

Steps into Calculus

Integrating Basic Functions

This guide will show you how to identify and integrate five basic functions.

Introduction

Many complicated functions can be broken down into combinations of these five elementary functions:

$$y = ax^n$$

$$y = a\sin(kx)$$

$$y = a\cos(kx)$$

$$y = ae^{kx}$$

$$y = a\ln(kx)$$

The factsheet: [Five Basic Functions](#) gives more information about these functions and the study guide: [More Complicated Functions](#) shows you how to use these basic functions to make other, more complicated functions. Identifying how a more complicated function is created from these simpler functions may help you to choose a suitable rule to integrate that function such as:

1. Term-by-term integration
2. Integration by parts (see study guide: [Integration by Parts](#))
3. Integral which give natural logarithms (see study guide: [Integration and Natural Logarithms](#))
4. Partial fractions or
5. Integration using a substitution

A crucial part of the methods given above is that, at some stage, they all involve integrating one of the basic functions. This study guide can help you with integrating these functions. You can think of each of the basic functions of x above as a template with **numerical constants** (a and n in the first and a and k in the other four). If the function you are integrating fits one of these templates then you can integrate it using the relevant rule given in this study guide. This guide shows you how to match the function you need to integrate with the relevant template by finding the values of the constants a , n or k .

All the integrals here are indefinite integrals and so will have $+ C$ at the end of the answer. The study guide: [What is Integration?](#) explains the terms and vocabulary of integration which you will see in this guide, including why the **constant of integration** c is added to the end of indefinite integrals.

The results in this study guide are presented without any mathematical proof. However, if you are interested, a [Learning Enhancement Tutor](#) would be happy to show you the mathematics behind them.

1. Integrating the power function $y = ax^n$

The study guide: [Integrating Using the Power Rule](#) covers this topic in depth and you should read this guide for help with using the following formulas, which together are known as the **power rule for integration**. The main results of the power rule for integration are:

If $y = a$	then $\int y dx = ax + c$
If $y = ax^n$ and $n \neq -1$	then $\int y dx = a \frac{x^{n+1}}{n+1} + c$
If $y = ax^{-1}$	then $\int y dx = a \ln x + c$

Remember, you can always check your integration by differentiating the result and seeing if you get the original function.

2. Integrating the trigonometric functions $y = a \sin kx$ and $y = a \cos kx$

The integrals of the basic sine and cosine functions are closely linked:

If $y = a \sin kx$	then $\int y dx = -\frac{a}{k} \cos kx + c$
If $y = a \cos kx$	then $\int y dx = \frac{a}{k} \sin kx + c$

You should make a special note of the minus sign which is present in the integral of sine but absent in the integral of cosine. This is the opposite of the corresponding rules for differentiation. It is a common mistake to confuse these two sets of rules. A good way of avoiding the confusion is to learn the rules for differentiating sine and cosine and then use them to check your integration by differentiating back.

As with integrating the power function, the key to successfully finding an integral using this rule is the identification of the constants a and k .

Example: Integrate $y = 3\sin(2x)$.

When attempting a problem like this, it is important to correctly identify the rule you need to use. Can you see that $y = 3\sin(2x)$ fits the first rule in this section with $a = 3$ and $k = 2$? If so, you can substitute these values into the rule to show that:

$$\int y dx = \int 3\sin(2x) dx = -\frac{3}{2}\cos(2x) + c$$

You can check this by differentiating $y = -\frac{3}{2}\cos(2x) + c$ by using the rule for differentiating cosine, see study guide: [Differentiating Basic Functions](#):

$$\frac{dy}{dx} = (-1) \cdot \left(-\frac{3}{2}\right) \cdot 2 \cdot \sin(2x) = 3\sin(2x)$$

which is the original function and confirms that the integration is correct.

When a and k are both equal to 1 you have two important special cases of the rule:

$$\int \sin x dx = -\cos x + c \quad \text{and} \quad \int \cos x dx = \sin x + c$$

3. Integrating the exponential function $y = ae^{kx}$

If $y = ae^{kx}$ then $\int y dx = \frac{a}{k}e^{kx} + c$

The key to success in integrating an exponential function of this type is identifying the constants a and k correctly so that it fits the template.

Example: Integrate $y = \frac{e^{-2x}}{5}$.

The function which needs to be integrated fits the pattern of the rule with $a = 1/5$ and $k = -2$ so the integral is:

$$\int y dx = \int \frac{e^{-2x}}{5} dx = \frac{1}{5} \cdot \frac{1}{-2} \cdot e^{-2x} + c = -\frac{1}{10}e^{-2x} + c$$

You can check this by differentiating $y = -\frac{1}{10}e^{-2x} + c$ by using the rule for differentiating the exponential function, see the study guide: [Differentiating Basic Functions](#):

$$\frac{dy}{dx} = (-2) \cdot \left(-\frac{1}{10}\right) \cdot e^{-2x} = \frac{1}{5}e^{-2x} = \frac{e^{-2x}}{5}$$

which is the original function and that means that your integration is correct.

The case when $a = 1$ and $k = 1$ leads to the interesting result:

$$\int e^x dx = e^x + c$$

It is an important property of the exponential function that it is identical to its integral plus the constant of integration.

4. Integrating the natural logarithm function $y = a \ln kx$.

The integral of the natural logarithm function is:

$$\int a \ln(kx) dx = a[x \ln(kx) - x] + c$$

but this is not seen as often as the other integrals in this guide. Its derivation, however, is an interesting application of integration by parts (see study guide: [Integration by Parts](#)).

Want to know more?

If you have any further questions about this topic you can make an appointment to see a [Learning Enhancement Tutor](#) in the [Student Support Service](#), as well as speaking to your lecturer or adviser.

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