Learning Enhancement Team

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Model Answers: Integrating $y = ax^n$

Integrating using the Power Rule study guide

You can do all the questions on this sheet using the rule:

$$\int ax^n dx = \frac{a}{n+1}x^{n+1} + c \qquad \text{for} \qquad n \neq -1$$

$$\int ax^{-1} dx = a\ln x + c \qquad \text{for} \qquad n = -1$$

See the study guide: *Integrating using the Power Rule* for help with this.

You might need to rearrange the integrals so that they are in the form of the two rules above, only then can you try to identify the a and the n.

1.

a) $x^4 + c$

The question fits the pattern $\int ax^n dx$ with $a = 4$ and $n = 3$ and so:

$$\int ax^n dx = \frac{4}{3+1}x^{3+1} + c = \frac{4}{4}x^4 + c = x^4 + c$$

b) $2x^3 + c$

The question fits the pattern $\int ax^n dx$ with $a = 6$ and $n = 2$ and so:

$$\int ax^n dx = \frac{6}{2+1}x^{2+1} + c = \frac{6}{3}x^3 + c = 2x^3 + c$$

c) $\frac{x^3}{18}+c$

By rewriting the integral as:

$$\int \frac{x^2}{6}dx = \int \frac{1}{6}x^2 dx$$

You can see that it fits the pattern $\int ax^n dx$ with $a=\frac{1}{6}$ and $n=2$ and so:

$$\int ax^n dx = \frac{1/6}{2+1}x^{2+1}+c = \frac{1}{3}\times\frac{1}{6}x^3+c = \frac{1}{18}x^3+c$$

d) $2x^2+c$

By rewriting the integral as:

$$\int 4x dx = \int 4x^1 dx$$

You can see that it fits the pattern $\int ax^n dx$ with $a=4$ and $n=1$ and so:

$$\int ax^n dx = \frac{4}{1+1}x^{1+1} = \frac{4}{2}x^2 = 2x^2$$

e) $x^5-\frac{3}{2}x^2+c$

The integral can be split up into two separate integrals:

$$\int 5x^4-3x dx = \int 5x^4 dx - \int 3x dx$$
$$= \frac{5}{4+1}x^{4+1}-\frac{3}{1+1}x^{1+1}+c$$
$$= \frac{5}{5}x^5-\frac{3}{2}x^2+c$$
$$= x^5-\frac{3}{2}x^2+c$$

f) $4x + c$

By rewriting the integral as:

$$\int 4dx = \int 4x^0 dx$$

you can see that it fits the pattern $\int ax^n dx$ with $a = 4$ and $n = 0$ and so:

$$\int ax^n dx = \frac{4}{0+1}x^{0+1} + c = \frac{4}{1}x^1 + c = 4x + c$$

g) $0.7x + c$

You could do this one in the same way as question f) or you can just notice that when integrating constants you may just multiply by *x* and add *c.*

h) $2\pi x + c$

You could do this one in the same way as question f) or you can just notice that when integrating constants you may just multiply by *x* and add *c.*

i) $-x^{-3} + c$ (You can also write the answer as $-\frac{1}{x^3} + c$)

You can rewrite the integral as:

$$\int \frac{3}{x^4} dx = \int 3x^{-4} dx$$

See the study guide: *Laws of Indices* if you find this difficult to understand.
Then you can see that it fits the pattern $\int ax^n dx$ with $a = 3$ and $n = -4$ and so:

$$\int ax^n dx = \frac{3}{-4+1}x^{-4+1} + c = \frac{3}{-3}x^{-3} + c = -x^{-3} + c$$

j) $-x^{-2}+c$ (You can also write the answer as $-\frac{1}{x^2}+c$)

You can rewrite the integral as:

$$\int \frac{2}{x^3}dx = \int 2x^{-3}dx$$

See the study guide: *Laws of Indices* if you find this difficult to understand.
You can see that it fits the pattern $\int ax^n dx$ with $a=2$ and $n=-3$ and so:

$$\int ax^n dx = \frac{2}{-3+1}x^{-3+1}+c = \frac{2}{-2}x^{-2}+c = -x^{-2}+c$$

k) $-x^{-1}+c$ (You can also write the answer as $-\frac{1}{x}+c$)

You can rewrite the integral as:

$$\int \frac{1}{x^2}dx = \int x^{-2}dx$$

See the study guide: *Laws of Indices* if you find this difficult to understand.
You can see that it fits the pattern $\int ax^n dx$ with $a=1$ and $n=-2$ and so:

$$\int ax^n dx = \frac{1}{-2+1}x^{-2+1}+c = \frac{1}{-1}x^{-1}+c = -x^{-1}+c$$

l) $\ln x + c$

You can rewrite the integral as:

$$\int \frac{1}{x}dx = \int x^{-1}dx$$

See the study guide: *Laws of Indices* if you find this difficult to understand.
You can see that it fits the pattern $\int ax^n dx$ with $a=1$ and $n=-1$.

Take care! This is the special case when the normal integration power rule does not work. So:

$$\int x^{-1}dx = \ln x + c$$

2.

All your answers to question 1 (except the last one) are in the form $ax^n + c$ and so can be differentiated using the power rule for differentiation. See the study guide *Differentiating using the Power Rule* if you need help with this. The answer to question l) is differentiated using the rule for differentiating logs found in the study guide *Differentiating Basic Functions.*

Once you have differentiated all your answers you should notice that they are the same as the original integrands (the function inside the integral) in question 1.

3.

This question involves differential equations and you are asked to find the solutions of these equations. In other words you integrate them as before and notice that dy/dx simply integrates to y plus a constant. In integration questions you only need to have one constant per question as you can combine them all together into a single constant. Here they are collected and added to the right-hand side of the answer

a) $y = x^4 + c$

Integrating both sides of the equation gives:

$$\int \frac{dy}{dx} dx = \int 4x^3 dx$$

Integration is the inverse of differentiation and so if you differentiate y you get dy/dx and if you integrate this you get back to y and so:

Left-hand side $\int \frac{dy}{dx} dx = y$

Right-hand side $\int 4x^3 dx = \frac{4}{3+1} x^{3+1} + c = x^4 + c$

You can check your answer by differentiating both sides of this equation and you should end up with the original differential equation.

b) $y = 4x + c$

Integrating both sides of the equation gives:

Left-hand side $\int \frac{dy}{dx} dx = y$

Right-hand side $\int 4dx = \frac{4}{1} x^{0+1} + c = 4x + c$

You can check your answer by differentiating both sides of this equation and you should end up with the original differential equation.

c) $y = \frac{x^2}{6} + c$

Integrating both sides of the equation gives:

Left-hand side $\int \frac{dy}{dx} dx = y$

Right-hand side $\int \frac{x}{3} dx = \int \frac{1}{3} x dx = \frac{1}{3} \times \frac{1}{2} x^{1+1} + c = \frac{1}{6} x^2 + c$

You can check your answer by differentiating both sides of this equation and you should end up with the original differential equation.

d) $y = \frac{4}{3} \ln x + c$

Integrating both sides of the equation gives:

Left-hand side $\int \frac{dy}{dx} dx = y$

Right-hand side $\int \frac{4}{3x} dx = \int \frac{4}{3} \frac{1}{x} dx = \frac{4}{3} \ln x + c$

You can check your answer by differentiating both sides of this equation and you should end up with the original differential equation.

4. Remember to check your answers by differentiating them. Do not be confused by the different variables.

a) $\lambda^4 + c$

You can see that it fits the pattern $\int ax^n dx$ with $\lambda = x$, $a = 4$ and $n = 3$ and so:

$$\int 4\lambda^3 d\lambda = \frac{4}{3+1}\lambda^{3+1} + c = \frac{4}{4}\lambda^4 + c = \lambda^4 + c$$

b) $2A^{\frac{3}{2}} + c$

You can rewrite the integral as:

$$\int 3\sqrt{A}\, dA = \int 3A^{\frac{1}{2}} dA$$

See the study guide: *Laws of Indices* if you find this difficult to understand.

You can see that it fits the pattern $\int ax^n dx$ with $x = A$, $a = 3$ and $n = \frac{1}{2}$ and so:

$$\int 3A^{\frac{1}{2}} dA = \frac{3}{1/2+1}A^{\frac{1}{2}+1} + c = 3 \div \frac{3}{2}A^{\frac{3}{2}} + c = 2A^{\frac{3}{2}} + c$$

c) $10A^{\frac{1}{2}} + c$ (The answer can be written as $10\sqrt{A} + c$)

You can rewrite the integral as:

$$\int \frac{5}{\sqrt{A}} dA = \int 5A^{-\frac{1}{2}} dA$$

See the study guide: *Laws of Indices* if you find this difficult to understand.

You can see that it fits the pattern $\int ax^n dx$ with $x = A$, $a = 5$ and $n = -\frac{1}{2}$ and so:

$$\int 5A^{-\frac{1}{2}} dA = \frac{5}{-1/2+1}A^{-\frac{1}{2}+1} + c = 5 \div \frac{1}{2}A^{\frac{1}{2}} + c = 10A^{\frac{1}{2}} + c$$

d) $-\frac{2}{3}s^{-1}+c$ (The answer can be written as $-\frac{2}{3s}+c$)

You can rewrite the integral as:

$$\int \frac{2}{3s^2}ds = \int \frac{2}{3}s^{-2}ds$$

See the study guide: *Laws of Indices* if you find this difficult to understand.

You can see that it fits the pattern $\int ax^n dx$ with $x = s$, $a = \frac{2}{3}$ and $n = -2$ and so:

$$\int \frac{2}{3}s^{-2}ds = \frac{2}{3}\frac{1}{-2+1}s^{-2+1} + c = -\frac{2}{3}s^{-1} + c$$

e) $5\ln x - \frac{1}{2}x^2 + c$

By dividing each of the two terms on the numerator of the integrand by *x*, you get:

$$\int \frac{5-x^2}{x}dx = \int 5x^{-1} - xdx$$

The integral can be split up into two separate integrals:

$$\int 5x^{-1} - 3dx = \int 5x^{-1}dx - \int 3dx$$

$$= 5\ln x - \frac{3}{0+1}x^{0+1} + c$$

$$= 5\ln x - 3x + c$$

f) $\frac{1}{3}v^3 - 3v^2 + 9v + c$

Expanding the brackets gives you:

$$\int (v-3)^2\,dv = \int v^2 - 6v + 9\,dv$$

If you find opening brackets difficult you can read the study guide: *Opening Brackets*.

The integral can be split up into three separate integrals:

$$\int v^2 - 6v + 9dv = \int v^2 dv - \int 6v dv + \int 9dv$$

$$= \frac{v^{2+1}}{2+1} - 6\frac{v^{1+1}}{1+1} + 9\frac{v^{0+1}}{0+1} + c$$

$$= \frac{v^3}{3} - 3v^2 + 9v + c$$

g) a constant *c*

The fractions add to give 0 and so the answer is just the constant of integration.