Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model Answers: Basic Operations with Vectors

Basic Operations with Vectors study guide

1. Trace around the parallelogram to help you find the vectors. You can see that when we have addition of two vectors and the second letter of the first vector and the first letter of the second vector are the same then the new vector is the first letter of the first vector and the second letter of the second vector. It does not matter that they go through a different route, the beginning and the end of the journey is important.

i. $\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$.

You go from A to B, then from B to C so the overall journey is A to C.

ii. $\overrightarrow{CD}+\overrightarrow{DA}=\overrightarrow{CA}$

You go from C to D, then from D to A so the overall journey is C to A.

iii. $\overrightarrow{AC}+\overrightarrow{CB}+\overrightarrow{BD}=\overrightarrow{AD}$

You go from A to C, then from C to B and then from so the overall journey is C to A.

iv. $\overrightarrow{AB}-\overrightarrow{CB}=\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$

You go from A to B, then from B to C (notice that the minus sign reverses the journey and so the overall journey is A to C.

2. In this question you have been given two vectors $\mathbf{a}=3\mathbf{i}-2\mathbf{j}$ and $\mathbf{b}=-\mathbf{i}-3\mathbf{j}$. The 2 Dimensional space vectors **a** and **b** are given in terms of the rectangular unit vectors **i** and **j**.

a.

i. $\mathbf{a}+\mathbf{b}=(3\mathbf{i}-2\mathbf{j})+(-\mathbf{i}-3\mathbf{j})=(3-1)\mathbf{i}+(-2-3)\mathbf{j}=2\mathbf{i}-5\mathbf{j}$

ii. $\mathbf{b}+\mathbf{a}=(-\mathbf{i}-3\mathbf{j})+(3\mathbf{i}-2\mathbf{j})=(-1+3)\mathbf{i}+(-3-2)\mathbf{j}=2\mathbf{i}-5\mathbf{j}$

iii. $\mathbf{a}-\mathbf{b}=(3\mathbf{i}-2\mathbf{j})-(-\mathbf{i}-3\mathbf{j})=(3-(-1))\mathbf{i}+(-2-(-3))\mathbf{j}=4\mathbf{i}+\mathbf{j}$

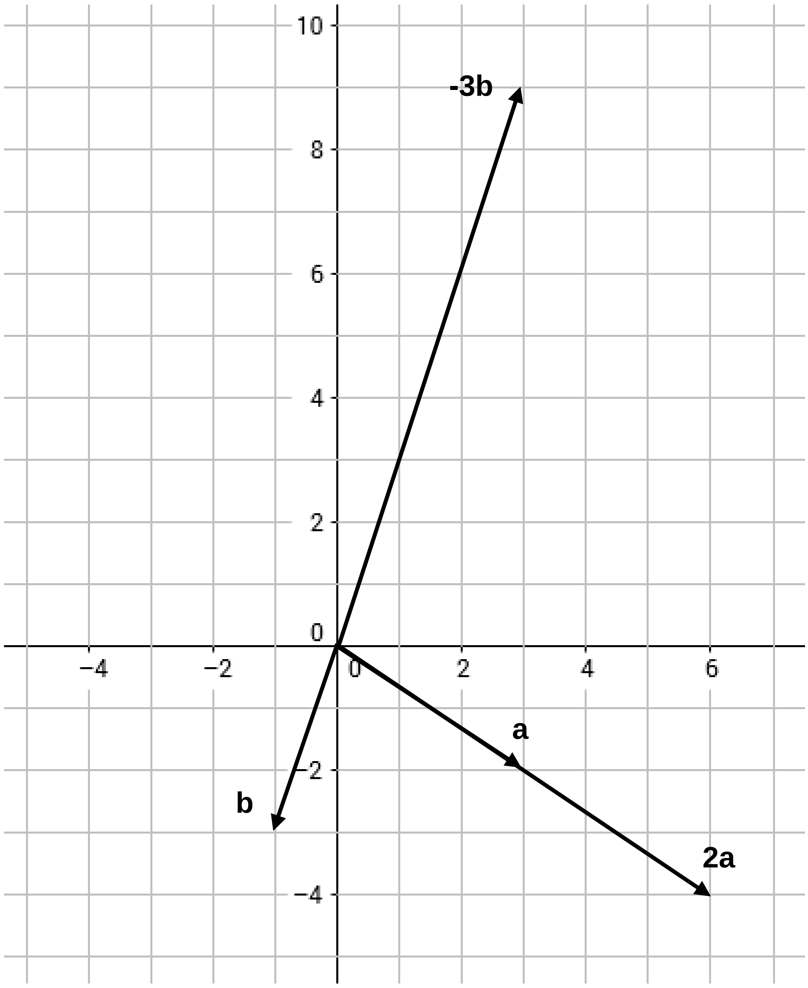
iv. $\mathbf{b}-\mathbf{a}=(-\mathbf{i}-3\mathbf{j})-(3\mathbf{i}-2\mathbf{j})=(-1-3)\mathbf{i}+(-3-(-2))\mathbf{j}=-4\mathbf{i}-1\mathbf{j}$

v. $2\mathbf{a}=(3\times 2)\mathbf{i}+(-2\times 2)\mathbf{j}=6\mathbf{i}-4\mathbf{j}$

vi. $2\mathbf{a}+\mathbf{b}=(6\mathbf{i}-4\mathbf{j})+(-\mathbf{i}-3\mathbf{j})=(6-1)\mathbf{i}+(-4-3)\mathbf{j}=5\mathbf{i}-7\mathbf{j}$

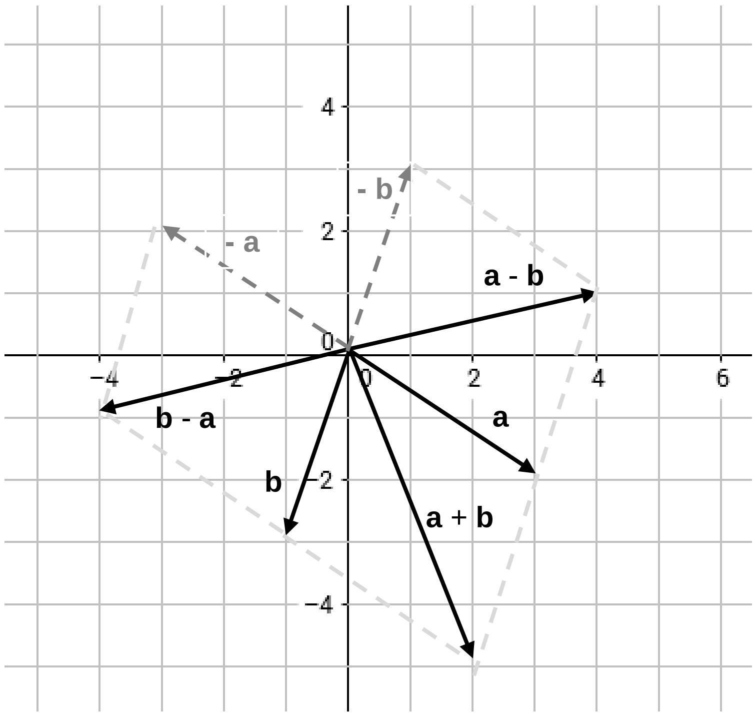
b.

i.-iv. Using graph paper you should get something similar to:



Looking at the image you drew you should see that **a** and 2**a** have the same direction but different magnitude and that **b** and $-3\mathbf{b}$ are on the same line with opposite directions and with different magnitude.

v.-viii. Using graph paper you should get something similar to:



Looking the image you drew you should see that $\mathbf{a}+\mathbf{b}$ is the diagonal of a parallelogram formed by the vectors **a** and **b**. And that $\mathbf{b}+\mathbf{a}$ is the same vector as $\mathbf{a}+\mathbf{b}$. Now, $\mathbf{a}-\mathbf{b}$ is the diagonal of the parallelogram formed by the **a** and $-\mathbf{b}$. Similarly, $\mathbf{b}-\mathbf{a}$ is the diagonal of the parallelogram formed by the vectors $-\mathbf{a}$ and **b**.

3.

i. $3\mathbf{a}=3\left(\tfrac{2}{3}\mathbf{i}-\tfrac{4}{6}\mathbf{j}-\tfrac{6}{9}\mathbf{k}\right)=\left(3\times\tfrac{2}{3}\right)\mathbf{i}+\left(3\times-\tfrac{4}{3}\right)\mathbf{j}+\left(3\times-\tfrac{6}{9}\right)\mathbf{k}=2\mathbf{i}-4\mathbf{j}-2\mathbf{k}$

ii. $-2\mathbf{b}=-2\left(-\tfrac{1}{2}\mathbf{i}-3\mathbf{j}+\tfrac{3}{4}\mathbf{k}\right)=\left(-2\times-\tfrac{1}{2}\right)\mathbf{i}+\left(-2\times-3\right)\mathbf{j}+\left(-2\times\tfrac{3}{4}\right)\mathbf{k}=\mathbf{i}+6\mathbf{j}-\tfrac{3}{2}\mathbf{k}$

iii. $4\mathbf{c}=4\left(\tfrac{9}{12}\mathbf{i}-\tfrac{3}{4}\mathbf{j}+\tfrac{1}{2}\mathbf{k}\right)=\left(4\times\tfrac{9}{12}\right)\mathbf{i}+\left(4\times-\tfrac{3}{4}\right)\mathbf{j}+\left(4\times\tfrac{1}{2}\right)\mathbf{k}=3\mathbf{i}-3\mathbf{j}+2\mathbf{k}$

iv. $3\mathbf{a}-2\mathbf{b}+4\mathbf{c}$ using i. ii. And iii. above you get:

$$\begin{aligned}3\mathbf{a}-2\mathbf{b}+4\mathbf{c}&=\left(2\mathbf{i}-4\mathbf{j}-2\mathbf{k}\right)+\left(\mathbf{i}+6\mathbf{j}-\tfrac{3}{2}\mathbf{k}\right)+\left(3\mathbf{i}-3\mathbf{j}+2\mathbf{k}\right)\\&=\left(2+1+3\right)\mathbf{i}+\left(-4+6-3\right)\mathbf{j}+\left(-2-\tfrac{3}{2}+2\right)\mathbf{k}=6\mathbf{i}-\mathbf{j}-\tfrac{3}{2}\mathbf{k}\end{aligned}$$

v. $12(\mathbf{a}+\mathbf{b}+\mathbf{c})$. Notice that the denominators of the coordinates of the vectors are divisors of 12. So you can multiply each of the vectors with 12 first. And instead of writing:

$$12(\mathbf{a}+\mathbf{b}+\mathbf{c}) = 12\left(\left(\tfrac{2}{3}\mathbf{i}-\tfrac{4}{6}\mathbf{j}-\tfrac{6}{9}\mathbf{k}\right)+\left(-\tfrac{1}{2}\mathbf{i}-3\mathbf{j}+\tfrac{3}{4}\mathbf{k}\right)+\left(\tfrac{9}{12}\mathbf{i}-\tfrac{3}{4}\mathbf{j}+\tfrac{1}{2}\mathbf{k}\right)\right)$$
$$= 12\left(\left(\tfrac{2}{3}-\tfrac{1}{2}+\tfrac{9}{12}\right)\mathbf{i}+\left(-\tfrac{4}{6}-3-\tfrac{3}{4}\right)\mathbf{j}+\left(-\tfrac{6}{9}+\tfrac{3}{4}+\tfrac{1}{2}\right)\mathbf{k}\right)$$

You can write:

$$12(\mathbf{a}+\mathbf{b}+\mathbf{c}) = 12\mathbf{a}+12\mathbf{b}+12\mathbf{c}$$
$$= 12\left(\tfrac{2}{3}\mathbf{i}-\tfrac{4}{6}\mathbf{j}-\tfrac{6}{9}\mathbf{k}\right)+12\left(-\tfrac{1}{2}\mathbf{i}-3\mathbf{j}+\tfrac{3}{4}\mathbf{k}\right)+12\left(\tfrac{9}{12}\mathbf{i}-\tfrac{3}{4}\mathbf{j}+\tfrac{1}{2}\mathbf{k}\right)$$
$$= (8\mathbf{i}-8\mathbf{j}-8\mathbf{k})+(-6\mathbf{i}-36\mathbf{j}-12\mathbf{k})+(9\mathbf{i}-9\mathbf{j}+6\mathbf{k})$$
$$= (8-6+9)\mathbf{i}+(-8-36-9)\mathbf{j}+(-8-12+6)\mathbf{k}$$
$$= 11\mathbf{i}-53\mathbf{j}-14\mathbf{k}$$

vi. $\mathbf{a}+\mathbf{b}+\mathbf{c}$ in order to add these vectors you can use your working from the above question. In v. you calculated $12(\mathbf{a}+\mathbf{b}+\mathbf{c})$ and found that:

$$12(\mathbf{a}+\mathbf{b}+\mathbf{c}) = 11\mathbf{i}-53\mathbf{j}-14\mathbf{k}$$

Multiplying the result by the scalar 1/12 you would get $\mathbf{a}+\mathbf{b}+\mathbf{c}$. So:

$$\mathbf{a}+\mathbf{b}+\mathbf{c} = \tfrac{1}{12}(12(\mathbf{a}+\mathbf{b}+\mathbf{c}))$$
$$= \tfrac{1}{12}(11\mathbf{i}-53\mathbf{j}-14\mathbf{k})$$
$$= \left(\tfrac{1}{12}\times 11\right)\mathbf{i}+\left(\tfrac{1}{12}\times -53\right)\mathbf{j}+\left(\tfrac{1}{12}\times -14\right)\mathbf{k}$$
$$= \tfrac{11}{12}\mathbf{i}-\tfrac{53}{12}\mathbf{j}-\tfrac{7}{6}\mathbf{k}$$

4. You have been given that $\mathbf{a} = 2\mathbf{i}+3\mathbf{j}-\mathbf{k}$, $\mathbf{b} = -\mathbf{i}+2\mathbf{k}$ and $\mathbf{c} = d\mathbf{i}+3\mathbf{j}+f\mathbf{k}$.

a. You want to find d and f so that $\mathbf{c} = \mathbf{a}+3\mathbf{b}$. You first need to calculate $\mathbf{a}+3\mathbf{b}$:

$$\mathbf{a}+3\mathbf{b} = (2\mathbf{i}+3\mathbf{j}-\mathbf{k})+3(-\mathbf{i}+0\mathbf{j}+2\mathbf{k})$$
$$= (2+(3\times -1))\mathbf{i}+(3+3\times 0)\mathbf{j}+(-1+3\times 2)\mathbf{k}$$
$$= -1\mathbf{i}+3\mathbf{j}+5\mathbf{k}$$

And you also know that $\mathbf{c} = d\mathbf{i}+3\mathbf{j}+f\mathbf{k}$. So, $d\mathbf{i}+3\mathbf{j}+f\mathbf{k} = -1\mathbf{i}+3\mathbf{j}+5\mathbf{k}$ and for the two vectors to be equal their coordinates have to be equal. So, $d = -1$ and $f = 5$.

b. You want to show that the magnitude of $\mathbf{c}$ is $\sqrt{35}$. From the previous question you have found $\mathbf{c} = -1\mathbf{i}+3\mathbf{j}+5\mathbf{k}$. So, the magnitude is:

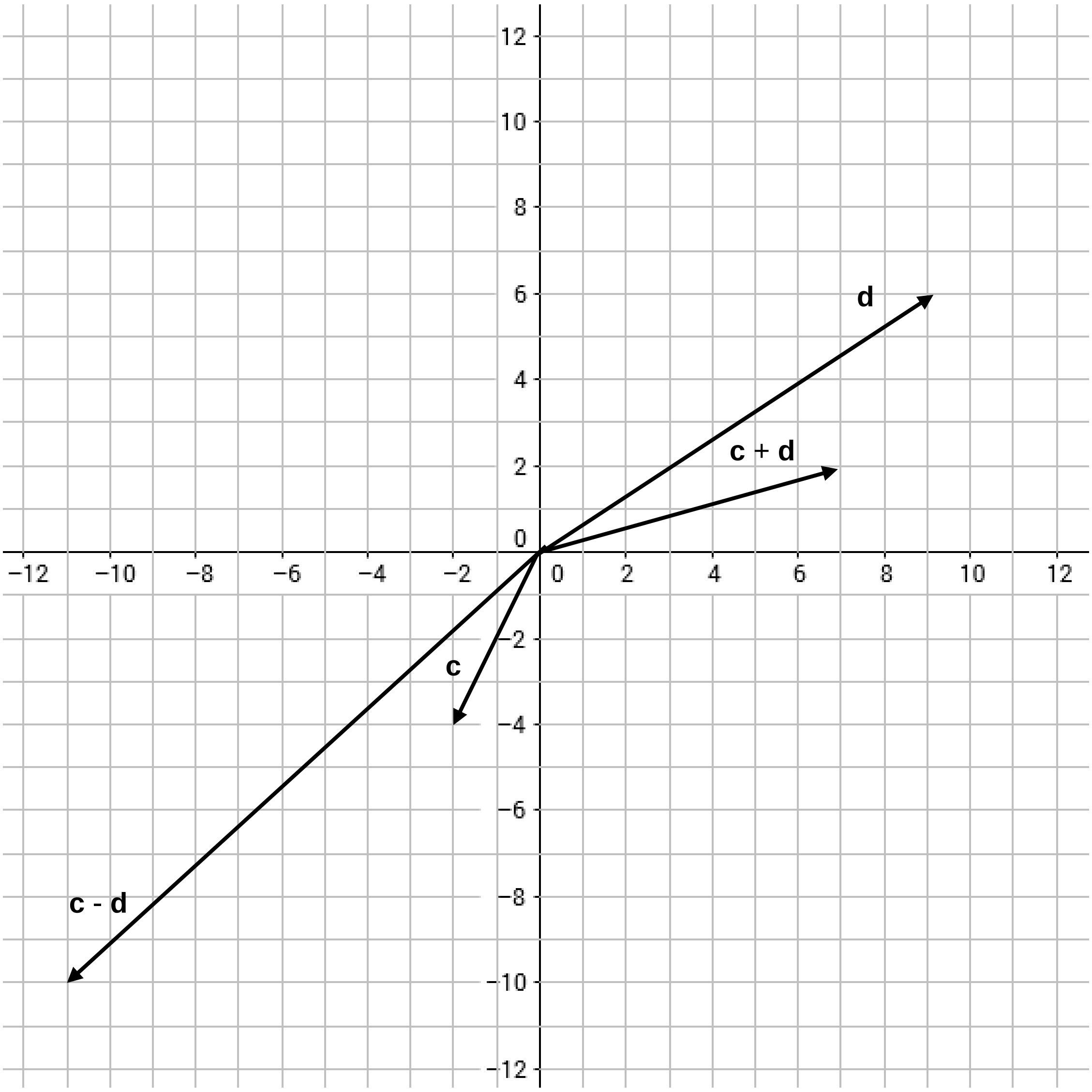
$$|\mathbf{c}| = \sqrt{c_1^2+c_2^2+c_3^2} = \sqrt{(-1)^2+(3)^2+(5)^2} = \sqrt{1+9+25} = \sqrt{35}$$

5. From the graph below you can see that $\mathbf{a} = \mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} + 2\mathbf{j}$

i. $\mathbf{d} = 3\mathbf{b} = 3(3\mathbf{i} + 2\mathbf{j}) = 9\mathbf{i} + 6\mathbf{j}$

ii. $\mathbf{c} - \mathbf{d} = -2\mathbf{a} - \mathbf{d} = -2(\mathbf{i} + 2\mathbf{j}) - (9\mathbf{i} + 6\mathbf{j}) = (-2-9)\mathbf{i} + (-4-6)\mathbf{j} = -11\mathbf{i} - 10\mathbf{j}$

iii. $\mathbf{c} + \mathbf{d} = -2\mathbf{a} + \mathbf{d} = -2(\mathbf{i} + 2\mathbf{j}) + (9\mathbf{i} + 6\mathbf{j}) = (-2+9)\mathbf{i} + (-4+6)\mathbf{j} = 7\mathbf{i} + 2\mathbf{j}$



This model answers are one of a series on mathematics produced by the Learning Enhancement Team with funding from the UEA Alumni Fund. Scan the QR-code with a smartphone app for more resources.

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