Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model Answers: Basics of Differential Equations

These are the model answers for the worksheet on Basics of Differential Equations.

Basics of Differential Equations worksheet	Basics of Differential Equations study guide

1.

a. $$\frac{dy}{dx} + y = 0$$

The highest order derivative is $\frac{dy}{dx}$ which is first order. So the differential equation is **first order**.

b. $$y' + 6xy = 0$$

The highest order derivative is y' which is first order, so this is a **first order** differential equation.

c. $$\frac{d^2y}{dx^2} = 3x^2y$$

This is a **second order** differential equation since the highest order derivative is $\frac{d^2y}{dx^2}$ which is second order.

d. $$y'' + y' - 4 = 0$$

This differential equation is **second order** since the highest order derivative is y'', which is second order.

e. $$\left(\frac{dy}{dx}\right)^2 = 3x^2y$$

This question looks similar to question 1 part (c). However it is important to recognise:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2} \qquad \text{and} \qquad \left(\frac{dy}{dx}\right)^2=\frac{dy}{dx}\cdot\frac{dy}{dx}, \qquad \text{so} \qquad \left(\frac{dy}{dx}\right)^2\neq\frac{d^2y}{dx^2},$$

in other words: **the square of the first derivative does not mean the same thing mathematically as the second derivative**.

So:

$$\left(\frac{dy}{dx}\right)^2=3x^2y$$

is a **first order** differential equation since the highest order derivative is $\frac{dy}{dx}$, which is first order.

f.

$$\frac{\partial z}{\partial x}-3x^2y=x$$

This is a **first order** *partial* differential equation since the highest order derivative is $\frac{\partial z}{\partial x}$, which is a first order partial derivative.

2.

a.

$$\frac{dy}{dx}+3y=0$$

This differential equation is:

Linear as *y* and its derivatives are only multiplied by constants. There are no terms that are nonlinear in *y* .

First order since the first order derivative $\frac{dy}{dx}$ is the highest order derivative.

Ordinary, as only ordinary derivatives are present.

b. $$\frac{\partial z}{\partial x}+3xz=0$$

This differential equation is:

Linear because *z* and its derivatives are only multiplied by constants or functions of *x*. There are no terms that are nonlinear in *z*.

Partial as partial derivatives are present.

First order since the first order partial derivative $\frac{\partial z}{\partial x}$ is the highest order derivative.

c.

$y'' - yy' = 0$

This differential equation is:

Nonlinear because there is a product of y and y'. This is a nonlinear function of y and its derivative.

Ordinary as only ordinary derivatives are present.

Second order due to the second order derivative y'' being the highest order derivative.

d. $\frac{\partial^2 u}{\partial x^2} + \frac{dv}{dt} = 0$

This differential equation is:

Linear because the derivatives are only multiplied by constants.

Partial as partial derivatives are present.

Second order due to the second order derivative $\frac{\partial^2 u}{\partial x^2}$ being the highest order derivative.

e. $x(y')^2 + y = 3x$

Remembering that $y'' \neq (y')^2$, this differential equation is:

Nonlinear because y' is squared. This is a nonlinear function of y'.

Ordinary as only ordinary derivatives are present.

First order since the highest order derivative is the first order derivative y'.

f. $y'' + 3y' = \cos(y)$

This differential equation is:

Nonlinear because $\cos(y)$ is a nonlinear function of y.

Ordinary as only ordinary derivatives are present.

Second order since the highest order derivative is the second order derivative y''.

3. You are given the second order ordinary differential equation: $y'' + 4y' + 3y = 0$.

You are told to first substitute $y = e^{-x}$ into the differential equation.

First you need to differentiate this with respect to x to get y', and then differentiate with respect to x again to get y''. Differentiating once gives:

$y' = -e^{-x}$ then differentiating again gives: $y'' = e^{-x}$

The next step is to substitute these into the ordinary differential equation (ODE):

$y'' + 4y' + 3y = 0$ becomes $e^{-x} - 4e^{-x} + 3e^{-x} = 0$

Both sides of the equation are equal to zero. So:

$y = e^{-x}$ **is** a function that **solves** $y'' + 4y' + 3y = 0$.

Next you are told to substitute $y = e^{-3x}$ into the equation.

You need to differentiate with respect to x to get y' and then differentiate with respect to x again to get y''. Differentiating once gives:

$y' = -3e^{-3x}$ then differentiating again gives: $y'' = 9e^{-3x}$

The next step is to substitute these into the ODE:

$y'' + 4y' + 3y = 0$ becomes $9e^{-3x} - 12e^{-3x} + 3e^{-3x} = 0$

and both sides of the equation are zero. So:

$y = e^{-3x}$ is also a function that **solves** $y'' + 4y' + 3y = 0$.

4. You are given the second order ordinary differential equation $y'' + y' - y = 0$, and are told to substitute $y = e^{-2x}$ into it.

First you need to differentiate this with respect to x to get y' and then differentiate with respect to x again to get y''. Differentiating once gives:

$y' = -2e^{-2x}$ and differentiating again gives $y'' = 4e^{-2x}$

The next step is to substitute these into the ordinary differential equation (ODE):

$y'' + y' - y = 0$ becomes $4e^{-2x} - 2e^{-2x} - e^{-2x} = e^{-2x} = 0$

but this is **not** a true statement since

$e^{-2x} \neq 0$

so $y = e^{-2x}$ is **not** a solution this ODE.