Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model answers: Basics of Matrices

Basics of Matrices study guide

1.

a. A has 2 rows and 2 columns so is a 2×2 (two by two) matrix.
B has 1 row and 2 columns so is a 1×2 (one by two) matrix.
C has 2 rows and 1 column so is a 2×1 (two by one) matrix.
D has 2 rows and 2 columns so is a 2×2 (two by two) matrix.
E has 2 rows and 3 columns so is a 2×3 (two by three) matrix.
F has 2 rows and 1 columns so is a 2×1 (two by one) matrix.

b. You can only add matrices together if they are the same size so A and D can be added together as both are 2×2 matrices. Also C and F can be added together as both are 2×1 matrices.

c. In the following answers the dot $\cdot$ represents multiplication.

i. $$3A = 3\cdot\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix} = \begin{bmatrix}3\cdot 4 & 3\cdot 1\\ 3\cdot(-1) & 3\cdot 2\end{bmatrix} = \begin{bmatrix}12 & 3\\ -3 & 6\end{bmatrix}$$

ii. $$\frac{1}{2}B = \frac{1}{2}\cdot\begin{bmatrix}5 & -2\end{bmatrix} = \begin{bmatrix}\frac{1}{2}\cdot 5 & \frac{1}{2}\cdot(-2)\end{bmatrix} = \begin{bmatrix}\frac{5}{2} & -1\end{bmatrix}$$

iii. $$-2C = -2\cdot\begin{bmatrix}3\\ -1\end{bmatrix} = \begin{bmatrix}(-2)\cdot 3\\ (-2)\cdot(-1)\end{bmatrix} = \begin{bmatrix}-6\\ 2\end{bmatrix}$$

iv. Firstly perform the multiplications:

$$12A = 12\cdot\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix} = \begin{bmatrix}12\cdot 4 & 12\cdot 1\\ 12\cdot(-1) & 12\cdot 2\end{bmatrix} = \begin{bmatrix}48 & 12\\ -12 & 24\end{bmatrix}$$

$$4D = 4\cdot\begin{bmatrix}6 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}4\cdot 6 & 4\cdot 0\\ 4\cdot 0 & 4\cdot 1\end{bmatrix} = \begin{bmatrix}24 & 0\\ 0 & 4\end{bmatrix}$$

Using these results you can perform the addition:

$$12A+4D=\begin{bmatrix}48 & 12\\ -12 & 24\end{bmatrix}+\begin{bmatrix}24 & 0\\ 0 & 4\end{bmatrix}=\begin{bmatrix}48+24 & 12+0\\ -12+0 & 24+4\end{bmatrix}=\begin{bmatrix}72 & 12\\ -12 & 28\end{bmatrix}$$

v. Firstly perform the multiplications:

$$5\pi C=5\pi\cdot\begin{bmatrix}3\\ -1\end{bmatrix}=\begin{bmatrix}5\pi\cdot 3\\ 5\pi\cdot(-1)\end{bmatrix}=\begin{bmatrix}15\pi\\ -5\pi\end{bmatrix}$$

$$2\pi F=2\pi\cdot\begin{bmatrix}-2\\ 5\end{bmatrix}=\begin{bmatrix}2\pi\cdot(-2)\\ 2\pi\cdot 5\end{bmatrix}=\begin{bmatrix}-4\pi\\ 10\pi\end{bmatrix}$$

Using these results, you can perform the subtraction:

$$5\pi C-2\pi F=\begin{bmatrix}15\pi\\ -5\pi\end{bmatrix}-\begin{bmatrix}-4\pi\\ 10\pi\end{bmatrix}=\begin{bmatrix}15\pi+4\pi\\ -5\pi-10\pi\end{bmatrix}=\begin{bmatrix}19\pi\\ -15\pi\end{bmatrix}$$

There is an alternative way of approaching this question. You have π as a common factor in the question and so $5\pi C-2\pi F$ can be written as, $\pi(5C-2F)$. So you can calculate $5C-2F$ and then multiply the result by π:

$$5C-2F=5\cdot\begin{bmatrix}3\\ -1\end{bmatrix}-2\cdot\begin{bmatrix}-2\\ 5\end{bmatrix}=\begin{bmatrix}15\\ -5\end{bmatrix}-\begin{bmatrix}-4\\ 10\end{bmatrix}=\begin{bmatrix}15+4\\ -5-10\end{bmatrix}=\begin{bmatrix}19\\ -15\end{bmatrix}$$

And so:

$$\pi(5C-2F)=\pi\cdot\begin{bmatrix}19\\ -15\end{bmatrix}=\begin{bmatrix}19\pi\\ -15\pi\end{bmatrix}$$

vi. Firstly perform the multiplications, using SNAlphabet if you need to:

$$xD=x\cdot\begin{bmatrix}6 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}x\cdot 6 & x\cdot 0\\ x\cdot 0 & x\cdot 1\end{bmatrix}=\begin{bmatrix}6x & 0\\ 0 & x\end{bmatrix}$$

$$yA=y\cdot\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix}=\begin{bmatrix}y\cdot 4 & y\cdot 1\\ y\cdot(-1) & y\cdot 2\end{bmatrix}=\begin{bmatrix}4y & y\\ -y & 2y\end{bmatrix}$$

Using these results you can perform the subtraction:

$$xD - yA = \begin{bmatrix} 6x & 0 \\ 0 & x \end{bmatrix} - \begin{bmatrix} 4y & y \\ -y & 2y \end{bmatrix} = \begin{bmatrix} 6x-4y & 0-y \\ 0+y & x-2y \end{bmatrix} = \begin{bmatrix} 6x-4y & -y \\ y & x-2y \end{bmatrix}$$

2. The key to understanding how to answer this question is that the elements of the answer C are found by adding the corresponding elements from A and B so:

$$\begin{bmatrix} 1 & 2 & 0 \\ a & b & c \end{bmatrix} + \begin{bmatrix} a & b & c \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} c & 5 & c \\ c & c & c \end{bmatrix}$$

Comparing corresponding elements:

First row, first column: $1 + a = c$
First row, second column: $2 + b = 5$
First row, third column: $0 + c = c$
Second row, first column: $a + 1 = c$
Second row, second column: $b + 2 = c$
Second row, third column: $c + 0 = c$

From the first row, second column you know that $2 + b = 5$ and so $b = 3$.

From the second row, second column you know that $b + 2 = c$.
As $b = 3$, $3 + 2 = c$ and so $c = 5$.

From first row, first column (or second row, first column) $1 + a = c$.
As $c = 5$, $1 + a = 5$ and so $a = 4$.

Check you answer by substituting your answers in:

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 5 \\ 1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \end{bmatrix}$$

This is true and so your answer is correct.