University of East Anglia
STUDENT SUPPORT SERVICE

Learning Enhancement Team

Model Answers: The Differential Operator

These are the model answers for the worksheet that has questions on the differential operator.

The Differential Operator Study Guide

a. $2x$

When you apply the differential operator to an expression such as $2x$ you write:

$$\frac{d}{dx}(2x)$$

The result is the derivative of $2x$ with respect to x. This is found by using the power rule of differentiation (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)* and so:

$$\frac{d}{dx}(2x) = 2$$

b. $-\frac{1}{4}x^6$

When you apply the differential operator to an expression such as $-\frac{1}{4}x^6$ you write:

$$\frac{d}{dx}\left(-\frac{1}{4}x^6\right)$$

The result is the derivative of $-\frac{1}{4}x^6$ with respect to x. This is found by using the power rule of differentiation (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)* and so:

$$\frac{d}{dx}\left(-\frac{1}{4}x^6\right)=-\frac{3}{2}x^5$$

c. $-\frac{x^3}{3}$

You can rewrite $-\frac{x^3}{3}$ as $-\frac{1}{3}x^3$.

When you apply the differential operator to an expression such as $-\frac{1}{3}x^3$ you write:

$$\frac{d}{dx}\left(-\frac{1}{3}x^3\right)$$

The result is the derivative of $-\frac{1}{3}x^3$ with respect to *x*. This is found by using the power rule of differentiation (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)* and so:

$$\frac{d}{dx}\left(-\frac{1}{3}x^3\right)=-x^2$$

d. $-e^x$

When you apply the differential operator to an expression such as $-e^x$ you write:

$$\frac{d}{dx}\left(-e^x\right)$$

The result is the derivative of $-e^x$ with respect to *x*. This is found by using the rules for differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions)* and so:

$$\frac{d}{dx}\left(-e^x\right)=-e^x$$

e. $\cos(x)$

When you apply the differential operator to an expression such as $\cos(x)$ you write:

$$\frac{d}{dx}(\cos(x))$$

The result is the derivative of $\cos(x)$ with respect to *x*. This is found by using the rules for differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions)* and so:

$$\frac{d}{dx}(\cos(x)) = -\sin(x)$$

f. $\ln(3x)$

When you apply the differential operator to an expression such as $\ln(3x)$ you write:

$$\frac{d}{dx}(\ln(3x))$$

The result is the derivative of $\ln(3x)$ with respect to *x*. This is found by using the rules for differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions)* and so:

$$\frac{d}{dx}(\ln(3x)) = \frac{1}{x}$$

2. Now look at the following equations and apply the differential operator to them.

a. $y = 2x$

To apply the differential operator to the equation $y = 2x$ you apply the operator to **both sides** of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}(2x)$$

The left-hand side shows that the operator has been **applied to** *y* resulting in the derivative of *y* with respect to *x*. Combining this with the result from 1a gives:

$$\frac{dy}{dx} = 2$$

which is the same as the result you would get by a direct application of the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)*.

b. $y = -\frac{1}{4}x^6$

To apply the differential operator to the equation $y = -\frac{1}{4}x^6$ you apply the operator to **both sides** of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(-\frac{1}{4}x^6\right)$$

The left-hand side shows that the operator has been **applied to** *y* resulting in the derivative of *y* with respect to *x*. Combining this with the result from 1b gives:

$$\frac{dy}{dx} = -\frac{3}{2}x^5$$

which is the same as the result you would get by a direct application of the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*).

c. $y = -\frac{x^3}{3}$

To apply the differential operator to the equation $y = -\frac{x^3}{3}$ you apply the operator to **both sides** of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(-\frac{1}{3}x^3\right)$$

The left-hand side shows that the operator has been **applied to** *y* resulting in the derivative of *y* with respect to *x*. Combining this with the result from 1c gives:

$$\frac{dy}{dx} = -x^2$$

which is the same as the result you would get by a direct application of the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*).

d. $y = -e^x$

To apply the differential operator to the equation $y = -e^x$ you apply the operator to **both sides** of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(-e^x\right)$$

The left-hand side shows that the operator has been **applied to** *y* resulting in the derivative of *y* with respect to *x*. Combining this with the result from 1d gives:

$$\frac{dy}{dx} = -e^{x}$$

which is the same as the result you would get by a direct application of the rules of differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions*).

e. $y = \cos(x)$

To apply the differential operator to the equation $y = \cos(x)$ you apply the operator to **both sides** of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}(\cos(x))$$

The left-hand side shows that the operator has been **applied to** *y* resulting in the derivative of *y* with respect to *x*. Combining this with the result from 1e gives:

$$\frac{dy}{dx} = -\sin(x)$$

which is the same as the result you would get by a direct application of the rules of differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions*).

f. $y = \ln(3x)$

To apply the differential operator to the equation $y = \ln(3x)$ you apply the operator to **both sides** of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}(\ln(3x))$$

The left-hand side shows that the operator has been **applied to** *y* resulting in the derivative of *y* with respect to *x*. Combining this with the result from 1f gives:

$$\frac{dy}{dx} = \frac{1}{x}$$

which is the same as the result you would get by a direct application of the rules of differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions*).

Looking at your answers in question 1 and question 2 you can see that your answers in question 1 are expressions which are the derivative of the expressions given, whereas your answers in question 2 are the derivatives of the function *y* with respect to *x*.

3. Applying the differential operator **twice** to the equations gives on the left-hand side the **second derivative** of *y* with respect to *x* which is written as:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}$$ second derivative as two applications of the differential operator

a. $y=-2x^3$

Applying the differential operator once:

To apply the differential operator to the equation $y=-2x^3$ you apply the operator to both sides of the equals sign and write:

$$\frac{d}{dx}(y)=\frac{d}{dx}\left(-2x^3\right)$$

The left-hand side shows that the operator has been applied to *y* resulting in the derivative of *y* with respect to *x*.

$$\frac{dy}{dx}=-6x^2$$

which is the same as the result you would get by a direct application of the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*).

Applying the differential operator twice:

From above you found that after one application of the differential operator you get $dy/dx=-6x^2$. Applying the operator to both sides of this result gives:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d}{dx}\left(-6x^2\right)$$

The right-hand side can be found using the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*):

$$\frac{d}{dx}\left(-6x^2\right) = -12x$$

Combining these results gives the second derivative of $y = -2x^3$:

$$\frac{d^2y}{dx^2} = -12x$$

b. $y = \frac{2x^7}{7}$

Applying the differential operator once:

To apply the differential operator to the equation $y = \frac{2x^7}{7}$ you apply the operator to both sides of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(\frac{2}{7}x^7\right)$$

The left-hand side shows that the operator has been applied to *y* resulting in the derivative of *y* with respect to *x*.

$$\frac{dy}{dx} = 2x^6$$

which is the same as the result you would get by a direct application of the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*).

Applying the differential operator twice:

From above you found that after one application of the differential operator you get $dy/dx = 2x^6$. Applying the operator to both sides of this result gives:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2x^6\right)$$

The right-hand side can be found using the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*):

$$\frac{d}{dx}\left(2x^6\right) = 12x^5$$

Combining these results gives the second derivative of $y = \frac{2x^7}{7}$:

$$\frac{d^2y}{dx^2} = 12x^5$$

c. $y = e^{2x}$

Applying the differential operator once:

To apply the differential operator to the equation $y = e^{2x}$ you apply the operator to both sides of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}\left(e^{2x}\right)$$

The left-hand side shows that the operator has been applied to *y* resulting in the derivative of *y* with respect to *x*.

$$\frac{dy}{dx} = 2e^{2x}$$

which is the same as the result you would get by differentiating the exponential function (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions*).

Applying the differential operator twice:

From above you found that after one application of the differential operator you get $dy/dx = 2e^{2x}$. Applying the operator to both sides of this result gives:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(2e^{2x}\right)$$

The right-hand side can be found using the rule for differentiating the exponential function:

$$\frac{d}{dx}\left(2e^{2x}\right) = 4e^{2x}$$

Combining these results gives the second derivative of $y = e^{2x}$:

$$\frac{d^2y}{dx^2} = 4e^{2x}$$

d. $y = \sin(2x)$

Applying the differential operator once:

To apply the differential operator to the equation $y = \sin(2x)$ you apply the operator to both sides of the equals sign and write:

$$\frac{d}{dx}(y) = \frac{d}{dx}(\sin(2x))$$

The left-hand side shows that the operator has been applied to *y* resulting in the derivative of *y* with respect to *x*.

$$\frac{dy}{dx} = 2\cos(2x)$$

which is the same as the result you would get by differentiating the trigonometric function (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions*).

Applying the differential operator twice:

From above you found that after one application of the differential operator you get $dy/dx = 2\cos(2x)$. Applying the operator to both sides of this result gives:

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2\cos(2x))$$

The right-hand side can be found using the rules for differentiating the cosine function:

$$\frac{d}{dx}(2\cos(2x)) = -4\sin(2x)$$

Combining these results gives the second derivative of $y = \sin(2x)$:

$$\frac{d^2y}{dx^2} = -4\sin(2x)$$

4.

a. $y + x = 2x^2 + 3x^3$

Usually you would rearrange this function for *y* and then differentiate it, however you can apply the differential operator to both sides instead:

$$\frac{d}{dx}(y+x)=\frac{d}{dx}(2x^2+3x^3)$$

You apply the operator to each term in turn on each side:

$$\frac{d}{dx}(y)+\frac{d}{dx}(x)=\frac{d}{dx}(2x^2)+\frac{d}{dx}(3x^3)$$

The first term is the derivative of *y* with respect to *x* and the other terms can be found using the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)* and so:

$$\frac{dy}{dx}+1=4x+9x^2$$

You can rearrange the result to find the derivative of *y* with respect to *x*:

$$\frac{dy}{dx}=4x+9x^2-1$$

b. $y-2x^{-3}+\frac{x^{-8}}{4}=2x^3+x^4$

Usually you would rearrange this function for *y* and then differentiate it, however you can apply the differential operator to both sides instead:

$$\frac{d}{dx}\left(y-2x^{-3}+\frac{x^{-8}}{4}\right)=\frac{d}{dx}(2x^3+x^4)$$

You apply the operator to each term in turn on each side:

$$\frac{d}{dx}(y)+\frac{d}{dx}(-2x^{-3})+\frac{d}{dx}\left(\frac{1}{4}x^8\right)=\frac{d}{dx}(2x^3)+\frac{d}{dx}(x^4)$$

The first term is the derivative of *y* with respect to *x* and the other terms can be found using the power rule (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)* and so:

$$\frac{dy}{dx}+6x^{-4}+\frac{1}{2}x^7=6x^2+4x^3$$

You can rearrange the result to find the derivative of *y* with respect to *x*:

$$\frac{dy}{dx}=6x^2+4x^3-6x^{-4}-\frac{1}{2}x^7$$

5.

a. x^2y

To apply the differential operator to this expression you need to notice that this is the product of a function of *x* and *y* and *y* is a function of *x*, so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) in combination with the differential operator:

$$\frac{d}{dx}\left(x^2y\right)=x^2\frac{d}{dx}(y)+y\frac{d}{dx}\left(x^2\right)$$
$$=x^2\frac{dy}{dx}+y(2x)$$
$$=x^2\frac{dy}{dx}+2xy$$

b. $\frac{x}{y}$

To apply the differential operator to this expression you need to notice that this is the quotient of a function of *x* and *y* and *y* is a function of *x*, so you use the quotient rule (to remind yourself how to use the quotient rule you can check the study guide: *The Quotient Rule*) in combination with the differential operator:

$$\frac{d}{dx}\left(\frac{x}{y}\right)=\frac{x\frac{d}{dx}(y)-y\frac{d}{dx}(x)}{y^2}$$
$$=\frac{x\frac{dy}{dx}-y}{y^2}$$
$$=\frac{x}{y^2}\frac{dy}{dx}-\frac{1}{y}\frac{dy}{dx}$$

c. $x^3\left(x^2+y^2\right)$

You can open the brackets and get $x^5+x^3y^2$. Applying the differential operator to this expression you get:

$$\frac{d}{dx}\left(x^5+x^3y^2\right)=\frac{d}{dx}\left(x^5\right)+\frac{d}{dx}\left(x^3y^2\right)$$

The result of the first term is the derivative of x^5 with respect to *x*. This is found by using the power rule of differentiation (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule*). And the second term is the product of a function of *x* and *y* and *y* is a function of *x*, so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) in combination with the differential operator:

$$\frac{d}{dx}\left(x^5+x^3y^2\right)=\frac{d}{dx}\left(x^5\right)+\frac{d}{dx}\left(x^3y^2\right)$$

$$=5x^4+x^3\frac{d}{dx}\left(y^2\right)+y^2\frac{d}{dx}\left(x^3\right)$$

$$=5x^4+x^3(2y)\frac{dy}{dx}+y^2\left(3x^2\right)$$

$$=5x^4+2x^3y\frac{dy}{dx}+3x^2y^2$$

d. $\dfrac{x+y^3}{x^2+y^2}$

To apply the differential operator to this expression you need to notice that this is the quotient of a function of *x* and *y* and *y* is a function of *x*, so you use the quotient rule (to remind yourself how to use the quotient rule you can check the study guide: *The Quotient Rule*) in combination with the differential operator:

$$\frac{d}{dx}\left(\frac{x+y^3}{x^2+y^2}\right)=\frac{\left(x+y^3\right)\frac{d}{dx}\left(x^2+y^2\right)-\left(x^2+y^2\right)\frac{d}{dx}\left(x+y^3\right)}{\left(x^2+y^2\right)^2}$$

$$=\frac{\left(x+y^3\right)\left(2x+\frac{d}{dx}\left(y^2\right)\right)-\left(x^2+y^2\right)\left(1+\frac{d}{dx}\left(y^3\right)\right)}{\left(x^2+y^2\right)^2}$$

$$=\frac{\left(x+y^3\right)\left(2x+2y\frac{dy}{dx}\right)-\left(x^2+y^2\right)\left(1+3y^2\frac{dy}{dx}\right)}{\left(x^2+y^2\right)^2}$$

6.

a. y^3

As y^3 is a composite function of *x* you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}\left(y^3\right) = \frac{dy}{dx}\frac{d}{dy}\left(y^3\right) = 3y^2\frac{dy}{dx}$$

b. $y^{\frac{3}{4}}$

As $y^{\frac{3}{4}}$ is a composite function of x you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}\left(y^{\frac{3}{4}}\right) = \frac{dy}{dx}\frac{d}{dy}\left(y^{\frac{3}{4}}\right) = \frac{3}{4}y^{-\frac{1}{4}}\frac{dy}{dx}$$

c. $\sin(3y)$

When you apply the differential operator to an expression such as $\sin(3y)$ you write:

$$\frac{d}{dx}(\sin(3y))$$

The result is a composite function and you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}(\sin(3y)) = \frac{dy}{dx}\frac{d}{dy}(\sin(3y)) = 3\cos(3y)\frac{dy}{dx}$$

d. $\cos(2y)$

When you apply the differential operator to an expression such as $\cos(2y)$ you write:

$$\frac{d}{dx}(\cos(2y))$$

The result is a composite function and you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}(\cos(2y)) = \frac{dy}{dx}\frac{d}{dy}(\cos(2y)) = 2\sin(2y)\frac{dy}{dx}$$

e. $-\ln(4y)$

When you apply the differential operator to an expression such as $-\ln(4y)$ you write:

$$\frac{d}{dx}(-\ln(4y))$$

The result is a composite function and you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}(-\ln(4y)) = \frac{dy}{dx}\frac{d}{dy}(-\ln(4y)) = -\frac{1}{y}\frac{dy}{dx}$$

f. $2e^{-4y}$

When you apply the differential operator to an expression such as $2e^{-4y}$ you write:

$$\frac{d}{dx}\left(2e^{-4y}\right)$$

The result is a composite function and you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}\left(2e^{-4y}\right) = \frac{dy}{dx}\frac{d}{dy}\left(2e^{-4y}\right) = -8e^{-4y}\frac{dy}{dx}$$

7.

a. When you apply the differential operator to an expression such as:

$y^2 - 2e^{-x} + x^2\ln(-3y) + \cos(2y)$ you write:

$$\frac{d}{dx}\left(y^2 - 2e^{-x} + x^2\ln(-3y) + \cos(2y)\right) = \frac{d}{dx}\left(y^2\right) + \frac{d}{dx}\left(-2e^{-x}\right) + \frac{d}{dx}\left(x^2\ln(-3y)\right) + \frac{d}{dx}(\cos(2y))$$

The first term that the differential operator is applied to is y^2, which is a composite function of x. You need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}\left(y^2\right)=\frac{dy}{dx}\frac{d}{dy}\left(y^2\right)=2y\frac{dy}{dx}$$

The second term the differential operator is applied to is $-2e^{-x}$ which is the derivative of $-2e^{-x}$ with respect to *x*. This is found by using the rules for differentiating basic functions (to remind yourself of how to differentiate basic functions you can check the study guide: *Differentiating Basic Functions)* and so:

$$\frac{d}{dx}\left(-2e^{-x}\right)=2e^{-x}$$

The third term the differential operator is applied to is $x^2\ln(-3y)$. To apply the differential operator to this expression you need to notice that this is the product of a function of *x* and *y* and *y* is a function of *x*, so you use the product rule (to remind yourself how to use the product rule you can check the study guide: *The Product Rule*) in combination with the differential operator:

$$\frac{d}{dx}\left(x^2\ln(-3y)\right)=x^2\frac{d}{dx}\left(\ln(-3y)\right)+\ln(-3y)\frac{d}{dx}\left(x^2\right)$$

$$=x^2\left(-\frac{1}{y}\right)\frac{dy}{dx}+\ln(-3y)(2x)$$

$$=-\frac{x^2}{y}\frac{dy}{dx}+2x\ln(-3y)$$

The fourth term is $\cos(2y)$. To apply the differential operator to this expression you need to notice that this is a composite function and you need to use the differential operator form of the chain rule (to remind yourself how to use the chain rule you can check the study guide: *The Chain Rule*) to perform the differentiation:

$$\frac{d}{dx}\left(\cos(2y)\right)=\frac{dy}{dx}\frac{d}{dy}\left(\cos(2y)\right)=2\sin(2y)\frac{dy}{dx}$$

So, from all the above the differential operator of the expression:

$y^2-2e^{-x}+x^2\ln(-3y)+\cos(2y)$ is:

$$\frac{d}{dx}\left(y^2-2e^{-x}+x^2\ln(-3y)+\cos(2y)\right)=\frac{d}{dx}\left(y^2\right)+\frac{d}{dx}\left(-2e^{-x}\right)+\frac{d}{dx}\left(x^2\ln(-3y)\right)+\frac{d}{dx}\left(\cos(2y)\right)$$

$$=2y\frac{dy}{dx}-2e^{-x}-\frac{x^2}{y}\frac{dy}{dx}+2x\ln(-3y)+2\sin(2y)\frac{dy}{dx}$$

b. Applying the differential operator to the equation $y^2 - 2e^{-x} + x^2\ln(-3y) + \cos(2y) = 1$ you get:

$$\frac{d}{dx}(y^2) + \frac{d}{dx}(-2e^{-x}) + \frac{d}{dx}(x^2\ln(-3y)) + \frac{d}{dx}(\cos(2y)) = \frac{d}{dx}(1)$$

Using your answer to 7a. you know that the left hand side of the equation is:

$$2y\frac{dy}{dx} - 2e^{-x} - \frac{x^2}{y}\frac{dy}{dx} + 2x\ln(-3y) + 2\sin(2y)\frac{dy}{dx}$$

Now for the right hand side of the equation. When you apply the differential operator to an expression such as 1 you write:

$$\frac{d}{dx}(1)$$

The result is the derivative of 1 with respect to *x*. This is found by using the power rule of differentiation (to remind yourself of how to use the power rule you can check the study guide: *Differentiating Using the Power Rule)* and so:

$$\frac{d}{dx}(1) = 0$$

Therefore, applying the differential operator to the equation $y^2 - 2e^{-x} + x^2\ln(-3y) + \cos(2y) = 1$ you get:

$$2y\frac{dy}{dx} - 2e^{-x} - \frac{x^2}{y}\frac{dy}{dx} + 2x\ln(-3y) + 2\sin(2y)\frac{dy}{dx} = 0$$

You can rearrange the result to find the derivative of y with respect to *x.* First, by adding to both sides $2e^{-x} - 2x\ln(-3y)$ to give:

$$\left(2y - \frac{x^2}{y} - 2\sin(2y)\right)\frac{dy}{dx} = 2e^{-x} - 2x\ln(-3y)$$

Finally, dividing by $2y - \frac{x^2}{y} - 2\sin(2y)$ gives:

$$\frac{dy}{dx} = \frac{2e^{-x} - 2x\ln(-3y)}{2y - \frac{x^2}{y} - 2\sin(2y)}$$

This enables you to find the derivative of y with respect to x, which you couldn't find in 7a.

c. Looking at the equation $-2e^{-x}+\cos(2y)=-y^2+1-x^2\ln(-3y)$. You can see that if you add to both sides $y^2+x^2\ln(-3y)$ you will get:

$$y^2-2e^{-x}+x^2\ln(-3y)+\cos(2y)=1$$

Which is the same as the equation you applied the differential operator in 7b. And so, applying the differential operator to $-2e^{-x}+\cos(2y)=-y^2+1-x^2\ln(-3y)$ would give:

$$-2e^{-x}+2\sin(2y)\frac{dy}{dx}=-2y\frac{dy}{dx}+\frac{x^2}{y}\frac{dy}{dx}-2x\ln(-3y)$$

Which you can rearrange and get to:

$$2y\frac{dy}{dx}-2e^{-x}-\frac{x^2}{y}\frac{dy}{dx}+2x\ln(-3y)+2\sin(2y)\frac{dy}{dx}=0$$

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