Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model Answers: Basics of Vectors

These are the model answers for the worksheet that has questions on the basics of vectors.

Basics of Vectors study guide

1.

i. The following vectors have the same direction:

$\overrightarrow{CD}$ and $\overrightarrow{IF}$ $\quad$ $\overrightarrow{BC}$, $\overrightarrow{FE}$ and $\overrightarrow{LM}$ $\quad$ $\overrightarrow{HG}$ and $\overrightarrow{JI}$

ii. The following vectors have the same magnitude:

$\overrightarrow{AB}$, $\overrightarrow{CD}$ and $\overrightarrow{IF}$ $\quad$ $\overrightarrow{BC}$, $\overrightarrow{FE}$ and $\overrightarrow{JI}$ $\quad$ $\overrightarrow{KN}$ and $\overrightarrow{KO}$

iii. The following vectors have opposite direction:

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ $\quad$ $\overrightarrow{AB}$ and $\overrightarrow{IF}$ $\quad$ $\overrightarrow{BC}$ and $\overrightarrow{JI}$

$\overrightarrow{BC}$ and $\overrightarrow{HG}$ $\quad$ $\overrightarrow{FE}$ and $\overrightarrow{JI}$ $\quad$ $\overrightarrow{FE}$ and $\overrightarrow{HG}$

$\overrightarrow{LM}$ and $\overrightarrow{JI}$ $\quad$ $\overrightarrow{LM}$ and $\overrightarrow{HG}$

iv. The following vectors have same direction and different magnitude:

$\overrightarrow{BC}$, and $\overrightarrow{LM}$ $\quad$ $\overrightarrow{FE}$ and $\overrightarrow{LM}$ $\quad$ $\overrightarrow{HG}$ and $\overrightarrow{JI}$

v. The following vectors have different direction and same magnitude:

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ $\quad$ $\overrightarrow{AB}$ and $\overrightarrow{IF}$ $\quad$ $\overrightarrow{BC}$ and $\overrightarrow{JI}$

$\overrightarrow{FE}$ and $\overrightarrow{JI}$ $\quad$ $\overrightarrow{KN}$ and $\overrightarrow{KO}$

vi. The following vectors have same direction and same magnitude:

$\overrightarrow{CD}$ and $\overrightarrow{IF}$ $\quad$ $\overrightarrow{BC}$ and $\overrightarrow{FE}$

vii. The following vectors have opposite direction and same magnitude:

$\overrightarrow{AB}$ and $\overrightarrow{CD}$ $\overrightarrow{AB}$ and $\overrightarrow{IF}$ $\overrightarrow{BC}$ and $\overrightarrow{JI}$

$\overrightarrow{FE}$ and $\overrightarrow{JI}$

The scale of the grid makes no difference as you are counting the units of the scale and the size of the units does not affect that. The coordinates of the points on the grid are not needed as the magnitude or the direction of the vectors do not depend on that.

2. In this question you need to find the magnitude of a vector. Remember that the magnitude of a vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ in 3 Dimensional (3D) space is given by:

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}\,.$$

In this question the vectors are in 2 Dimensional (2D) space meaning that the vectors are of the form $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$ and the magnitude is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$.

i. For the vector $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $a_1 = 2$ and $a_2 = 3$. Using these values in the equation for the magnitude in 2 Dimensional space $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$ gives:

$$|\mathbf{a}| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13}$$

So the magnitude (or length) of $\mathbf{a}$ is $\sqrt{13} = 3.61$ to 2 d.p.

ii. For the vector $\mathbf{b} = 1\mathbf{i} + -4\mathbf{j}$, $b_1 = 1$ and $b_2 = -4$. Using these values in the equation for the magnitude in 2 Dimensional space $|\mathbf{b}| = \sqrt{b_1^2 + b_2^2}$ gives:

$$|\mathbf{b}| = \sqrt{1^2 + (-4)^2} = \sqrt{1 + 16} = \sqrt{17}$$

So the magnitude (or length) of $\mathbf{b}$ is $\sqrt{17} = 4.12$ to 2 d.p.

iii. For the vector $\mathbf{c} = -2\mathbf{i} + \mathbf{j}$, $c_1 = -2$ and $c_2 = 1$. Using these values in the equation for the magnitude in 2 Dimensional space $|\mathbf{c}| = \sqrt{c_1^2 + c_2^2}$ gives:

$$|\mathbf{c}| = \sqrt{(-2)^2 + 1^2} = \sqrt{4 + 1} = \sqrt{5}$$

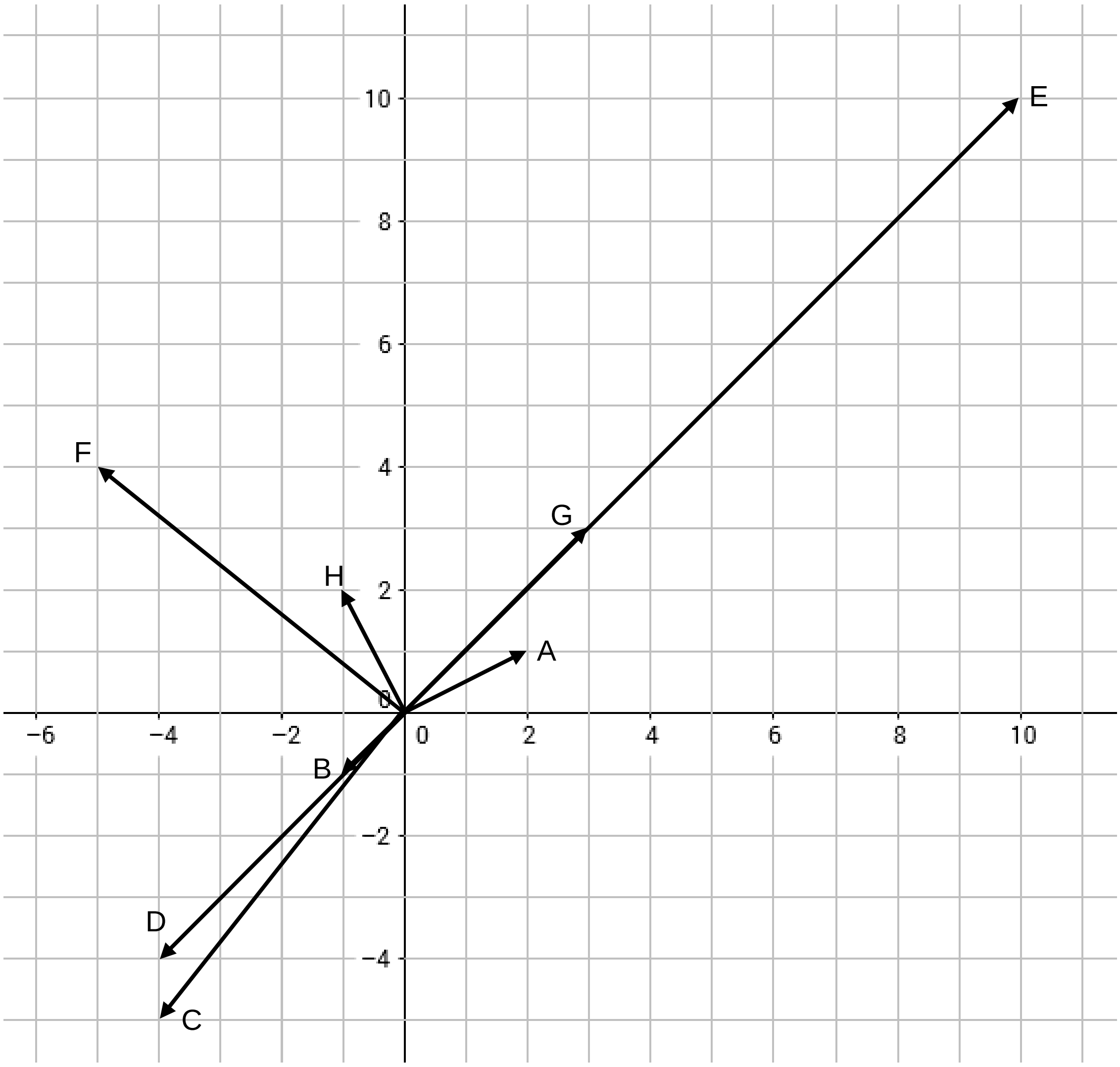
So the magnitude (or length) of $\mathbf{c}$ is $\sqrt{5} = 2.24$ to 2 d.p.

iv. For the vector $\mathbf{d} = -3\mathbf{i} - 5\mathbf{j}$, $d_1 = -3$ and $d_2 = -5$. Using these values in the equation for the magnitude in 2 Dimensional space $|\mathbf{d}| = \sqrt{d_1^2 + d_2^2}$ gives:

$$|\mathbf{d}| = \sqrt{(-3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}$$

So the magnitude (or length) of $\mathbf{d}$ is $\sqrt{34} = 5.83$ to 2 d.p.

3. Using graph paper to draw the 2-dimensional vectors you should get something like:



a. The vectors that have the same direction are:

$\overrightarrow{OE}$ and $\overrightarrow{OG}$ $\overrightarrow{OB}$ and $\overrightarrow{OD}$

b. The vectors that have the opposite direction are:

$\overrightarrow{OE}$ and $\overrightarrow{OB}$ $\overrightarrow{OE}$ and $\overrightarrow{OD}$ $\overrightarrow{OG}$ and $\overrightarrow{OB}$ $\overrightarrow{OG}$ and $\overrightarrow{OD}$

c. The vectors that have the same magnitude are:

$\overrightarrow{OA}$ and $\overrightarrow{OH}$ as:

$$\left|\overrightarrow{OA}\right| = \sqrt{2^2 + 1^2} = \sqrt{4+1} = \sqrt{5} \text{ and } \left|\overrightarrow{OH}\right| = \sqrt{(-1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}$$

And $\overrightarrow{OF}$ and $\overrightarrow{OC}$ as:

$$\left|\overrightarrow{OF}\right| = \sqrt{(-5)^2 + (4)^2} = \sqrt{25+16} = \sqrt{41} \text{ and } \left|\overrightarrow{OC}\right| = \sqrt{(-4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$$

4. In this question you need to find the magnitude and corresponding unit vector of a vector. Remember that the a unit vector of a vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ is given by:

$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|} = \frac{a_1}{|\mathbf{a}|}\mathbf{i} + \frac{a_2}{|\mathbf{a}|}\mathbf{j} + \frac{a_3}{|\mathbf{a}|}\mathbf{k}$$

i. For the vector $\mathbf{a} = -\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $a_1 = -1$, $a_2 = 3$ and $a_3 = 3$. Using these values in the equation for the magnitude from question 2 gives:

$$|\mathbf{a}| = \sqrt{(-1)^2 + 3^2 + 3^2} = \sqrt{1+9+9} = \sqrt{19}$$

So the magnitude (or length) of $\mathbf{a}$ is $\sqrt{19} = 4.36$ to 2 d.p. The unit vector of $\mathbf{a}$ is:

$$\hat{\mathbf{a}} = \frac{a_1}{|\mathbf{a}|}\mathbf{i} + \frac{a_2}{|\mathbf{a}|}\mathbf{j} + \frac{a_3}{|\mathbf{a}|}\mathbf{k} = \frac{-1}{\sqrt{19}}\mathbf{i} + \frac{3}{\sqrt{19}}\mathbf{j} + \frac{3}{\sqrt{19}}\mathbf{k}$$

You can always check that your answer for a unit vector is correct by showing the magnitude of the unit vector is 1.

$$|\hat{\mathbf{a}}| = \sqrt{\left(\frac{-1}{\sqrt{19}}\right)^2 + \left(\frac{3}{\sqrt{19}}\right)^2 + \left(\frac{3}{\sqrt{14}}\right)^2} = \sqrt{\frac{1}{19} + \frac{9}{19} + \frac{9}{19}} = \sqrt{\frac{19}{19}} = \sqrt{1} = 1$$

ii. For the vector $\mathbf{b} = 3\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $b_1 = 3$, $b_2 = 1$ and $b_3 = -4$ Using these values in the equation for the magnitude from question 2 gives:

$$|\mathbf{b}| = \sqrt{3^2 + 1^2 + (-4)^2} = \sqrt{9+1+16} = \sqrt{26}$$

So the magnitude (or length) of $\mathbf{b}$ is $\sqrt{26} = 5.10$ to 2 d.p. The unit vector of $\mathbf{b}$ is:

$$\hat{\mathbf{b}} = \frac{b_1}{|\mathbf{b}|}\mathbf{i} + \frac{b_2}{|\mathbf{b}|}\mathbf{j} + \frac{b_3}{|\mathbf{b}|}\mathbf{k} = \frac{3}{\sqrt{26}}\mathbf{i} + \frac{1}{\sqrt{26}}\mathbf{j} + \frac{-4}{\sqrt{26}}\mathbf{k}$$

iii. For the vector $\mathbf{c} = \frac{1}{2}\mathbf{i} - \mathbf{j} + 0\mathbf{k}$, $c_1 = \frac{1}{2}$, $c_2 = -1$ and $c_3 = 0$. Using these values in the equation for the magnitude from question 2 gives:

$$|\mathbf{c}| = \sqrt{\left(\tfrac{1}{2}\right)^2 + (-1)^2 + 0^2} = \sqrt{\tfrac{1}{4} + 1 + 0} = \sqrt{\tfrac{5}{4}} = \frac{\sqrt{5}}{2}$$

So the magnitude (or length) of $\mathbf{c}$ is $\frac{\sqrt{5}}{2} = 1.12$ to 2 d.p. The unit vector of $\mathbf{c}$ is:

$$\hat{\mathbf{c}} = \frac{c_1}{|\mathbf{c}|}\mathbf{i} + \frac{c_2}{|\mathbf{c}|}\mathbf{j} + \frac{c_3}{|\mathbf{c}|}\mathbf{k} = \frac{\frac{1}{2}}{\frac{\sqrt{5}}{2}}\mathbf{i} + \frac{-1}{\frac{\sqrt{5}}{2}}\mathbf{j} + \frac{0}{\frac{\sqrt{5}}{2}}\mathbf{k} = \frac{\sqrt{5}}{5}\mathbf{i} + \frac{-2\sqrt{5}}{5}\mathbf{j} + 0\mathbf{k}$$

iv. For the vector $\mathbf{d} = \sqrt{2}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$, $d_1 = \sqrt{2}$, $d_2 = -\frac{2}{3}$ and $d_3 = \frac{1}{3}$. Using these values in the equation for the magnitude from question 2 gives:

$$|\mathbf{d}| = \sqrt{\left(\sqrt{2}\right)^2 + \left(-\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2} = \sqrt{2 + \tfrac{4}{9} + \tfrac{1}{9}} = \sqrt{\tfrac{23}{9}} = \frac{\sqrt{23}}{3}$$

So the magnitude (or length) of $\mathbf{d}$ is $\frac{\sqrt{23}}{3} = 1.60$ to 2 d.p. The unit vector of $\mathbf{d}$ is:

$$\hat{\mathbf{d}} = \frac{d_1}{|\mathbf{d}|}\mathbf{i} + \frac{d_2}{|\mathbf{d}|}\mathbf{j} + \frac{d_3}{|\mathbf{d}|}\mathbf{k} = \frac{\sqrt{2}}{\frac{\sqrt{23}}{3}}\mathbf{i} + \frac{-\frac{2}{3}}{\frac{\sqrt{23}}{3}}\mathbf{j} + \frac{\frac{1}{3}}{\frac{\sqrt{23}}{3}}\mathbf{k} = \frac{3\sqrt{46}}{23}\mathbf{i} - \frac{2}{23}\mathbf{j} + \frac{1}{23}\mathbf{k}$$

v. For the vector $\mathbf{e} = \frac{3}{2}\mathbf{i} - \mathbf{j} + \frac{\sqrt{11}}{2}\mathbf{k}$, $e_1 = \frac{3}{2}$, $e_2 = -1$ and $e_3 = \frac{\sqrt{11}}{2}$. Using these values in the equation for the magnitude from question 2 gives:

$$|\mathbf{e}| = \sqrt{\left(-\tfrac{3}{2}\right)^2 + (-1)^2 + \left(\tfrac{\sqrt{11}}{2}\right)^2} = \sqrt{\tfrac{9}{4} + 1 + \tfrac{11}{4}} = \sqrt{1 + \tfrac{20}{4}} = \sqrt{1+5} = \sqrt{6}$$

So the magnitude (or length) of $\mathbf{e}$ is $\sqrt{6} = 2.45$ to 2 d.p. The unit vector of $\mathbf{e}$ is:

$$\hat{\mathbf{e}} = \frac{e_1}{|\mathbf{e}|}\mathbf{i} + \frac{e_2}{|\mathbf{e}|}\mathbf{j} + \frac{e_3}{|\mathbf{e}|}\mathbf{k} = \frac{\frac{3}{2}}{\sqrt{6}}\mathbf{i} + \frac{-1}{\sqrt{6}}\mathbf{j} + \frac{\frac{\sqrt{11}}{2}}{\sqrt{6}}\mathbf{k} = \frac{\sqrt{6}}{2}\mathbf{i} - \frac{\sqrt{6}}{6}\mathbf{j} + \frac{\sqrt{66}}{12}\mathbf{k}$$

These model answers are one of a series on mathematics produced by the Learning Enhancement Team with funding from the UEA Alumni Fund. Scan the QR-code with a smartphone app for more resources.

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