Learning Enhancement Team

Steps into Discrete Mathematics

The Dean of Students' Office
Learning Enhancement

Operations on Sets

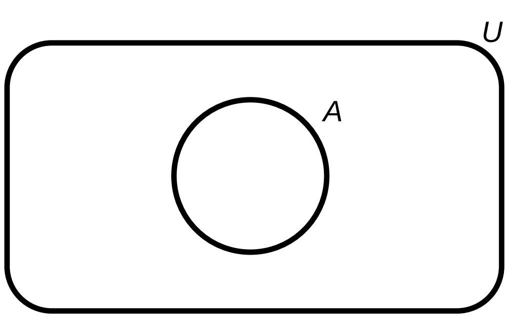
This guide introduces a variety of operations you can do on sets. Specifically it discusses complement, union, intersection, relative complement, symmetric difference and Cartesian product. It uses the Venn diagram as a medium to visualise these operations where appropriate.

Introduction

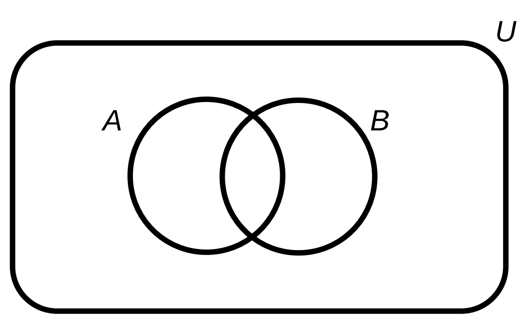
The study guide: *Basics of Sets* explains that there are few topics in mathematics that are more fundamental than **sets**. It introduces many of the common words and symbols you would expect to see when learning the mathematics of sets. If you are unfamiliar with sets you should read that guide before continuing with this one.

Venn diagrams

This guide employs **Venn diagrams** to help visualise the ideas of operations on sets. These diagrams were introduced by John Venn in the late 19th century as a way of visualising logic. However they serve equally well for looking at sets. A Venn diagram consists of a rectangle (called the **universe**, labelled U) around a number of appropriately labelled interlocking shapes which represent specific sets. For representations of 1, 2 and 3 sets these shapes are circles but for higher numbers of sets, the shapes become more elaborate. Shaded areas highlight areas of importance.



Venn diagram for a single set A



Venn diagram for sets A and B

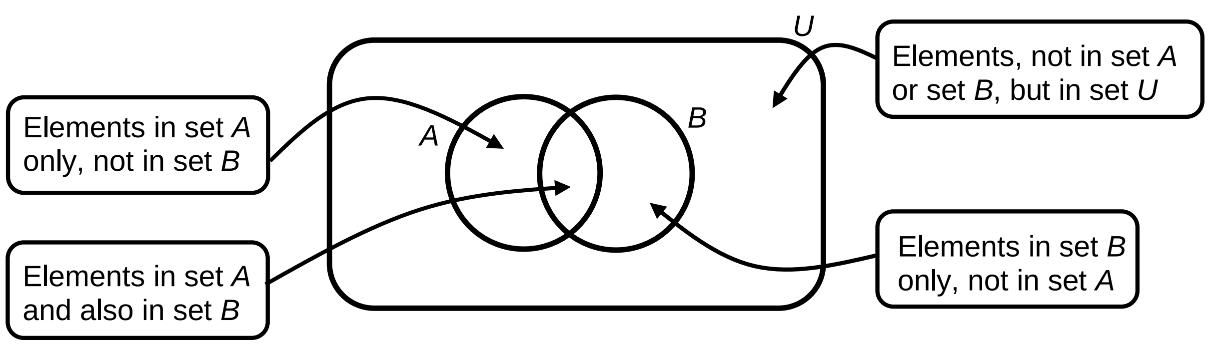
You can look on-line for examples which illustrate more than two sets, but this guide only makes use of the two diagrams on the first page.

Example: Construct a Venn diagram to illustrate a universe of natural numbers less than 10, set *A* comprises odd numbers less than 10 and set *B* comprises prime numbers less than 10.

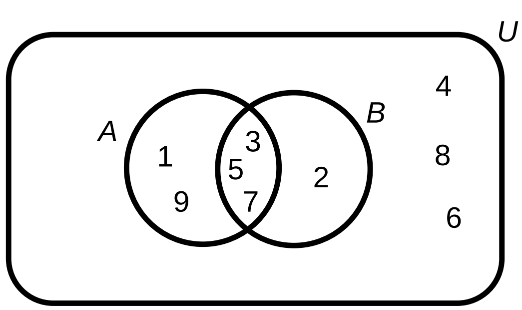
Firstly let's write out the sets:

$$U = \{1,2,3,4,5,6,7,8,9\}$$
$$A = \{1,3,5,7,9\}$$
$$B = \{2,3,5,7\}$$

The areas in the 2-set Venn diagram are populated by specific elements of the sets *U*, *A* and *B*.



When you construct a Venn diagram you should start in the innermost part first (here the elements in both *A* and *B*) and work your way out (elements in *A* only, elements in *B* only) until you are left with elements that are not in *A* or *B* but are in *U*. This gives:



Operations on sets

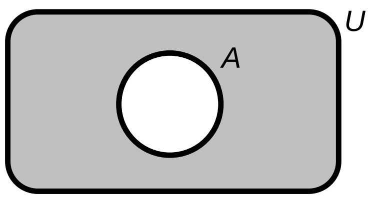
The following discussions are about the different things you can do using sets, these are called **operations** and will be illustrated using the sets *U*, *A* and *B* (introduced in the previous section). There are many laws which govern these operations and they are listed in the factsheet: *Sets*.

1. Complement

The **complement** of a set is the way that mathematicians describe the set of elements that are **not** in that particular set.

> The **complement** of a set *A* is written A^c
> and is the set of elements not in *A* but in *U.*

The complement of set *A* is shaded grey in the Venn diagram below:



In terms of sets, as $U=\{1,2,3,4,5,6,7,8,9\}$ and $A=\{1,3,5,7,9\}$, $A^c=\{2,4,6,8\}$.

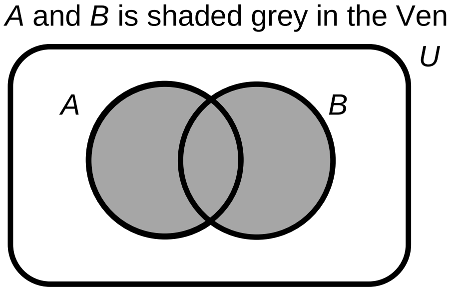
The complement of the universe is the empty set and vice-versa: $U^c=\varnothing$ and $\varnothing^c=U$.

2. Union

You can describe the set of *all* the elements that are in set *A* or set *B* as the **union** of those two sets. Any elements that are in both sets only appear once in the union set.

> The **union** of sets *A* and *B* is written $A \cup B$ and is
> the set of all the elements that appear in sets *A* or *B.*

A way to remember the symbol for union $\cup$ is that it looks like the first letter of the word "union". The union of *A* and *B* is shaded grey in the Venn diagram below.



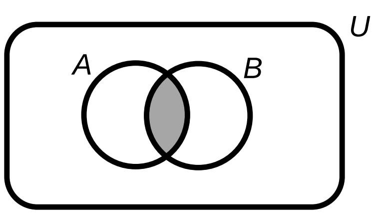
In terms of the sets $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,3,5,7,9\}$ and $B = \{2,3,5,7\}$, $A \cup B$ are all the elements that appear in *A* or *B* so $A \cup B = \{1,2,3,5,7,9\}$. Note that 3, 5 and 7 only appear once even though they are elements of both sets.

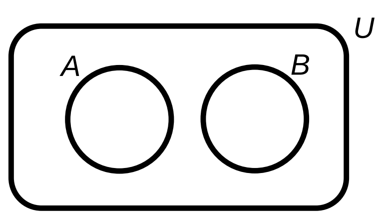
3. Intersection

You can describe the set of the elements that are in both set *A* and set *B* as the **intersection** of those two sets. If there are no elements in the intersection of two sets, the sets are said to be **disjoint**.

> The **intersection** of sets *A* and *B* is written $A \cap B$ and is the set of all the elements that appear in both set *A* and set *B*.

The intersection of *A* and *B* is shaded grey in the Venn diagram on the left below and disjoint sets are illustrated in the Venn diagram on the right.

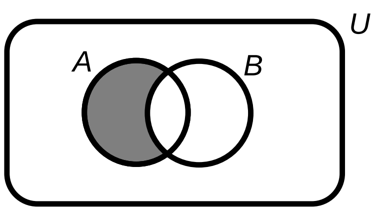




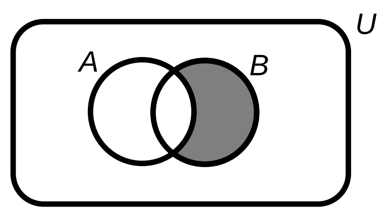
In terms of the sets $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,3,5,7,9\}$ and $B = \{2,3,5,7\}$, $A \cap B$ are all the elements that appear in both *A* and *B* so $A \cap B = \{3,5,7\}$. If *A* and *B* are disjoint then $A \cap B = \varnothing$.

4. Relative complement or difference

The elements of a set that are in that set only and no other (apart from the universe) are defined by the **relative complement** operation, sometimes called the **difference**. So the relative complement of set *B* with respect to set *A* are all the elements in *A* that do not appear in *B*. This is often called the difference of *A* and *B* and is written in many ways: $A \setminus B$, $A - B$ or $A \sim B$ are all common. You should note that $A \setminus B$ and $B \setminus A$ are not equal to each other, as shown in the Venn diagrams below.



$A \setminus B$ is shaded

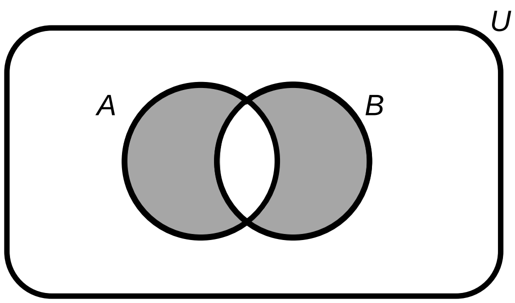


$B \setminus A$ is shaded

In terms of the sets $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,3,5,7,9\}$ and $B=\{2,3,5,7\}$, $A \setminus B=\{1,9\}$ and $B \setminus A=\{2\}$.

5. Symmetric difference

The **symmetric difference** of two sets is the set of elements that appear in either set but not in both. The symmetric difference of sets *A* and *B* is written $A \oplus B$ and is shaded grey in the Venn diagram below:



In terms of the sets $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,3,5,7,9\}$ and $B=\{2,3,5,7\}$, $A \oplus B=\{1,2,9\}$.

You can think of the symmetric difference as the relative complement of intersection of *A* and *B* to the union of *A* or *B*. Symbolically:

$$A \oplus B=(A \cup B) \setminus (A \cap B)$$

6. Cartesian product

The final set operation in this guide is the **Cartesian product**. This differs from the others because it cannot be drawn as a Venn diagram. To understand the Cartesian product, you first need to understand the idea of an **ordered pair**. An ordered pair is made up of two elements either chosen from different sets or the same set. Let's consider taking an element from set *A* and an element from set *B* to make an ordered pair. You can make the rule that the element from set *A* is chosen first and the element from set *B* is chosen second. By imposing an order you are making an ordered pair. For example you can write your ordered pair brackets, separated by a comma:

$(1,2)$ ordered pair: 1 is from *A* and 2 is from set *B*

The **Cartesian product** of sets *A* and *B* **is the set of all possible ordered pairs** that can be made by choosing an element from *A* first and then an element from *B* second. The set is denoted by $A \times B$.

For the sets $A = \{1,3,5,7,9\}$ and $B = \{2,3,5,7\}$, the Cartesian product is:

$$A \times B = \{(1,2),(1,3),(1,5),(1,7),(3,2),(3,3),(3,5),(3,7),$$
$$(5,2),(5,3),(5,5),(5,7),(7,2),(7,3),(7,5),(7,7),$$
$$(9,2),(9,3),(9,5),(9,7)\}$$

Cartesian products play an important role in the mathematics of **relations**, see study guide: *Basics of Relations*. A mathematical relation from one set to another is always a subset of the Cartesian product of those sets.

Further guidance and information

If you have any further questions about this topic, or would like to discuss any other aspects of mathematics, you can talk to your lecturer or Personal Adviser, or make an appointment to see a **Learning Enhancement Tutor** in the **Dean of Students' Office.**

Telephone: 01603 592761
Email: dos.help@uea.ac.uk
Website: http://www.uea.ac.uk/dos/let

There are further resources on many other aspects of mathematics, statistics and science available from the Dean of Students' Office and on its website. These include questions to practise, model solutions and webcasts illustrating essential skills.

This guidance leaflet is one of a series on mathematics produced by the Dean of Students' Office at the University of East Anglia.

Scan the QR-code with a smartphone for a webcast of this study guide.

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