Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Steps into Discrete Mathematics

Basics of Matrices

This guide gives an introduction to matrix algebra, including how to write and name a matrix, how to perform matrix addition and subtraction and how to multiply a matrix by a scalar.

Introduction

A **matrix** is something you will often come across in mathematics. They are a very powerful tool in handling large amounts of data as the mathematical rules for combining and manipulating **matrices** (the plural of matrix) are easily carried out by a computer. In computer graphics, matrices are used to project a 3-dimensional model onto a 2-dimensional screen, which creates realistic-seeming motion. In physics they are used to simplify complicated calculations in quantum mechanics. In fact, wherever complicated sets of equations need to be solved matrices are used.

A matrix is made up of **rows** and **columns** of numbers or symbols (called **elements**) which are usually arranged within **square brackets**. The symbol for a matrix is an **italic capital** letter. This makes them easier to refer to since writing matrices in full, especially if they are large, is time consuming. For example:

$$M = \begin{bmatrix} 3 & 7 & 4 \\ 5 & 2 & 0 \end{bmatrix}$$

M is a matrix with two rows and three columns. It has six elements 3, 7, 4, 5, 2 and 0.

Matrices come in different shapes and sizes and you can describe them in terms of their **dimension** or **size**. The dimension of a matrix is given by two numbers *m* and *n* and is written as $m \times n$, this is not *m* multiplied by *n* but is said as "***m* by *n***"). Specifically ***m*** is the **number of rows** (rows are horizontal) in the matrix and ***n*** is the **number of columns** (columns are vertical).

Size of a matrix = Number of Rows × Number of Columns

The matrix *M* above has two rows and three columns and so has a size of 2×3. You can say that *M* is a "two by three" matrix.

Example: Describe the size or dimension of the following matrices:

$$A = \begin{bmatrix} 2 & 3 \\ 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ -3 \end{bmatrix} \quad C = [3] \quad D = \begin{bmatrix} 4 & 1 & -3 & 8 & 9 \\ 3 & 0 & -6 & 6 & 2 \\ 5 & -1 & 0 & -2 & 3 \end{bmatrix} \quad E = \begin{bmatrix} 2 & 0 & 7 \\ -1 & 5 & 2 \\ 5 & -4 & 3 \\ 2 & 6 & 1 \end{bmatrix}$$

A has two rows and two columns and so is a 2×2 (two by two) matrix.

B has two rows and one column and so is a 2×1 (two by one) matrix.

C has one row and one column and so is a 1×1 (one by one) matrix.

D has three rows and five columns and so is a 3×5 (three by five) matrix.

E has four rows and three columns and so is a 4×3 (four by three) matrix.

Some special types of matrices

1. Square matrices

A square matrix has the same number of rows and columns. These are very common types of matrices and form the basis for more advances studies of matrices. For example if:

$$A = \begin{bmatrix} 2 & 3 \\ 7 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 8 & -6 \\ -1 & 7 & 2 \end{bmatrix}$$

then *A* is a 2×2 (two by two) square matrix and *B* is a 3×3 (three by three) square matrix.

2. Row matrices and column matrices

A **row matrix** is one that only has one row (but any number of columns) and a **column matrix** is one which has only one column (but any number of rows). For example:

$$A = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & -5 & 8 \end{bmatrix}$$

A is a **column matrix** as it contains one column only. It is a 2×1 matrix.

B is a **row matrix** as it only has one row. It is a 1×4 matrix.

Row and column matrices are common in many subjects where mathematics is used as they are often used to represent **vectors**.

3. Diagonal matrices

An important part of a square matrix is the **principal diagonal** which goes from the top left element to the bottom right element. For example the principal diagonal in the matrix below comprises the elements 1, 8 and 2 and is highlighted below:

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 8 & -6 \\ -1 & 7 & 2 \end{bmatrix}$$

If all the other elements in a square matrix, other than those in the principal diagonal are zero then the matrix is called a **diagonal matrix**. For example, by making all the other elements in the matrix above equal to zero you get the diagonal matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

4. Identity matrices

If all the elements in the principal diagonal of a diagonal matrix are equal to one then the square matrix is called an **identity matrix**. There is an identity matrix for each size of square matrix. An identity matrix is denoted by I_n, where the subscript n is the number of rows (or columns) in the matrix. For example:

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Identity matrices play an important role in the multiplication of matrices and advanced matrix methods.

Adding and subtracting matrices

The most important thing to remember about adding and subtracting matrices is that you can **only** add and subtract matrices which are the same size; that is matrices that have the same number of rows and columns as each other. The result of addition or subtraction of matrices is also a matrix of the same size. It is important, when you are beginning to understand the basics of matrices, whether you can or as importantly cannot perform the operation you need. You should also work out what size of matrix you expect your answer to be. This allows you to *sketch* your answer out; use a matrix of the size you require and section it up into where the elements should go and use it to work out the actual final answer (see below).

When you are adding and/or subtracting matrices, each element in the result is found by adding (or subtracting as required) the corresponding elements in the matrices. Take the two matrices:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix}$$

You can calculate $A + B$ as both A and B are the same size (two by two). The result is also a two by two matrix whose elements are found as follows:

Top left: Top left of A + Top left of B $= 2 + -1 = 1$

Top right: Top right of A + Top right of B $= -1 + 4 = 3$

Bottom left: Bottom left of A + Bottom left of B $= 0 + 5 = 5$

Bottom right: Bottom right of A + Bottom right of B $= 3 + -3 = 6$

So:

$$A + B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (2 + -1) & (-1 + 4) \\ (0 + 5) & (3 + 3) \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 5 & 6 \end{bmatrix}$$

Sketch

Example: Given that:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 1 & 1 \\ 5 & 2 & 3 \end{bmatrix}$$

Find, where possible, (i) $A - B$ (ii) $C - D$ (iii) $A + C$.

(i) $A - B$ is possible as both A and B are 2×2 matrices so:

$$A - B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} -1 & 4 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} (2 - -1) & (-1-4) \\ (0-5) & (3-3) \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -5 & 0 \end{bmatrix}$$

(ii) $C - D$ is possible as both C and D are 2×3 matrices so:

$$\begin{bmatrix} 1 & -3 & 4 \\ 2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 1 \\ 5 & 2 & 3 \end{bmatrix} = \begin{bmatrix} (1-0) & (-3-1) & (4-1) \\ (2-5) & (1-2) & (1-3) \end{bmatrix} = \begin{bmatrix} 1 & -4 & 3 \\ -3 & -1 & -2 \end{bmatrix}$$

(iii) $A + C$ is not possible as A (2×2) and C (2×3) are different sizes.

Multiplying a matrix by a scalar

A **scalar** is a single number. To multiply a matrix by a scalar you simply multiply every element in the matrix by the scalar, this very different to **matrix multiplication** which is discussed at length in the study guide: *Multiplying Matrices*. The result of multiplying a matrix by a scalar is a matrix of the same size.

Example: Given the matrices:

$$A = \begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -10 & 2 \\ 0 & -4 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Find (i) $3A$, (ii) $-\frac{1}{2}B$, (iii) λI (iv) $xA - yB$.

(Throughout these following answers the dot · is used to imply multiplication.)

(i) $$3A = 3\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 \cdot 2 & 3 \cdot 1 \\ 3 \cdot (-1) & 3 \cdot 4 \end{bmatrix} = \begin{bmatrix} 6 & 3 \\ -3 & 12 \end{bmatrix}$$

(ii) $$-\tfrac{1}{2}B = -\tfrac{1}{2}\begin{bmatrix}-10 & 2\\ 0 & -4\end{bmatrix} = \begin{bmatrix}\left(-\tfrac{1}{2}\right)\cdot(-10) & \left(-\tfrac{1}{2}\right)\cdot 2\\ \left(-\tfrac{1}{2}\right)\cdot 0 & \left(-\tfrac{1}{2}\right)\cdot(-4)\end{bmatrix} = \begin{bmatrix}5 & -1\\ 0 & 2\end{bmatrix}$$

(iii) Here the Greek letter lambda (λ) is used to denote *any* scalar so:

$$\lambda I = \lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}\lambda\cdot 1 & 0\\ 0 & \lambda\cdot 1\end{bmatrix} = \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$$

Notice that when you multiply this (or any) identity matrix by any scalar you get a diagonal matrix.

(iv) Again, in this question x and y are symbols used to describe any given scalars.

$$xA - yB = x\begin{bmatrix}2 & 1\\ -1 & 4\end{bmatrix} - y\begin{bmatrix}-10 & 2\\ 0 & -4\end{bmatrix} = \begin{bmatrix}2\cdot x & 1\cdot x\\ (-1)\cdot x & 4\cdot x\end{bmatrix} - \begin{bmatrix}(-10)\cdot y & 2\cdot y\\ 0\cdot y & (-4)\cdot y\end{bmatrix}$$
$$= \begin{bmatrix}2x + 10y & x - 2y\\ -x & 4x + 4y\end{bmatrix}$$

Want to know more?

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