^{2x} - ye^{xy}\right)}{\left(xe^{xy} + 6e^{6y}\right)}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

d. $\ln(xy) + \sin(2x - y) = 0$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}(\ln(xy)) + \frac{d}{dx}(\sin(2x - y)) = \frac{d}{dx}(0)$$

The first term involves differentiating $\ln(xy)$ which is a composition of functions and so you use the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) and so:

$$\frac{d}{dx}(\ln(xy)) = \frac{1}{xy}\frac{d}{dx}(xy)$$

$$= \frac{1}{xy}\left(x\frac{d}{dx}(y) + y\frac{d}{dx}(x)\right)$$

$$= \frac{1}{xy}\left(x\frac{d}{dx}(y) + y\right)$$

As y is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}(\ln(xy)) = \frac{1}{xy}\left(x\frac{dy}{dx}+y\right) = \frac{1}{y}\frac{dy}{dx}+\frac{1}{x}$$

The second term involves differentiating $\sin(2x-y)$ which is a composition of functions and so you use the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) and so:

$$\frac{d}{dx}(\sin(2x-y)) = \cos(2x-y)\frac{d}{dx}(2x-y)$$

$$= \cos(2x-y)\left(\frac{d}{dx}(2x)+\frac{d}{dx}(-y)\right)$$

$$= \cos(2x-y)\left(2+\frac{d}{dx}(-y)\right)$$

As y is a composite function of x. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}(\sin(2x-y)) = \cos(2x-y)\left(2-\frac{dy}{dx}\right)$$

The right hand side is the derivative of 0 with respect to x (which is zero). Combining these results together gives the derivative of $\ln(xy)+\sin(2x-y)=0$ as:

$$\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}+\cos(2x-y)\left(2-\frac{dy}{dx}\right)=0$$

You can rearrange for dy/dx by subtracting $\left(\frac{1}{x}+2\cos(2x-y)\right)$ from each side to give:

$$\frac{1}{y}\frac{dy}{dx}-\cos(2x-y)\frac{dy}{dx} = -\frac{1}{x}-2\cos(2x-y)$$

Finally, dividing by $\left(\frac{1}{y}-\cos(2x-y)\right)$ gives:

$$\frac{dy}{dx} = \frac{-\frac{1}{y} - 2\cos(2x - y)}{\frac{1}{y} - \cos(2x - y)}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

e. $\cos(x + 2y) - \cos x - \cos^2 y = 0$

As you cannot use basic methods to rearrange this function to get $y = f(x)$, you must differentiate implicitly. Firstly, apply the differential operator to each term:

$$\frac{d}{dx}(\cos(x + 2y)) + \frac{d}{dx}(-\cos x) + \frac{d}{dx}(-\cos^2 y) = \frac{d}{dx}(0)$$

The first term involves differentiating $\cos(x + 2y)$ which is a composition of functions and so you use the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) and so:

$$\frac{d}{dx}(\cos(x + 2y)) = -\sin(x + 2y)\frac{d}{dx}(x + 2y)$$

$$= -\sin(x + 2y)\left(\frac{d}{dx}(x) + \frac{d}{dx}(2y)\right)$$

$$= -\sin(x + 2y)\left(1 + \frac{d}{dx}(2y)\right)$$

As $2y$ is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form ay^n the derivative is given by $any^{n-1}\frac{dy}{dx}$. So:

$$\frac{d}{dx}(\cos(x + 2y)) = -\sin(x + 2y)\left(1 - 2\frac{dy}{dx}\right)$$

The second term is a basic function of *x* (to remind yourself how to differentiate basic functions of *x* you can check the study guide: *Differentiating Basic Functions*) and so:

$$\frac{d}{dx}(-\cos x) = \sin x$$

The third term involves differentiating $-\cos^2 y$ and so you use the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain*

Rule) and so:

$$\frac{d}{dx}\left(-\cos^2 y\right) = -2\cos y\frac{d}{dx}(\cos y)$$

As cosy is a composite function of *x*. The rules on the first page of the study guide: *Implicit Differentiation* tell you that if the function is of the form $a\cos ky$ the derivative is given by $-ak\sin ky\frac{dy}{dx}$. So:

$$\frac{d}{dx}\left(-\cos^2 y\right) = -2\cos y\left(-\sin y\frac{dy}{dx}\right) = 2\cos y\sin y\frac{dy}{dx}$$

The right hand side is the derivative of 0 with respect to *x* (which is zero). Combining these results together gives the derivative of $\cos(x+2y)-\cos x-\cos^2 y=0$ as:

$$-\sin(x+2y)\left(1-2\frac{dy}{dx}\right)+\sin x+2\cos y\sin y\frac{dy}{dx}=0$$

You can rearrange for dy/dx by subtracting $(-\sin(x+2y)+\sin x)$ from each side to give:

$$-2\sin(x+2y)\frac{dy}{dx}+2\cos y\sin y\frac{dy}{dx}=-\sin x+\sin(x+2y)$$

Finally, dividing by $(-2\sin(x+2y)+2\cos y\sin y)$ gives:

$$\frac{dy}{dx}=\frac{-\sin x+\sin(x+2y)}{-2\sin(x+2y)+2\cos y\sin y}$$

Note that the derivative is a function of both *y* and *x*, this is very common when you use implicit differentiation.

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