Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model answers: Operations on Sets

Operations on Sets study guide

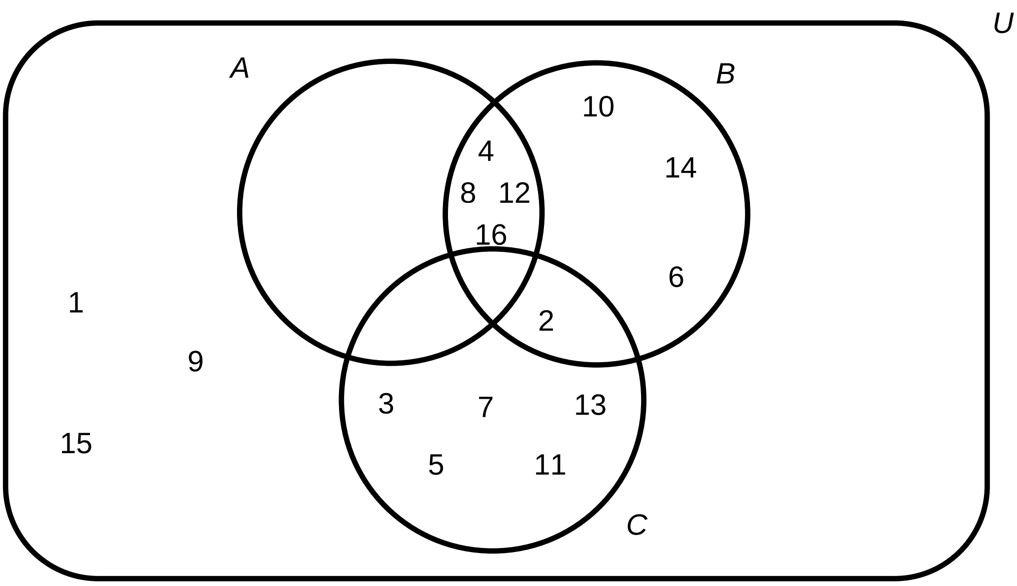
1. a) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16\}$.

$A = \{4, 8, 12, 16\}$

$B = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$C = \{2, 3, 5, 7, 11, 13\}$

The Venn diagram illustrating *U*, *A*, *B*, and *C* is illustrated below.



b) Remember the definitions of each of the operations from the study guide before starting.

i) $A^c = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15\}$

Remember that these are the elements of *U* that are not in *A*. You can see that this is all of *U* save for 4, 8, 12 and 16.

ii) $B \cup C = \{2,3,4,5,6,7,8,10,11,12,13,14,16\}$

Remember that these are the elements of *U* that appear in sets *B* or *C*. Note that although 2 appears in both *B* and *C*, it only appears once in the union.

iii) $A \cap C = \varnothing$

Remember that these are the elements of *U* that appear in both sets *A* and *C*. You can see that no element appears in both *A* and *C* and so this intersection is empty.

iv) $B \setminus C = \{4,6,8,10,12,14,16\}$

Remember that these are the elements of *U* that appear in set *B* but not in set *C*.

v) $C \setminus B = \{3,5,7,11,13\}$

Remember that these are the elements of *U* that appear in the set *C* but not the set *B*. You can see that $B \setminus C$ and $C \setminus B$ are not the same set.

vi) $C \oplus B = \{3,4,5,6,7,8,10,11,12,13,14,16\}$

Remember that these are the elements of *U* that appear in the set C, or the set *B*, but not in both sets. You can notice that this set is also the union of the sets $B \setminus C$ and $C \setminus B$.

vii) $B \cap \left(C^c\right) = \{4,6,8,10,12,14,16\}$

You can see here that $C^c = \{1,4,6,8,9,10,12,14,15,16\}$, and then $B \cap \left(C^c\right)$ is those elements that appear in both *B* and C^c. Also notice that this set is equal to $B \setminus C = \{4,6,8,10,12,14,16\}$.

viii) $A \cup (B \cup C) = \{2,3,4,5,6,7,8,10,11,12,13,14,16\}$

You can use your answer $B \cup C = \{2,3,4,5,6,7,8,10,11,12,13,14,16\}$ from part ii), and then $A \cup (B \cup C)$ is those elements that appear in the sets *A* or $B \cup C$. However, as all elements of *A* appear in $B \cup C$, the union of *A* with this set is just the same as $B \cup C$.

ix) $A \cup (B \cap C) = \{2,4,8,12,16\}$

You can see here that $B \cap C = \{2\}$, and then $A \cup (B \cap C)$ is those elements that appear in the sets *A* or $B \cap C$.

x) $\left(B^c\right) \cap C = \{3,5,7,11,13\}$

You can see here that $B^c = \{1,3,5,7,9,11,13,15\}$, and then $\left(B^c\right) \cap C$ is those elements that appear in both C and B^c. You can also notice that this set is equal to $C \setminus B = \{3,5,7,11,13\}$.

xi) $(A \oplus B) \cup C = \{2,3,5,6,7,10,11,13,14\}$

You can see here that $(A \oplus B) = \{2,6,10,14\}$, and then $(A \oplus B) \cup C$ is those elements that appear in the sets C or $A \oplus B$.

xii) $(A \cup B) \setminus (B \cup C) = \varnothing$

Here $A \cup B = \{2,4,6,8,10,12,14,16\}$ and $B \cup C = \{2,3,4,5,6,7,8,10,11,12,13,14,16\}$
Note that every element of $A \cup B$ also appear in $B \cup C$. So there are no elements in $A \cup B$ that are not in $B \cup C$, and so you can write that $(A \cup B) \setminus (B \cup C) = \varnothing$.

2. a)

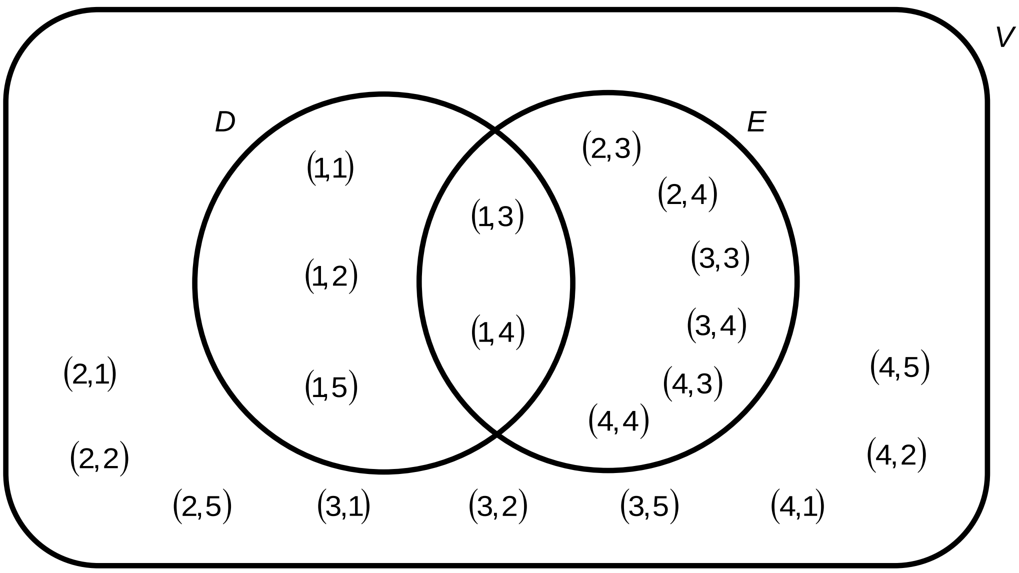
$$V = X \times Y = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(2,5),$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5)\}$$

Remember here that the order of the product does matter!

b) $D = \{(1,1),(1,2),(1,3),(1,4),(1,5)\}$

$E = \{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4),(4,3),(4,4)\}$

The Venn diagram is given below.



c)

i) $D^c = \{(2,1),(2,2),(2,3),(2,4),(2,5),(3,1),(3,2),(3,3),(3,4),(3,5),(4,1),(4,2),(4,3),(4,4),(4,5)\}$

Remember that these are the elements of *V* that are not in *D*. You can see that this is all elements that appear in *V* except for those in *D*.

ii) $E^c = \{(1,1),(1,2), (1,5),(2,1),(2,2),(2,5),(3,1),(3,2),(3,5),(4,1),(4,2),(4,5)\}$

Remember that these are the elements of *V* that are not in *E*. You can see that this is all elements that appear in *V* except for those in *E*.

iii) $D \cup E = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(3,3),(3,4),(4,3),(4,4)\}$

Remember that these are the elements of *V* that appear in sets *D* or *E*. Note that although $(1,3)$ and $(1,4)$ appear in both *D* and *E*, they only appear once in the union.

iv) $D \cap E = \{(1,3),(1,4)\}$

Remember that these are the elements of *V* that appear in both sets *D* and *E*

v) $D \setminus E = \{(1,1),(1,2),(1,5)\}$

Remember that these are the elements of *V* that appears in the set *D* but not the set *E*.

vi) $E \setminus D = \{(2,3),(2,4),(3,3),(3,4),(4,3),(4,4)\}$

Remember that these are the elements of *V* that appears in the set *D* but not the set *E*. Like in question 1, you can see that $D \setminus E$ and $E \setminus D$ are not the same set.

vii) $D \oplus E = \{(1,1),(1,2),(1,5),(2,3),(2,4),(3,3),(3,4),(4,3),(4,4)\}$

Remember that these are the elements of *V* that appears in the set *D*, or the set *E*, but not in both sets. You can notice that this set is also the union of the sets $D \setminus E$ and $E \setminus D$. You can also see that $D \oplus E = (D \cup E) \setminus (D \cap E)$, confirming a result from the study guide.

viii)

$$D \times D = \{((1,1),(1,1)),\ ((1,1),(1,2)),\ ((1,1),(1,3)),\ ((1,1),(1,4)),\ ((1,1),(1,5)),$$
$$((1,2),(1,1)),\ ((1,2),(1,2)),\ ((1,2),(1,3)),\ ((1,2),(1,4)),\ ((1,2),(1,5)),$$
$$((1,3),(1,1)),\ ((1,3),(1,2)),\ ((1,3),(1,3)),\ ((1,3),(1,4)),\ ((1,3),(1,5)),$$
$$((1,4),(1,1)),\ ((1,4),(1,2)),\ ((1,4),(1,3)),\ ((1,4),(1,4)),\ ((1,4),(1,5)),$$
$$((1,5),(1,1)),\ ((1,5),(1,2)),\ ((1,5),(1,3)),\ ((1,5),(1,4)),\ ((1,5),(1,5))\}$$

Every element of $D \times D$ here is an ordered pair consisting of ordered pairs.

In particular, the elements of $D \times D$ are *not* ordered quadruples, e.g. $(1,3,1,5)$.

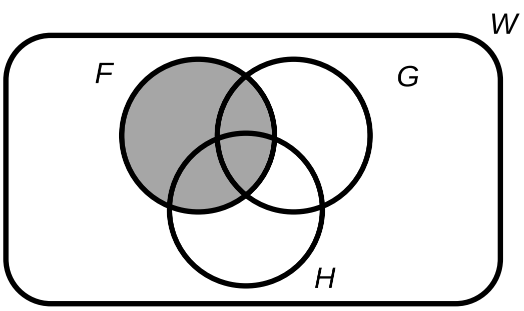
It is important to remember that order matters in a Cartesian product; so $((1,3),(1,2))$ is not the same element as $((1,2),(1,3))$.

3. a) You can write $W = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19\}$ for clarity.

i) $F \cup (G \cap H) = \{1,3,4,6,8,9,10,11,16\}$

You can see here that $G \cap H = \{3,9,10,11\}$ and then $F \cup (G \cap H)$ are those elements that appear in the sets F or $G \cap H$.

The Venn diagram is given below.



ii) $(F \cup G) \cap (F \cup H) = \{1,3,4,6,8,9,10,11,16\}$

You can see here that:

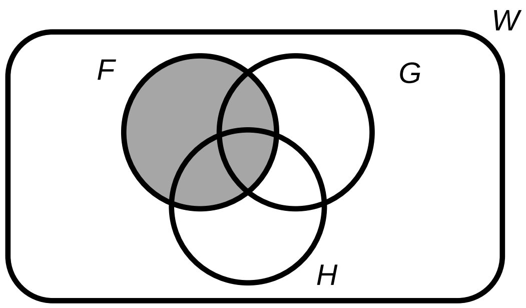
$F \cup G = \{1,2,3,4,6,7,8,9,10,11,14,16\}$

and:

$F \cup H = \{1,3,4,5,6,7,8,9,10,11,12,16,19\}$

So $(F \cup G) \cap (F \cup H) = \{1, 3, 4, 6, 8, 9, 10, 11, 16\}$ are those elements that appear in both $(F \cup G)$ and $(F \cup H)$.

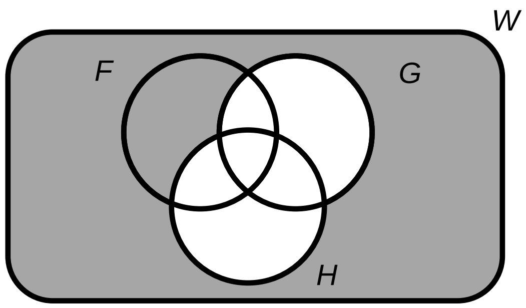
The Venn diagram is given below.



iii) $(G \cup H)^c = \{6, 13, 15, 17, 18, 19\}$

You can see that $G \cup H = \{1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19\}$, and so $(G \cup H)^c$ are those elements of W that do not appear in $G \cup H$.

The Venn diagram is given below.



iv) $G^c \cap H^c = \{6, 13, 15, 17, 18, 19\}$

You can see here that:

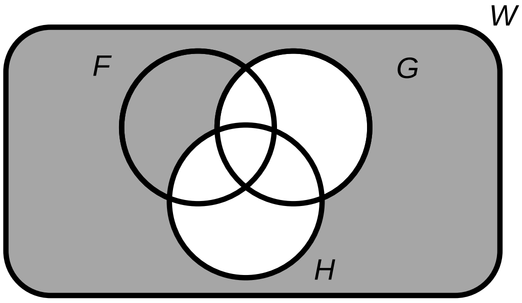
$G^c = \{1, 5, 6, 7, 12, 13, 15, 17, 18, 19\}$

and:

$H^c = \{2, 4, 6, 8, 13, 14, 15, 16, 17, 18\}$

So $G^c \cap H^c = \{1, 3, 4, 6, 8, 9, 10, 11, 16\}$ are those elements that appear in both G^c and H^c.

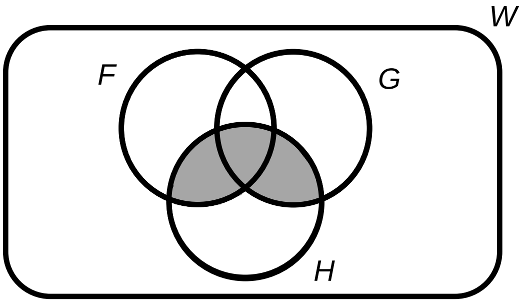
The Venn diagram is given below.



v) $H \cap (G \cup F) = \{1,3,9,10,11\}$

You can see here that $G \cup F = \{1,2,3,4,6,7,8,9,10,11,14,16\}$ and then $H \cap (G \cup F)$ are those elements that appear in both H and $G \cup F$.

The Venn diagram is given below.



vi) $(H \cap G) \cup (F \cap H) = \{1,3,9,10,11\}$

You can see here that:

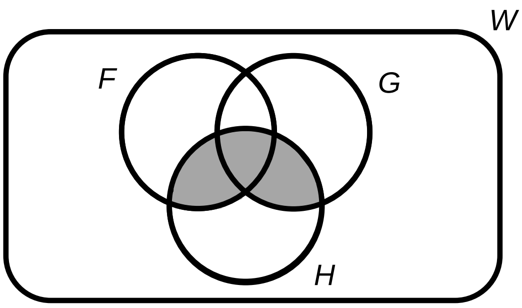
$H \cap G = \{3,9,10,11\}$

and:

$F \cap H = \{1,10\}$

So $(H \cap G) \cup (F \cap H) = \{1,3,9,10,11\}$ are those elements that appear in the sets $(H \cap G)$ or $(F \cap H)$.

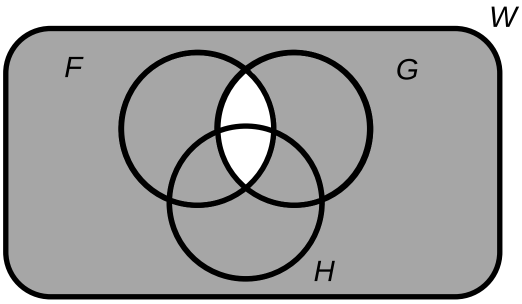
The Venn diagram is given below.



vii) $(G \cap F)^c = \{1,2,3,5,6,7,11,12,13,14,15,17,18,19\}$

You can see that $G \cap F = \{4,8,9,10,16\}$, and so $(G \cap F)^c$ are those elements of W that do not appear in $G \cap F$.

The Venn diagram is given below.



viii) $F^c \cup G^c = \{1,2,3,5,6,7,11,12,13,14,15,17,18,19\}$

You can see here that:

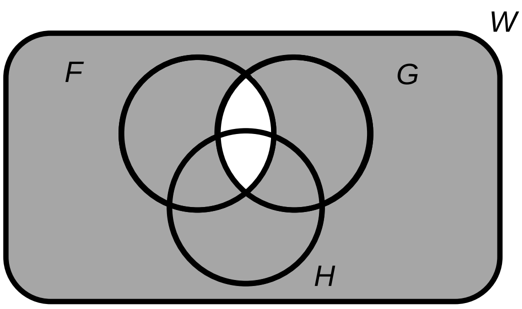
$F^c = \{2,3,5,7,11,12,13,14,15,17,18,19\}$

and:

$G^c = \{1,5,6,7,12,13,15,17,18,19\}$

So $F^c \cup G^c = \{1,2,3,5,6,7,11,12,13,14,15,17,18,19\}$ are those elements that appear in the sets F^c or G^c.

The Venn diagram is given below.



b) Notice that:

$F \cup (G \cap H) = (F \cup G) \cap (F \cup H) = \{1,3,4,6,8,9,10,11,16\}$

$(G \cup H)^c = G^c \cap H^c = \{6,13,15,17,18,19\}$

$H \cap (G \cup F) = (H \cap G) \cup (F \cap H) = \{1,3,9,10,11\}$

$(G \cap H)^c = G^c \cup H^c = \{1,2,3,5,6,7,11,12,13,14,15,17,18,19\}$

So i) = ii), iii) = iv), v) = vi) and vii) = viii).

You can also see this in the Venn diagrams, but it is not a concrete argument to see that these sets are equal.

c) Remember from 1a) that

$U = \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$.
$A = \{4,8,12,16\}$
$B = \{2,4,6,8,10,12,14,16\}$
$C = \{2,3,5,7,11,13\}$

i) $A \cup (B \cap C) = \{2,4,8,12,16\}$

You can see here that $B \cap C = \{2\}$, and then $A \cup (B \cap C)$ is those elements that appear in the sets A or $B \cap C$.

(You may have also noticed that this is exactly the same question as 1ix))

ii) $(A \cup B) \cap (A \cup C) = \{2,4,8,12,16\}$

You can use:

$A \cup B = \{2,4,6,8,10,12,14,16\}$

from the working of 1xii), and you can see that:

$A \cup C = \{2,3,4,5,7,8,11,12,13,16\}$

So $(A \cup B) \cap (A \cup C) = \{2,4,8,12,16\}$ are those elements that appear in both $(A \cup B)$ and $(A \cup C)$.

iii) $(B \cup C)^c = \{1,9,15\}$

You can use your answer $B \cup C = \{2,3,4,5,6,7,8,10,11,12,13,14,16\}$ from Q1ii), and so $(B \cup C)^c$ are those elements of U that do not appear in $B \cup C$.

iv) $B^c \cap C^c = \{1,9,15\}$

You can use:

$B^c = \{1,3,5,7,9,11,13,15\}$

from the working of 1x), and you can also use:

$C^c = \{1,4,6,8,9,10,12,14,15,16\}$

from the working of 1vii) to see that $B^c \cap C^c = \{1,9,15\}$ are those elements that appear in both B^c and C^c.

v) $C \cap (B \cup A) = \{2\}$

You can use $B \cup A = A \cup B = \{2,4,6,8,10,12,14,16\}$ and then $C \cap (B \cup A) = \{2\}$ are those elements that appear in both C and $B \cup A$.

vi) $(C \cap B) \cup (A \cap C) = \{2\}$

You can use:

$C \cap B = B \cap C = \{2\}$

from the working of 3i) and you can also use:

$A \cap C = \varnothing$

your answer from 1iii) to see that $(C \cap B) \cup (A \cap C) = \{2\}$ are those elements that appear in the sets $B \cap C$ or $(A \cap C)$.

vii) $(B \cap A)^c = \{1,2,3,5,6,7,9,10,11,13,14,15\}$

You can see that $B \cap A = \{4,8,12,16\}$, and so $(B \cap A)^c$ are those elements of U that do not appear in $B \cap A$.

viii) $B^c \cup A^c = \{1,2,3,5,6,7,9,10,11,13,14,15\}$

You can use:

$B^c = \{1,3,5,7,9,11,13,15\}$

from the working of 1x) and you can also use:

$A^c = \{1,2,3,5,6,7,9,10,11,13,14,15\}$

the answer from 1i) to show that $B^c \cup A^c = \{1,2,3,5,6,7,9,10,11,13,14,15\}$ are those elements that appear in the sets B^c or A^c.

Notice that:

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C) = \{2,4,8,12,16\}$

$(B \cup C)^c = B^c \cap C^c = \{1,9,15\}$

$C \cap (B \cup A) = (C \cap B) \cup (A \cap C) = \{2\}$

$(B \cap A)^c = B^c \cup A^c = \{1,2,3,5,6,7,9,10,11,13,14,15\}$

So these do agree with the observations made in 3b). In fact, these results are true for any sets. See factsheet: *Sets* for more identities like these.

These model answers are one of a series on mathematics produced by the Learning Enhancement Team with funding from the UEA Alumni Fund. Scan the QR-code with a smartphone app for more resources.

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