Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Model Answers: Integrating Basic Functions

Integrating Basic Functions study guide

The following rules will be referred to throughout these model answers.

rule	function	integral
1	$y = a$	$\int a\,dx = ax + c$
2	$y = ax^n$ (if $n \neq -1$)	$\int ax^n dx = \frac{ax^{n+1}}{n+1} + c$
3	$y = \frac{a}{x} = ax^{-1}$ (if $n = -1$)	$\int \frac{a}{x} dx = a \ln x + c$
4	$y = a \sin kx$	$\int a \sin kx\,dx = -\frac{a}{k} \cos kx + c$
5	$y = a \cos kx$	$\int a \cos kx\,dx = \frac{a}{k} \sin kx + c$
6	$y = ae^{kx}$	$\int ae^{kx} dx = \frac{a}{k} e^{kx} + c$

It is also very helpful to have a good knowledge of the laws of indices and how to multiply and divide algebraic fractions (see study guides: *Laws of Indices* and *Multiplying and Dividing Algebraic Fractions*).

Before you look at the model answers remember:

Integration is the inverse of differentiation.

So you can check all your answers by differentiating them to see if you get the original function. You do not need to look up the answers.

If you are not sure about how to differentiate the answers then read the study guide: *Differentiating Basic Functions.*

1.

(a) If $y = 7$ then $\int 7dx = 7x + c$

You can use rule (1) with $a = 7$ to show that:

$$\int 7dx = 7x + c$$

(b) If $y = \frac{1}{7}$ then $\int \frac{1}{7}dx = \frac{1}{7}x + c$

You can use rule (1) with $a = \frac{1}{7}$ to show that:

$$\int \frac{1}{7}dx = \frac{1}{7}x + c$$

(c) If $y = -7x$ then $\int -7xdx = -\frac{7}{2}x^2 + c$

You can use rule (2) with $a = -7$ and $n = 1$ to show that:

$$\int -7xdx = \int -7x^1dx = -\frac{7}{2}x^2 + c$$

(d) If $y = \frac{1}{7}x$ then $\int \frac{1}{7}xdx = \frac{x^2}{14} + c$

You can use rule (2) with $a = \frac{1}{7}$ and $n = 1$ to show that:

$$\int \frac{1}{7}xdx = \int \frac{1}{7}x^1dx = \frac{1}{7}\frac{x^2}{2} + c = \frac{x^2}{14} + c$$

(e) If $y = \frac{x}{7}$ then $\int \frac{x}{7}dx = \frac{x^2}{14} + c$

Notice that $\frac{x}{7} = \frac{1}{7}x$ and so you can use rule (2) with $a = \frac{1}{7}$ and $n = 1$ to show that:

$$\int \frac{x}{7}dx = \int \frac{1}{7}x^1dx = \frac{1}{7}\frac{x^2}{2} + c = \frac{x^2}{14} + c$$

You should also notice that this question is identical to the previous question.

(f) If $y = \frac{3x}{7}$ then $\int \frac{3x}{7} dx = \frac{3x^2}{14} + c$

Notice that $\frac{3x}{7} = \frac{3}{7}x$ and so you can use rule (2) with $a = \frac{3}{7}$ and $n = 1$ to show that:

$$\int \frac{3x}{7} dx = \int \frac{3}{7} x^1 dx = \frac{3}{7}\frac{x^2}{2} + c = \frac{3x^2}{14} + c$$

(g) If $y = -7x^2$ then $\int -7x^2 dx = -\frac{7x^3}{3} + c$

You can use rule (2) with $a = -7$ and $n = 2$ to show that:

$$\int -7x^2 dx = -7\frac{x^3}{3} + c = -\frac{7x^2}{3} + c$$

(h) If $y = 7x^3$ then $\int 7x^3 dx = \frac{7x^4}{4} + c$

You can use rule (2) with $a = 7$ and $n = 3$ to show that:

$$\int 7x^3 dx = 7\frac{x^4}{4} + c = \frac{7x^4}{4} + c$$

(i) If $y = -7x^{-2}$ then $\int -7x^{-2} dx = \frac{7}{x} + c$

You can use rule (2) with $a = -7$ and $n = -2$ to show that:

$$\int -7x^{-2} dx = -7\frac{x^{-1}}{-1} + c = 7x^{-1} + c = \frac{7}{x} + c$$

(j) If $y = -\frac{7}{x^2}$ then $\int -\frac{7}{x^2} dx = \frac{7}{x} + c$

Notice that $-\frac{7}{x^2} = -7x^{-2}$ and so you can use rule (2) with $a = -7$ and $n = -2$ to show that:

$$\int -\frac{7}{x^2} dx = \int -7x^{-2} dx = -7\frac{x^{-1}}{-1} + c = 7x^{-1} + c = \frac{7}{x} + c$$

You should also notice that this is exactly the same as the previous question.

(k) If $y = -\frac{7}{3x^2}$ then $\int -\frac{7}{3x^2} dx = \frac{7}{3x} + c$

Notice that $-\frac{7}{3x^2} = -\frac{7}{3}x^{-2}$ and so you can use rule (2) with $a = -\frac{7}{3}$ and $n = -2$:

$$\int -\frac{7}{3x^2} dx = \int -\frac{7}{3}x^{-2}dx = -\frac{7}{3}\left(\frac{x^{-1}}{-1}\right) + c = \frac{7}{3}x^{-1} + c = \frac{7}{3x} + c$$

(l) If $y = \frac{7}{x}$ then $\int \frac{7}{x} dx = 7\ln x + c$

Notice that $\frac{7}{x} = 7x^{-1}$ and so you can use rule (3) with $a = 7$ to show that:

$$\int \frac{7}{x} dx = \int 7x^{-1}dx = 7\ln x + c$$

(m) If $y = (7x^3)^2$ then $\int (7x^3)^2 dx = 7x^7 + c$

Notice that $(7x^3)^2 = 49x^6$ and so you can use rule (2) with $a = 49$ and $n = 6$ to show that:

$$\int (7x^3)^2 dx = \int 49x^6 dx = 49\frac{x^7}{7} + c = 7x^7 + c$$

(n) If $y = (-7x^3)^2$ then $\int (-7x^3)^2 dx = 7x^7 + c$

Notice that $(-7x^3)^2 = 49x^6$ and so you can use rule (2) with $a = 49$ and $n = 6$ to show that:

$$\int (7x^3)^2 dx = \int 49x^6 dx = 49\frac{x^7}{7} + c = 7x^7 + c$$

Which is the same as the previous question.

(o) If $y = \sqrt{x}$ then $\int \sqrt{x} dx = \frac{2x^{3/2}}{3} + c$

Notice that $\sqrt{x} = x^{1/2}$ and so you can use rule (2) with $a = 1$ and $n = \frac{1}{2}$ to show that:

$$\int \sqrt{x} dx = \int x^{1/2} dx = \frac{x^{3/2}}{3/2} + c = \frac{2x^{3/2}}{3} + c$$

(p) If $y = 7\sqrt{x}$ then $\int 7\sqrt{x}dx = \frac{14x^{3/2}}{3} + c$

Notice that $7\sqrt{x} = 7x^{1/2}$ and so you can use rule (2) with $a = 7$ and $n = \frac{1}{2}$ to show that:

$$\int 7\sqrt{x}dx = \int 7x^{1/2}dx = \frac{7x^{3/2}}{3/2} + c = \frac{14x^{3/2}}{3} + c$$

(q) If $y = \frac{1}{7x}$ then $\int \frac{1}{7x}dx = \frac{1}{7}\ln x + c$

Notice that $\frac{1}{7x} = \frac{1}{7}x^{-1}$ and so you can use rule (3) with $a = \frac{1}{7}$ to show that:

$$\int \frac{1}{7x}dx = \int \frac{1}{7}x^{-1}dx = \frac{1}{7}\ln x + c$$

(r) If $y = \frac{1}{x^7}$ then $\int \frac{1}{x^7}dx = -\frac{1}{6x^6} + c$

Notice that $\frac{1}{x^7} = x^{-7}$ and so you can use rule (2) with $a = 1$ and $n = -7$ to show that:

$$\int \frac{1}{x^7}dx = \int x^{-7}dx = \frac{x^{-6}}{-6} + c = -\frac{1}{6x^6} + c$$

(s) If $y = -\frac{7}{\sqrt{x}}$ then $\int -\frac{7}{\sqrt{x}}dx = -14\sqrt{x} + c$

Notice that $-\frac{7}{\sqrt{x}} = -7x^{-1/2}$ and so you can use rule (2) with $a = -7$ and $n = -\frac{1}{2}$:

$$\int -\frac{7}{\sqrt{x}}dx = \int -7x^{-1/2}dx = -7\frac{x^{1/2}}{1/2} + c = -14x^{1/2} + c = -14\sqrt{x} + c$$

(t) If $y = -\frac{1}{7\sqrt[3]{x}}$ then $\int -\frac{1}{7\sqrt[3]{x}}dx = -\frac{3x^{2/3}}{14} + c$

Notice that $-\frac{1}{7\sqrt[3]{x}} = -\frac{1}{7}x^{-1/3}$ and so you can use rule (2) with $a = -\frac{1}{7}$ and $n = -\frac{1}{3}$:

$$\int -\frac{1}{7\sqrt[3]{x}}dx = \int -\frac{1}{7}x^{-1/3}dx = -\frac{1}{7}\left(\frac{x^{2/3}}{2/3}\right) + c = -\frac{1}{7}\left(\frac{3x^{2/3}}{2}\right) + c = -\frac{3x^{2/3}}{14} + c$$

2.

(a) If $y = 7\sin x$ then $\int 7\sin x dx = -7\cos x + c$

You can use rule (4) with $a = 7$ and $k = 1$ to show that:

$$\int 7\sin x dx = -7\cos x + c$$

(b) If $y = \cos(7x)$ then $\int \cos(7x)dx = \frac{1}{7}\sin(7x) + c$

You can use rule (5) with $a = 1$ and $k = 7$ to show that:

$$\int \cos(7x)dx = \frac{1}{7}\sin(7x) + c$$

(c) If $y = 3\sin(7x)$ then $\int 3\sin(7x)dx = -\frac{3}{7}\cos(7x) + c$

You can use rule (4) with $a = 3$ and $k = 7$ to show that:

$$\int 3\sin(7x)dx = -\frac{3}{7}\cos(7x) + c$$

(d) If $y = \sin\left(\frac{x}{7}\right)$ then $\int \sin\left(\frac{x}{7}\right)dx = -7\cos\left(\frac{x}{7}\right) + c$

You can use rule (4) with $a = 1$ and $k = \frac{1}{7}$ to show that:

$$\int \sin\left(\frac{x}{7}\right)dx = \int \sin\left(\frac{1}{7}x\right)dx = -\frac{1}{1/7}\cos\left(\frac{x}{7}\right) + c = -7\cos\left(\frac{x}{7}\right) + c$$

(e) If $y = \frac{\sin x}{7}$ then $\int \frac{\sin x}{7}dx = -\frac{\cos x}{7} + c$

You can use rule (4) with $a = \frac{1}{7}$ and $k = 1$ to show that:

$$\int \frac{\sin x}{7}dx = \int \frac{1}{7}\sin x dx = -\frac{1}{7}\cos x + c = -\frac{\cos x}{7} + c$$

(f) If $y = 7\cos(-x)$ then $\int 7\cos(-x)dx = -7\sin(-x) + c$

You can use rule (5) with $a = 7$ and $k = -1$ to show that:

$$\int 7\cos(-x)dx = -7\sin(-x) + c$$

(g) If $y = -\cos\left(\frac{x}{7}\right)$ then $\int -\cos\left(\frac{x}{7}\right)dx = -7\sin\left(\frac{x}{7}\right) + c$

You can use rule (5) with $a = -1$ and $k = \frac{1}{7}$ to show that:

$$\int -\cos\left(\frac{x}{7}\right)dx = \int -\cos\left(\frac{1}{7}x\right)dx = -\frac{1}{1/7}\sin\left(\frac{1}{7}x\right) + c = -7\sin\left(\frac{x}{7}\right) + c$$

(h) If $y = \frac{-\cos(-3x)}{7}$ then $\int \frac{-\cos(-3x)}{7}dx = \frac{\sin(-3x)}{21} + c$

You can use rule (5) with $a = -\frac{1}{7}$ and $k = -3$ to show that:

$$\int \frac{-\cos(-3x)}{7}dx = \int -\frac{1}{7}\cos(-3x)dx = -\frac{1}{7}\frac{\sin(-3x)}{-3} + c = \frac{\sin(-3x)}{21} + c$$

(i) If $y = \cos\left(\frac{3x}{7}\right)$ then $\int \cos\left(\frac{3x}{7}\right)dx = \frac{7\sin\left(\frac{3x}{7}\right)}{3} + c$

You can use rule (5) with $a = 1$ and $k = \frac{3}{7}$ to show that:

$$\int \cos\left(\frac{3x}{7}\right)dx = \int \cos\left(\frac{3}{7}x\right)dx = \frac{\sin\left(\frac{3}{7}x\right)}{3/7} + c = \frac{7\sin\left(\frac{3x}{7}\right)}{3} + c$$

(j) If $y = -\frac{\sin(x/7)}{3}$ then $\int -\frac{\sin(x/7)}{3}dx = \frac{7\cos(x/7)}{3} + c$

You can use rule (4) with $a = -\frac{1}{3}$ and $k = \frac{1}{7}$ to show that:

$$\int -\frac{\sin(x/7)}{3}dx = \int -\frac{1}{3}\sin\left(\frac{1}{7}x\right)dx = -\frac{1}{3}\left(\frac{-\cos(x/7)}{1/7}\right) + c = \frac{7\cos(x/7)}{3} + c$$

(k) If $y = \frac{3\cos x}{7}$ then $\int \frac{3\cos x}{7} dx = \frac{3\sin x}{7} + c$

You can use rule (5) with $a = \frac{3}{7}$ and $k = 1$ to show that:

$$\int \frac{3\cos x}{7} dx = \int \frac{3}{7}\cos x dx = \frac{3}{7}\sin x + c = \frac{3\sin x}{7} + c$$

(l) If $y = 7e^x$ then $\int 7e^x dx = 7e^x + c$

You can use rule (6) with $a = 7$ and $k = 1$ to show that:

$$\int 7e^x dx = 7e^x + c$$

(m) If $y = e^{-7x}$ then $\int e^{-7x} dx = -\frac{e^{-7x}}{7} + c$

You can use rule (6) with $a = 1$ and $k = -7$ to show that:

$$\int e^{-7x} dx = -\frac{e^{-7x}}{7} + c$$

(n) If $y = \frac{e^{-x}}{7}$ then $\int \frac{e^{-x}}{7} dx = -\frac{e^{-x}}{7} + c$

You can use rule (6) with $a = \frac{1}{7}$ and $k = -1$ to show that:

$$\int \frac{e^{-x}}{7} dx = \int \frac{1}{7} e^{-x} dx = \frac{1}{7}\frac{e^{-x}}{-1} + c = -\frac{e^{-x}}{7} + c$$

(o) If $y = e$ then $\int e dx = ex + c$

Remember that e is just a number (a constant) and so you can use rule (1) with $a = e$:

$$\int e dx = ex + c$$

(p) If $y = \frac{1}{7e^x}$ then $\int \frac{1}{7e^x} dx = -\frac{1}{7e^x} + c$

Notice that $\frac{1}{7e^x} = \frac{1}{7}e^{-x}$ and so you can use rule (6) with $a = \frac{1}{7}$ and $k = -1$ to show that:

$$\int \frac{e^{-x}}{7} dx = \int \frac{1}{7}e^{-x} dx = \frac{1}{7}\frac{e^{-x}}{-1} + c = -\frac{e^{-x}}{7} + c$$

This is exactly the same as 2(n).

(q) If $y = \frac{7}{e^{3x}}$ then $\int \frac{7}{e^{3x}} dx = -\frac{7}{3e^{3x}} + c$

Notice that $\frac{7}{e^{3x}} = 7e^{-3x}$ and so you can use rule (6) with $a = 7$ and $k = -3$ to show that:

$$\int \frac{7}{e^{3x}} dx = \int 7e^{-3x} dx = 7\left(\frac{e^{-3x}}{-3}\right) + c = -\frac{7}{3e^{3x}} + c$$

(r) If $y = (e^x)^2$ then $\int (e^x)^2 dx = \frac{e^{2x}}{2} + c$

Notice that $(e^x)^2 = e^{2x}$ and so you can use rule (6) with $a = 1$ and $k = 2$ to show that:

$$\int (e^x)^2 dx = \int e^{2x} dx = \frac{e^{2x}}{2} + c$$

(s) If $y = \left(\frac{1}{e^x}\right)^2$ then $\int \left(\frac{1}{e^x}\right)^2 dx = -\frac{1}{2e^{2x}} + c$

Notice that $\left(\frac{1}{e^x}\right)^2 = (e^{-x})^2 = e^{-2x}$ and so you can use rule (6) with $a = 1$ and $k = -2$:

$$\int \left(\frac{1}{e^x}\right)^2 dx = \int e^{-2x} dx = \frac{e^{-2x}}{-2} + c = -\frac{1}{2e^{2x}} + c$$

(t) If $y = \ln 7$ then $\int \ln 7 dx = (\ln 7)x + c$

Remember that $\ln 7$ is just a number (a constant) and so you can use rule (1) with $a = \ln 7$:

$$\int \ln 7 dx = (\ln 7)x + c$$

3.

(a) If $y = 7x^2 + x - \frac{1}{3}$ then $\int\left(7x^2 + x - \frac{1}{3}\right)dx = \frac{7x^3}{3} + \frac{x^2}{2} - \frac{x}{3} + c$

Using term-by-term integration:

1st term: Rule (2) with $a = 7$ and $n = 2$

2nd term: Rule (2) with $a = 1$ and $n = 1$

3rd term: Rule (1) with $a = -\frac{1}{3}$

So $\int\left(7x^2 + x - \frac{1}{3}\right)dx = \frac{7x^3}{3} + \frac{x^2}{2} - \frac{x}{3} + c$

(b) If $y = x^3 - 3x^2 - 5x + 1$ then $\int\left(x^3 - 3x^2 - 5x + 1\right)dx = \frac{x^4}{4} - x^3 - \frac{5x^2}{2} + x + c$

Using term-by-term integration:

1st term: Rule (2) with $a = 1$ and $n = 3$

2nd term: Rule (2) with $a = -3$ and $n = 2$

3rd term: Rule (2) with $a = -5$ and $n = 1$

4th term: Rule (1) with $a = 1$

So $\int\left(x^3 - 3x^2 - 5x + 1\right)dx = \frac{x^4}{4} - \frac{3x^3}{3} - \frac{5x^2}{2} + x + c = \frac{x^4}{4} - x^3 - \frac{5x^2}{2} + x + c$

(c) If $y = \sin x + \cos x$ then $\int(\sin x + \cos x)dx = -\cos x + \sin x + c$

Using term-by-term integration:

1st term: Rule (4) with $a = 1$ and $k = 1$

2nd term: Rule (5) with $a = 1$ and $k = 1$

So $\int(\sin x + \cos x)dx = -\cos x + \sin x + c$

(d) If $y = 7\cos x - \sin(7x)$ then $\int(7\cos x - \sin(7x))dx = 7\sin x + \frac{\cos(7x)}{7} + c$

Using term-by-term integration:

1st term: Rule (5) with $a = 7$ and $k = 1$

2nd term: Rule (4) with $a = -1$ and $k = 7$

So $\int(7\cos x - \sin(7x))dx = 7\sin x - \left(-\frac{\cos(7x)}{7}\right) + c = 7\sin x + \frac{\cos(7x)}{7} + c$

(e) If $y = e^x - e^{-x} + \frac{1}{2}$ then $\int\left(e^x - e^{-x} + \frac{1}{2}\right)dx = e^x + e^{-x} + \frac{x}{2} + c$

Using term-by-term integration:

1st term: Rule (6) with $a = 1$ and $k = 1$

2nd term: Rule (6) with $a = 1$ and $k = -1$

3rd term: Rule (1) with $a = \frac{1}{2}$

So $\int\left(e^x - e^{-x} + \frac{1}{2}\right)dx = e^x - \left(-e^{-x}\right) + \frac{x}{2} + c = e^x + e^{-x} + \frac{x}{2} + c$

(f) If $y = 7 - \ln 3$ then $\int(7 - \ln 3)dx = (7 - \ln 3)x + c$

Using term-by-term integration:

1st term: Rule (1) with $a = 7$

2nd term: Rule (1) with $a = -\ln 3$

So $\int(7 - \ln 3)dx = 7x - (\ln 3)x + c = (7 - \ln 3)x + c$

(g) If $y = \frac{x^2 + 2x + 2}{7}$ then $\int\left(\frac{x^2 + 2x + 2}{7}\right)dx = \frac{x^3}{21} + \frac{x^2}{7} + \frac{2x}{7} + c$

Notice that $\frac{x^2 + 2x + 2}{7} = \frac{x^2}{7} + \frac{2x}{7} + \frac{2}{7}$ so use term-by-term integration:

1st term: Rule (2) with $a = \frac{1}{7}$ and $n = 2$

2nd term: Rule (2) with $a = \frac{2}{7}$ and $n = 1$

3rd term: Rule (1) with $a = \frac{2}{7}$

So $\int\left(\frac{x^2 + 2x + 2}{7}\right)dx = \int\left(\frac{1}{7}x^2 + \frac{2}{7}x + \frac{2}{7}\right)dx = \frac{1}{7}\frac{x^3}{3} + \frac{2}{7}\frac{x^2}{2} + \frac{2}{7}x + c = \frac{x^3}{21} + \frac{x^2}{7} + \frac{2x}{7} + c$

(h) If $y = (x+1)^2$ then $\int (x+1)^2 dx = \frac{x^3}{3} + x^2 + x + c$

Notice that $(x+1)^2 = x^2 + 2x + 1$ so use term-by-term integration:

1st term: Rule (2) with $a = 1$ and $n = 2$

2nd term: Rule (2) with $a = 2$ and $n = 1$

3rd term: Rule (1) with $a = 1$

So $\int (x+1)^2 dx = \int (x^2 + 2x + 1) dx = \frac{x^3}{3} + \frac{2x^2}{2} + x + c = \frac{x^3}{3} + x^2 + x + c$

(i) If $y = (e^x - e^{-x})^2 + 2$ then $\int (e^x - e^{-x})^2 + 2\, dx = \frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} + c$

Notice that $(e^x - e^{-x})^2 = e^{2x} + e^{-2x} - 2$ so $y = e^{2x} + e^{-2x}$ and you may use term-by-term integration:

1st term: Rule (6) with $a = 1$ and $k = 2$

2nd term: Rule (6) with $a = 1$ and $k = -2$

So $\int e^{2x} + e^{-2x} dx = \frac{1}{2} e^{2x} - \frac{1}{2} e^{-2x} + c$

(j) If $y = \cos(2x) - \sin(2x)$ then $\int \cos(2x) - \sin(2x) dx = \frac{1}{2}\sin(2x) + \frac{1}{2}\cos(2x) + c$

Using term-by-term integration:

1st term: Rule (5) with $a = 1$ and $k = 2$

2nd term: Rule (4) with $a = -1$ and $k = 2$

So $\int \cos(2x) - \sin(2x) dx = \frac{1}{2}\sin(2x) + \frac{1}{2}\cos(2x) + c$

(k) If $y = \frac{e^{2x}+e^{-2x}}{3}$ then $\int \frac{e^{2x}+e^{-2x}}{3}\,dx = \frac{e^{2x}-e^{-2x}}{6}+c$

You can rewrite y as $\frac{1}{3}e^{2x}+\frac{1}{3}e^{-2x}$ and then use term-by-term integration:

1st term: Rule (6) with $a=\frac{1}{3}$ and $k=2$

2nd term: Rule (6) with $a=\frac{1}{3}$ and $k=-2$

So $\int \frac{1}{3}e^{2x}+\frac{1}{3}e^{-2x}\,dx = \frac{1}{3}\frac{1}{2}e^{2x}+\frac{1}{3}\frac{1}{-2}e^{-2x}+c = \frac{1}{6}e^{2x}-\frac{1}{6}e^{-2x}+c = \frac{e^{2x}-e^{-2x}}{6}+c$

(l) If $y = \frac{x^2+2x-1}{x^2}$ then $\int \frac{x^2+2x-1}{x^2}\,dx = x+2\ln(x)+\frac{1}{x}+c$

You can rewrite y as $1+\frac{2}{x}-\frac{1}{x^2}$ and then use term-by-term integration:

1st term: Rule (1) with $a=1$

2nd term: Rule (3) with $a=2$

3rd term: Rule (2) with $a=-1$ and $k=-2$

So $\int 1+\frac{2}{x}-\frac{1}{x^2}\,dx = x+2\ln(x)+\frac{1}{x}+c$