Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Worksheet: What is Integration?

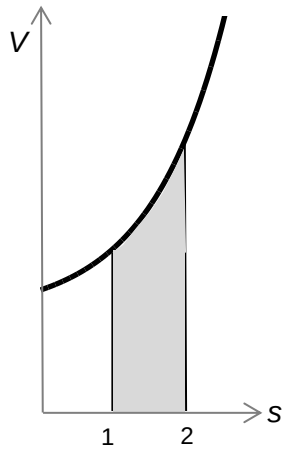
In questions 1 to 3 of this worksheet you will explore how definite integrals represent areas under graphs and then you will approximate one of those areas. Then in questions 4 to 6 you will check integration by differentiating and, since you know that integration is the inverse of differentiation, you will be able to use it to evaluate the exact area from question 3.

Model solutions to this sheet

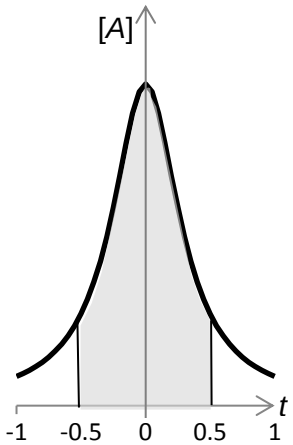
What is Integration? study guide

1. Write down the definite integral that represents the area under the following graphs (you do not have to evaluate the answer):

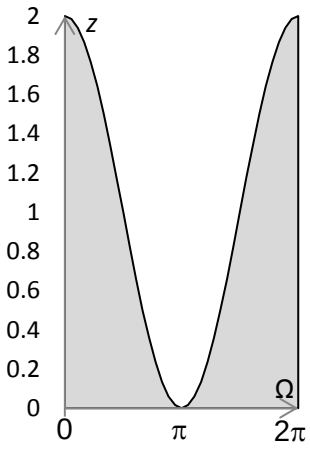
(a) $V = 4 + e^s$



(b) $[A] = (1 + 10t^2)^{-1}$



(c) $z = 1 + \cos(\Omega)$



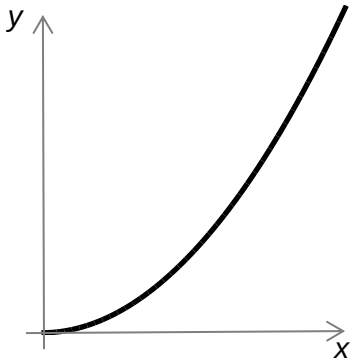
2. Sketch graphs to show the areas defined by the following definite integrals. (All of the functions have graphs that are straight lines, see the study guide: *Sketching Straight Lines* for help in sketching them).

(a) $\int_0^4 s\,ds$

(b) $\int_0^3 9\,d\lambda$

(c) $\int_{-5}^{-2} (7 - q)\,dq$

3. The graph to the right shows $y = x^2$ for $0 \le x \le 4$:



(a) Write down the definite integral that gives the area between the lines $x = 0$ and $x = 3$, the graph of the function and the *x*-axis.

(b) Shade this area on the graph.

(c) Approximate this area by working out the area of a suitable triangle of base length 3 (remember that the area of a triangle is half the base multiplied by the height).

(d) Now use three triangles and two rectangles, all of base length 1, to approximate the area. Compare your result to part (c).

4. By differentiating the answers, show that the following indefinite integrals are correct. Here *c* is a constant.

(a) $\int 4x\,dx = 2x^2 + c$ (b) $\int 1\,ds = s + c$

(c) $\int 4u - 6\,du = 2u^2 - 6u + c$ (d) $\int y^{-1/2}\,dy = 2y^{1/2} + c$

5. Find $f(x)$ if:

(a) $\int f(x)\,dx = 2x^3 + 5$ (b) $\int f(x)\,dx = 2x^3 - 0.7889223$

(c) $\int f(x)\,dx = 3x^4$ (d) $\int f(x)\,dx = 3x^{4/3} + 4x - \frac{1}{4}$

6. Can you work out $\int x^2\,dx$? Have a guess and then test it by differentiating it back.

Once you are sure you have got it right, set the constant of integration *c* to 0 and substitute $x = 3$ into your result. This should give you the definite integral and the exact area in question 3. How does your result compare to the approximate areas you calculated in parts (c) and (d) of question 3?