Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model answers: Basics of Sets

Basics of Sets study guide

1. i) $A = \{\text{iron}, \text{shield}, \text{hammer}, \text{green}, \text{bow}, \text{spider}\}$.

   Remember that you should be using curly brackets (braces) to enclose sets rather than parentheses (normal brackets). The distinction is very important!

   ii) $B = \{2, 3, 5, 7, 11, 13, 17, 19, 23, 29\}$

   iii) $C = \{2, 4, 6, \ldots\}$

   or $C = \{c : c \text{ is a positive, even, whole number}\}$

   or $C = \{c \in \mathbb{Z} : c > 0, c \text{ divides by } 2\}$

   All three of these answers for C above are acceptable; but remember that 0 is neither positive nor negative!

   iv) $D = \{-2, -1, 0, 1, 2, 3, 4\}$

   You can see that 5 is not included in D as the question wanted whole numbers strictly less than 5.

2.

a) 2 is an element of $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$.

Using proper mathematical notation, $2 \in \mathbb{N}$, $2 \in \mathbb{Z}$, and $2 \in \mathbb{Q}$.

You can see that $2 \in \mathbb{Q}$ as, for example, $2/1 = 2$ is an element of $\mathbb{Q}$.

i) $-8 \notin \mathbb{N}$, $-8 \in \mathbb{Z}$, and $-8 \in \mathbb{Q}$.

You can see that $\mathbb{N}$ only contains positive whole numbers and so $-8 \notin \mathbb{N}$. However, -8 is still a whole number and so $-8 \in \mathbb{Z}$. For $\mathbb{Q}$ you can use the same argument you used to show that $2 \in \mathbb{Q}$ to show that $-8 \in \mathbb{Q}$. So $-8/1 = -8 \in \mathbb{Q}$.

ii) $\frac{2}{3} \notin \mathbb{N}$, $\frac{2}{3} \notin \mathbb{Z}$, $\frac{2}{3} \in \mathbb{Q}$.

You can see that $2/3$ is not a whole number and so $2/3 \notin \mathbb{N}$, $2/3 \notin \mathbb{Z}$. However, $2/3$ is in the form a/b where *a* and *b* are whole numbers, so $2/3 \in \mathbb{Q}$.

iii) $0 \notin \mathbb{N}$, $0 \in \mathbb{Z}$, and $0 \in \mathbb{Q}$.

You can remember that $\mathbb{N}$ only contains positive whole numbers and that 0 is neither positive nor negative; so $0 \notin \mathbb{N}$. However, 0 is still a whole number and so $0 \in \mathbb{Z}$. You can use the same argument for 0 as you used for 2 to see that $0/1 = 0 \in \mathbb{Q}$.

iv) $\pi \notin \mathbb{N}$, $\pi \notin \mathbb{Z}$, $\pi \notin \mathbb{Q}$.

As π is not a whole number $\pi \notin \mathbb{N}$ and $\pi \notin \mathbb{Z}$. In fact π is an **irrational number**, it cannot be written in the form a/b where *a* and *b* are whole numbers and so $\pi \notin \mathbb{Q}$.

b) i) $E \subseteq \mathbb{N}$, $E \subseteq \mathbb{Z}$, and $E \subseteq \mathbb{Q}$.

You can see that $E = \{1,2,3,4,5,6,7\}$ contains only positive whole numbers, and so $E \subseteq \mathbb{N}$. You can see that $E \subseteq \mathbb{Z}$, and $E \subseteq \mathbb{Q}$ as both $\mathbb{Z}$ and $\mathbb{Q}$ contain all the whole numbers as elements. This is illustrated in the study guide: *Basics of Sets*.

ii) $F \not\subseteq \mathbb{N}$, $F \not\subseteq \mathbb{Z}$, and $F \subseteq \mathbb{Q}$.

The answers here rely on the fact that $2/3 \notin \mathbb{N}$ and $2/3 \notin \mathbb{Z}$, but $2/3 \in F$, and so $F \not\subseteq \mathbb{N}$ and $F \not\subseteq \mathbb{Z}$. However, all elements of *F* are elements of $\mathbb{Q}$ and so $F \subseteq \mathbb{Q}$.

iii) $G \not\subseteq \mathbb{N}$, $G \subseteq \mathbb{Z}$, and $G \subseteq \mathbb{Q}$.

You can see from the question that *G* contains only whole numbers, so you can write that $G \subseteq \mathbb{Z}$, and $G \subseteq \mathbb{Q}$. Remember that multiples of 4 can be negative and so $-4 \in G$. As $\mathbb{N}$ only contains positive whole numbers, you can see that $G \not\subseteq \mathbb{N}$.

iv) $H \subseteq \mathbb{N}$, $H \subseteq \mathbb{Z}$, and $H \subseteq \mathbb{Q}$.

Remember from the study guide that every set contains the empty set $\varnothing$ as a subset. As $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{Q}$ are all sets, they all contain the empty set as a subset.

3. $|A| = 6$. $|B| = 10$. $|C| = \infty$. $|D| = 7$. $|E| = 7$. $|F| = 4$. $|G| = \infty$. $|H| = 0$.

You can work these out by simply counting the number of elements in the sets. As *C* and *G* both contain infinitely many elements, the cardinalities of these are infinite. As *H* is the empty set, it has no elements in it and so the cardinality is 0.

Both *C* and *G* satisfy Cantor's definition of a countably infinite set. To justify this, you can see that $C \subseteq \mathbb{N}$ and $G \subseteq \mathbb{Z}$. Both of these sets satisfy Cantor's definition of a countably infinite set. This is because any infinite subset *X* of a countably infinite set *Y* must also satisfy Cantor's definition of a countably infinite set. You can see that if $X \subseteq Y$ and *X* was not countably infinite, then *Y* must not be countably infinite either. So as $C \subseteq \mathbb{N}$ and $G \subseteq \mathbb{Z}$, they must both satisfy Cantor's definition of a countably infinite set.

The set of all real numbers $\mathbb{R}$ is an uncountably infinite set and so has larger cardinality than everything you have seen in this worksheet.

4.

a) $P(S) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, S\}$

There are 16 elements in *P* and so $|P(S)| = 16$.

b) i) $2 \notin P(S)$.

Remember that $P(S)$ contains all subsets of *S*. As 2 is an element of *S* and not a subset of *S*, you can see that $2 \notin P(S)$.

ii) $S \in P(S)$.

As every set contains itself as a subset, you can see that $S \in P(S)$.

iii) $\{5\} \notin P(S)$.

You can see here that *S* does not contain the element 5. This means that $\{5\}$ is not a subset of *S* and so you can write $\{5\} \notin P(S)$.

iv) $\varnothing \in P(S)$.

As every set contains the empty set as a subset, you can see that $\varnothing \in P(S)$.

c) i) $\{2,3\} \not\subseteq P(S)$.

You can see here that $\{2,3\}$ is a subset of S and so it is an element of $P(S)$. Since it is an element of $P(S)$ it is not a subset of $P(S)$ and so you can write $\{2,3\} \not\subseteq P(S)$.

ii) $\{\{1\},\{2\},\{3\}\} \subseteq P(S)$.

You can see that $\{1\}$, $\{2\}$ and $\{3\}$ are all elements of $P(S)$, and that $\{\{1\},\{2\},\{3\}\}$ is a set of all these elements. As this set only contains elements of $P(S)$, you can write that $\{\{1\},\{2\},\{3\}\} \subseteq P(S)$.

iii) $\{\{4\}\} \subseteq P(S)$.

You can see that $\{4\}$, is an element of $P(S)$, and that $\{\{4\}\}$ is a set containing only this element. As this set only contains elements of $P(S)$, you can write that $\{\{4\}\} \subseteq P(S)$.

iv) $\{1,\{2\}\} \not\subseteq P(S)$.

You know that 1 is not an element of $P(S)$ from a similar argument to 3.b)i), and so you can write that $\{1,\{2\}\} \not\subseteq P(S)$.

These model answers are one of a series on mathematics produced by the Learning Enhancement Team with funding from the UEA Alumni Fund. Scan the QR-code with a smartphone app for more resources.