Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model Answers: What is Integration?

What is Integration? study guide

1. This is an exercise in notation. You need to put the function represented by the graph between the integration sign and the appropriate "*dx*", you also need find the correct limits from the graph.

(a) $\int_{1}^{2} 4+e^{s}dS$

The function is $4+e^{s}$.

The shaded area on the graph is between $S=1$ and $S=2$ and so these are the lower and upper limits on the integral respectively.

The independent variable in the function is *S* and so there is a *dS* at the end.

(b) $\int_{-0.5}^{0.5} \left(1+10t^{2}\right)^{-1}dt$

The function is $\left(1+10t^{2}\right)^{-1}$.

The shaded area on the graph is between $t=-0.5$ and $t=0.5$ and so these are the lower and upper limits on the integral respectively.

The independent variable in the function is *t* and so there is a *dt* at the end.

(c) $\int_{0}^{2\pi} 1+\cos(\Omega)d\Omega$

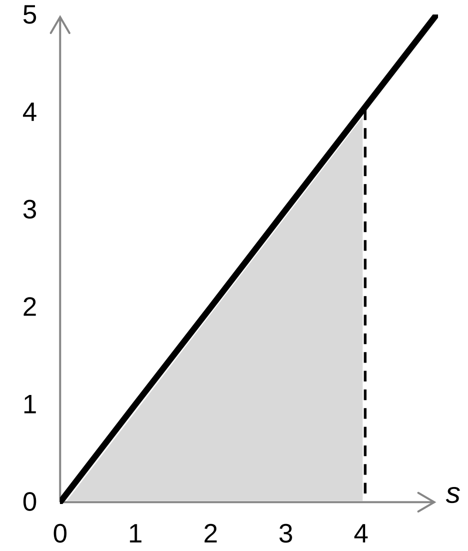
The function is $1+\cos(\Omega)$.

The shaded area on the graph is between $\Omega=0$ and $\Omega=2\pi$ and so these are the lower and upper limits on the integral respectively.

The independent variable in the function is Ω and so there is a $d\Omega$ at the end

2. You need to do the reverse of the previous question. You need to draw axes with the correct independent variable on the horizontal axis, get the function from between the integral sign and the appropriate "*dx*", then sketch the function (see the study guide: *Sketching Straight Lines*) and finally put vertical lines where the limits are and shade the area.

(a)



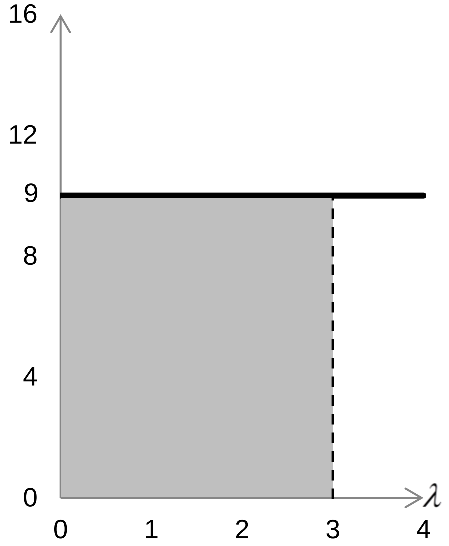
The *ds* at the end of the integral tells you that *s* is the independent variable and so this goes on the horizontal axis.

The function inside the integral is just *s* which means the graph must be $y = s$ which is a straight line going through the origin.

The limits are 0 and 4 and so the shaded area is between vertical lines at $s = 0$ and $s = 4$.

You shade in the area between the vertical lines and under the line.

(b)



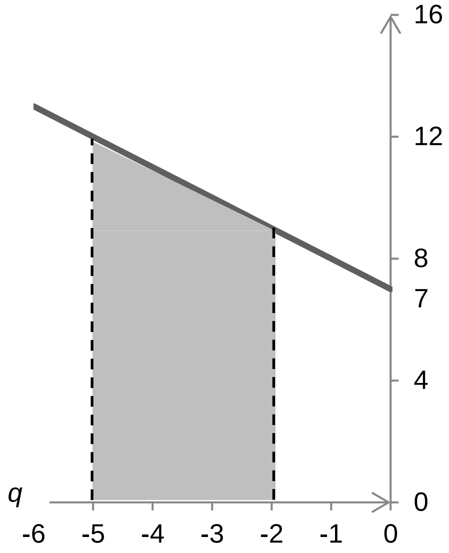
The $d\lambda$ at the end of the integral tells you that λ is the independent variable and so this goes on the horizontal axis

The function inside the integral is just 9 which means the graph must be $y = 9$ which is a horizontal line at 9.

The limits are 0 and 3 and so the shaded area is between vertical lines at $\lambda = 0$ and $\lambda = 3$.

You shade in the area between the vertical lines and under the line.

(c)



The *dq* at the end of the integral tells you that *q* is the independent variable and so this goes on the horizontal axis.

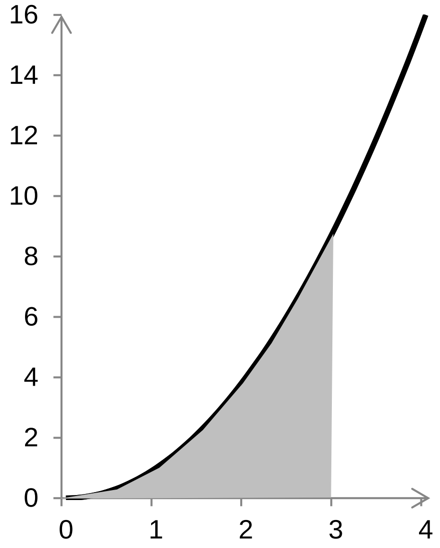
The function inside the integral is $7-q$ which means the graph must be $y=7-q$ which is a straight line of gradient -1 going through the point $(0,7)$.

The limits are -5 and -2 and so the shaded area is between vertical lines at $q=-2$ and $q=-5$.

You shade in the area between the vertical lines and under the straight line.

3.

(a) and (b)

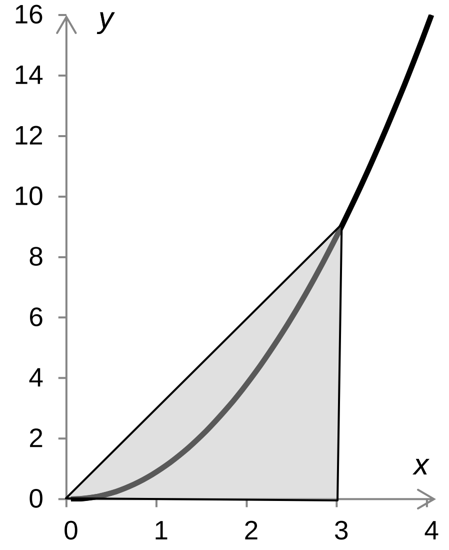


The required integral is $\int_0^3 x^2 dx$.

This is because the function represented by the curve is x^2 and the shaded area on the graph is between the limits $x=0$ and $x=3$.

The independent variable in the function is *x* and so there is a *dx* at the end

(c)



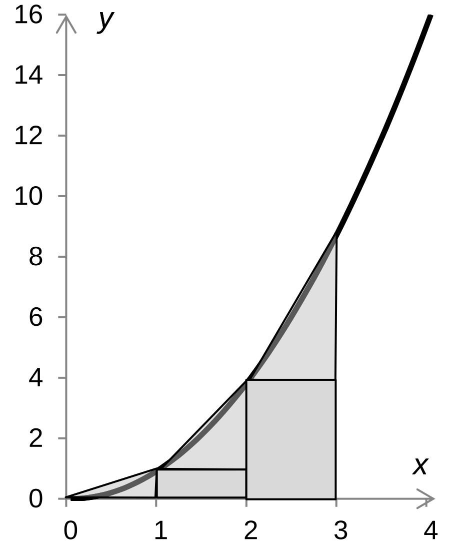
The approximate area is 13.5.

The shaded area is roughly triangular with a base of length 3.

The height of the triangle when $x = 3$ is $y = 3^2 = 9$ and so the area of the triangle is:

$$\frac{1}{2} \times \text{ base } \times \text{ height} = \frac{1}{2} \times 3 \times 9 = 13.5$$

(d)



The approximate area is .9.5.

Going from left to right:

The first triangle has area $\frac{1}{2} \times 1 \times 1^2 = 0.5$

The first rectangle has area $1 \times 1^2 = 1$

The second triangle has area $\frac{1}{2} \times 1 \times \left(2^2 - 1\right) = 1.5$

The second rectangle has area $1 \times 2^2 = 4$

The third triangle has area $\frac{1}{2} \times 1 \times \left(9 - 4\right) = 2.5$

Adding these together gives $0.5 + 1 + 1.5 + 4 + 2.5 = 9.5$ which ought to be a better approximation to the area than the one triangle approximation of 13.5. See question 6 for the actual area.

4. All these are in the form of a power of x and so you must use the power rule. See study guide: *Differentiating using the Power Rule*.

(a) Differentiating $2x^2 + c$ gives $4x$ which is the function in the integral.

(b) Differentiating $s + c$ gives 1 which is the function in the integral.

(c) Differentiating $2u^2 - 6u + c$ gives $4u - 6$ which is the function in the integral.

(d) Differentiating $2y^{1/2} + c$ gives $y^{-1/2}$ which is the function in the integral.

5. This is just like the previous question. Integration is the inverse of differentiation and so if you differentiate both sides of the equation you will get the answer.

(a) $f(x) = 6x^2$

Differentiating $\int f(x)dx$ gives $f(x)$. Differentiating $2x^3 + 5$ gives $6x^2$ and so $f(x) = 6x^2$.

(b) $f(x) = 6x^2$

Differentiating $\int f(x)dx$ gives $f(x)$. Differentiating $2x^3 - 0.7889223$ gives $6x^2$ and so $f(x) = 6x^2$. Even though the constant term is different from part (a) of this question, the answer is the same.

(c) $f(x) = 12x^3$

Differentiating $\int f(x)dx$ gives $f(x)$. Differentiating $3x^4$ gives $12x^3$ and so $f(x) = 12x^3$.

(d) $f(x) = 4x^{1/3} - 4$

Differentiating $\int f(x)dx$ gives $f(x)$. Differentiating $3x^{4/3} + 4x - \frac{1}{4}$ gives $4x^{1/3} + 4$ and so $f(x) = 4x^{1/3} + 4$.

6. $\int x^2 dx = \frac{1}{3}x^3 + c$

Check it by differentiating back. The only way to get back to x^2 is to differentiate x^3 but x^3 differentiates to $3x^2$ and so you need to divide by 3 to compensate. So:

$$\int_0^3 x^2 dx = 9$$

With $c = 0$ and $x = 3$, the integral becomes $\frac{1}{3}x^3 + 0 = \frac{1}{3} \cdot 3^3 = 9$.

You can see how, by breaking the area up into smaller parts, you closer to the actual area of the region which is 9.