Basic Operations with Vectors

This guide gives pictorial and algebraic explanations of how to add and subtract vectors and also how to multiply a vector by a scalar.

Introduction

Vectors are a very important idea in all areas of science. For example much of modern physics relies on vectors and the theory that describes them. **A vector can be thought of as describing a journey from one point to another** and in fact the word '*vector*' comes from the Latin for a passenger or carrier. Importantly a vector describes a measure which has both a magnitude (size) and a direction associated with it. In mathematics something which has a **magnitude** but not a direction is called a **scalar**. Vectors and scalars are related but different.

Vectors have magnitude and direction

Scalars have only magnitude

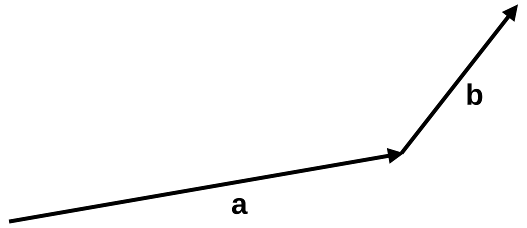
The study guide: *Basics of Vectors* explains the concept of vectors and scalars in more detail, along with the fundamental language used to describe them. If you are not familiar with vectors then you should read this guide before proceeding. You should pay particular attention to the description of a vector in terms of the **rectangular unit vectors**:

i - the unit vector (a single step) in the positive *x*-direction
j - the unit vector (a single step) in the positive *y*-direction
k - the unit vector (a single step) in the positive *z*-direction

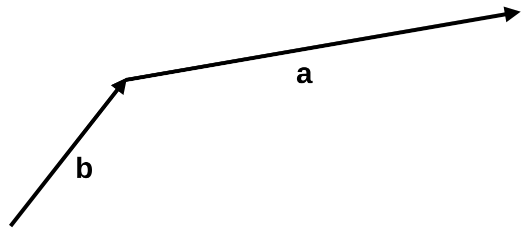
Describing a vector in this way leads to a simple method for their addition and subtraction.

Adding and subtracting vectors

When you add two vectors you can visualise the process as laying them end-to-end.

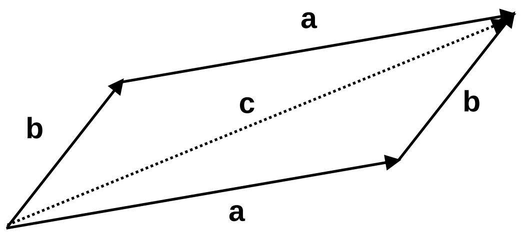


For example if you wish to add the vectors **a** and **b** you can think of it as vector **b** beginning at the end of vector **a**, $\mathbf{a}+\mathbf{b}$ is illustrated to the left.



You can also add **b** and **a**, where the vector **a** begins at the end of vector **b**, $\mathbf{b}+\mathbf{a}$ is illustrated to the left.

By combining both these pictures you can see that it does not matter which way around you add the vectors as the journeys they describe begin and end at the same place, they simply take different routes. The vector which described the overall journey represented by the addition of vectors is called the **resultant vector**. Here, if you call the resultant vector **c**, you can combine the two diagrams above to see that:



This diagram shows that, mathematically:

$$\mathbf{a}+\mathbf{b}=\mathbf{b}+\mathbf{a}=\mathbf{c}$$

The diagram also shows that if you add the number of steps in the *x*–direction in **a** and **b** you get the number of steps in the *x*-direction of the resultant **c**. This is also true for the *y*-direction (and also the *z*-direction). By expressing **a** and **b** as the general vectors:

$$\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k} \qquad \text{and} \qquad \mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$$

you can calculate the number of $\mathbf{i}$'s (which represent steps in the *x*-direction) in the resultant **c** by adding a_1 and b_1, the number of $\mathbf{j}$'s (which represent steps in the *y*-direction) by adding a_2 and b_2 and the number of $\mathbf{k}$'s (which represent steps in the *z*-direction) by adding a_3 and b_3. You can write this mathematically as:

$$\mathbf{a}+\mathbf{b}=\mathbf{c}=(a_1+b_1)\mathbf{i}+(a_2+b_2)\mathbf{j}+(a_3+b_3)\mathbf{k}$$

Example: Given that $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}+6\mathbf{k}$, what is $\mathbf{a}+\mathbf{b}$?

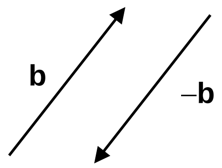
In this question $a_1=1$, $a_2=2$, $a_3=3$, $b_1=-2$, $b_2=0$ and $b_3=6$ and so:

$$\mathbf{a}+\mathbf{b}=(1+(-2))\mathbf{i}+(2+0)\mathbf{j}+(3+6)\mathbf{k}=-\mathbf{i}+2\mathbf{j}+9\mathbf{k}$$

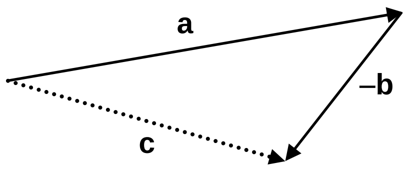
In other words, the resultant vector from adding **a** and **b** is $\mathbf{c}=-\mathbf{i}+2\mathbf{j}+9\mathbf{k}$. Similarly:

$$\mathbf{b}+\mathbf{a}=((-2)+1)\mathbf{i}+(0+2)\mathbf{j}+(6+3)\mathbf{k}=-\mathbf{i}+2\mathbf{j}+9\mathbf{k}$$

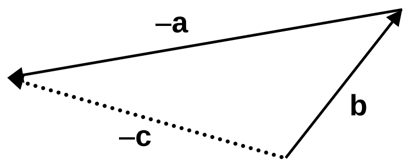
Subtraction of vectors can be thought of in a similar way.



Remember that a vector which has the same magnitude but opposite direction to another vector is the negative of that vector.



If you wish to subtract the vector **b** from vector **a** you can think of adding the vector –**b** to **a**. To the left is a representation of $\mathbf{a}+(-\mathbf{b})=\mathbf{a}-\mathbf{b}=\mathbf{c}$



You can also subtract **a** from **b** and show that the resultant is –**c**. To the left is a representation of $\mathbf{b}+(-\mathbf{a})=\mathbf{b}-\mathbf{a}=-\mathbf{c}$

Again by expressing **a** and **b** as the general vectors $\mathbf{a}=a_1\mathbf{i}+a_2\mathbf{j}+a_3\mathbf{k}$ and $\mathbf{b}=b_1\mathbf{i}+b_2\mathbf{j}+b_3\mathbf{k}$, you can write a general formula for subtracting vectors as:

$$\mathbf{a}-\mathbf{b}=(a_1-b_1)\mathbf{i}+(a_2-b_2)\mathbf{j}+(a_3-b_3)\mathbf{k}$$

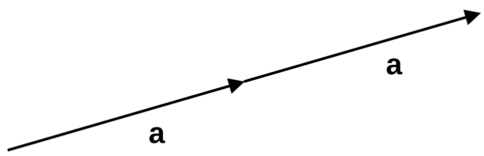
Example: Given that $\mathbf{a}=\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $\mathbf{b}=-2\mathbf{i}+6\mathbf{k}$ what is $\mathbf{a}-\mathbf{b}$?

Here $a_1=1$, $a_2=2$, $a_3=3$, $b_1=-2$, $b_2=0$ and $b_3=6$ and so:

$$\mathbf{a}-\mathbf{b}=(1-(-2))\mathbf{i}+(2-0)\mathbf{j}+(3-6)\mathbf{k}=3\mathbf{i}+2\mathbf{j}-3\mathbf{k}$$

Multiplying a vector and a scalar

You can visualise adding a vector to itself by laying multiples of that vector end-to-end. As you have seen in the previous section, this also represents addition of vectors.



For example $\mathbf{a}+\mathbf{a}$ is shown to the left. The resultant of this is $2\mathbf{a}$.

Similarly if you add another **a** you get $\mathbf{a}+\mathbf{a}+\mathbf{a}$ which has a resultant $3\mathbf{a}$ and so on. In this way you can see that successive additions of **a** represent multiplying the vector by a constant or **scalar**. You are not restricted to whole numbers and for example 0.5**a** would represent a journey to half way along **a**.

When you multiply a general vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ by any scalar m you affect the values of the coefficients of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ which are a_1, a_2 and a_3 respectively. It is important to note that $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are not affected themselves. You can write this mathematically as:

$$m\mathbf{a} = (a_1 m)\mathbf{i} + (a_2 m)\mathbf{j} + (a_3 m)\mathbf{k}$$

In other words you multiply the scalar parts of **a** by the scalar m.

Example: Given that $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 6\mathbf{k}$, what is (i) $2\mathbf{a}$ (ii) $-3\mathbf{b}$ (iii) $2\mathbf{a} - 3\mathbf{b}$?

(i) Using $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$:

$$2\mathbf{a} = (1\times 2)\mathbf{i} + (2\times 2)\mathbf{j} + (3\times 2)\mathbf{k} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$$

(ii) Using $b_1 = -2$, $b_2 = 0$ and $b_3 = 6$:

$$-3\mathbf{b} = (-2\times -3)\mathbf{i} + (0\times -3)\mathbf{j} + (6\times -3)\mathbf{k} = 6\mathbf{i} - 18\mathbf{k}$$

(iii) Using the result from (i) and (ii):

$$2\mathbf{a} - 3\mathbf{b} = 2\mathbf{a} + (-3\mathbf{b}) = (2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}) + (6\mathbf{i} - 18\mathbf{k}) = 8\mathbf{i} + 4\mathbf{j} - 12\mathbf{k}$$

Want to know more?

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