Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model Answers: The Quotient Rule

Differentiating Basic Functions study guide

More Complicated Functions study guide

Quotient Rule study guide

1.

(a) $y = \dfrac{x^5}{\sin x}$ is suitable for the quotient rule with $u = x^5$ and $v = \sin x$.

(b) $y = \dfrac{x^5}{7}$ can be differentiated using the power rule as the function can be rewritten as $y = \frac{1}{7}x^5$.

(c) $y = \dfrac{3e^x}{\cos x}$ is suitable for the quotient rule with $u = 3e^x$ and $v = \cos x$.

(d) $y = \dfrac{\sin(3x)}{\cos(5x)}$ is suitable for the quotient rule with $u = \sin(3x)$ and $v = \cos(5x)$.

(e) $y = \dfrac{-\sqrt{t}}{5\cos t}$ is suitable for the quotient rule with $u = -\sqrt{t} = -t^{1/2}$ and $v = 5\cos t$.

(f) $y = \dfrac{3}{e^x}$ can be rewritten as $y = 3e^{-x}$ and so can be differentiated using a basic function rule.

(g) $y = \dfrac{\sin\theta}{\theta}$ can be rewritten as $y = \theta^{-1}\sin\theta$ and so can be differentiated using the product rule.

(h) $y = \dfrac{5}{6}\dfrac{(3x-5)}{e^x}$ can be rewritten as $y = \frac{5}{6}(3x-5)e^{-x}$ and so can be differentiated using the product rule. You may also find the quotient rule useful too.

(i) $y = \frac{x^2 - 1}{x + 1}$ you can factorise the numerator as it is the difference of 2 squares to see that $x^2 - 1 = (x+1)(x-1)$ and so, after cancelling down $y = x - 1$ which can be differentiated using the power rule.

(j) $y = \tan\theta$ as $\tan\theta = \frac{\sin\theta}{\cos\theta}$ you can use the quotient rule the to differentiate this function with $u = \sin\theta$ and $v = \cos\theta$.

(k) $y = \frac{3\sin 2\theta}{4\sin 2\theta}$ you can cancel down the $\sin 2\theta$ factor and so $y = \frac{3}{4}$ which can be differentiated using the power rule.

(l) $y = \frac{x^3 - 7x + 3}{2}$ you can split this function into three terms and so $y = \frac{x^3}{2} - \frac{7x}{2} + \frac{3}{2}$. Each of the terms can be differentiated using the power rule.

The following section will use the quotient rule to differentiate a, c, d, e and j. The derivatives of the other functions will be quoted and you can follow the links to help with these processes if you need to.

(a) For $y = \frac{x^5}{\sin x}$, $\frac{dy}{dx} = \frac{x^4(5\sin x - x\cos x)}{\sin^2 x}$

and $\frac{dy}{dx} = \frac{2^4(5\sin 2 - 2\cos 2)}{\sin^2 2} = 104.09$ when $x = 2$.

$y = \frac{x^5}{\sin x}$ is suitable for the quotient rule with $u = x^5$ and $v = \sin x$.

Answer Space	*Exploring Space*
$y = \frac{x^5}{\sin x}$	$u = x^5$
$y = \frac{u}{v}$	$v = \sin x$
So as $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$v^2 = \sin^2 x$
$= \frac{(\sin x)\cdot(5x^4) - (x^5)\cdot(\cos x)}{\sin^2 x}$	$\frac{du}{dx} = 5x^4$
$= \frac{x^4(5\sin x - x\cos x)}{\sin^2 x}$	$\frac{dv}{dx} = \cos x$

So the derivative of $y = \frac{x^5}{\sin x}$ is $\frac{dy}{dx} = \frac{x^4(5\sin x - x\cos x)}{\sin^2 x}$.

When $x = 2$, $\frac{dy}{dx} = \frac{2^4(5\sin 2 - 2\cos 2)}{\sin^2 2} = 104.09$ to 2 decimal places. (Remember to use radians when performing calculus calculations which involve trigonometric functions.)

(b) The derivative of $y = \frac{x^5}{7}$ is $\frac{dy}{dx} = \frac{5x^4}{7}$, and $\frac{dy}{dx} = \frac{80}{7}$ when $x = 2$.

See study guide: *Differentiation of Basic Functions* for more details.

(c) For $y = \frac{3e^x}{\cos x}$, $\frac{dy}{dx} = \frac{3e^x(\cos x + \sin x)}{\cos^2 x}$

and $\frac{dy}{dx} = \frac{3e^2(\cos 2 + \sin 2)}{\cos^2 2} = 63.12$ when $x = 2$.

$y = \frac{3e^x}{\cos x}$ is suitable for the quotient rule with $u = 3e^x$ and $v = \cos x$.

Answer Space	*Exploring Space*
$y = \frac{3e^x}{\cos x}$	$u = 3e^x$
$y = \frac{u}{v}$	$v = \cos x$
So as $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$v^2 = \cos^2 x$
$= \frac{(\cos x)\cdot(3e^x) - (3e^x)\cdot(-\sin x)}{\cos^2 x}$	$\frac{du}{dx} = 3e^x$
$= \frac{3e^x(\cos x + \sin x)}{\cos^2 x}$	$\frac{dv}{dx} = -\sin x$

So the derivative of $y = \frac{3e^x}{\cos x}$ is $\frac{dy}{dx} = \frac{3e^x(\cos x + \sin x)}{\cos^2 x}$.

When $x = 2$, $\frac{dy}{dx} = \frac{3e^2(\cos 2 + \sin 2)}{\cos^2 2} = 63.12$ to 2 decimal places. (Remember to use radians when performing calculus calculations which involve trigonometric functions.)

(d) For $y = \frac{\sin(3x)}{\cos(5x)}$, $\frac{dy}{dx} = \frac{3\cos 5x \cos 3x + 5\sin 5x \sin 3x}{\cos^2 5x}$

and $\frac{dy}{dx} = \frac{3\cos 10 \cos 6 + 5\sin 10 \sin 6}{\cos^2 10} = -2.35$ when $x = 2$.

$y = \frac{\sin(3x)}{\cos(5x)}$ is suitable for the quotient rule with $u = \sin(3x)$ and $v = \cos(5x)$.

Answer Space	*Exploring Space*
$y = \frac{\sin(3x)}{\cos(5x)}$	$u = \sin(3x)$
$y = \frac{u}{v}$	$v = \cos(5x)$
So as $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$v^2 = \cos^2(5x)$
$= \frac{(\cos 5x)\cdot(3\cos 3x) - (\sin 3x)\cdot(-5\sin 5x)}{\cos^2 5x}$	$\frac{du}{dx} = 3\cos(3x)$
$= \frac{3\cos 5x \cos 3x + 5\sin 5x \sin 3x}{\cos^2 5x}$	$\frac{dv}{dx} = -5\sin(5x)$

So the derivative of $y = \frac{\sin(3x)}{\cos(5x)}$ is $\frac{dy}{dx} = \frac{3\cos 5x \cos 3x + 5\sin 5x \sin 3x}{\cos^2 5x}$.

When $x = 2$, $\frac{dy}{dx} = \frac{3\cos 10 \cos 6 + 5\sin 10 \sin 6}{\cos^2 10} = -2.35$ to 2 decimal places.

(Remember to use radians when performing calculus calculations which involve trigonometric functions.)

(e) For $y = \frac{-\sqrt{t}}{5\cos t}$, $\frac{dy}{dt} = \frac{-\cos t + 2t\sin t}{10\sqrt{t}\cos^2 t}$

and $\frac{dy}{dt} = \frac{-\cos 2 + 4\sin 2}{10\sqrt{2}\cos^2 2} = 1.66$ when $t = 2$.

$y = \frac{-\sqrt{t}}{5\cos t}$ is suitable for the quotient rule with $u = -\sqrt{t} = -t^{1/2}$ and $v = 5\cos t$.

Answer Space	*Exploring Space*
$y = \frac{-\sqrt{t}}{5\cos t}$	$u = -t^{1/2}$
$y = \frac{u}{v}$	$v = 5\cos t$
So as $\frac{dy}{dt} = \frac{v\frac{du}{dt} - u\frac{dv}{dt}}{v^2}$	$v^2 = 25\cos^2 t$
$= \frac{(5\cos t)\cdot\left(-\frac{1}{2\sqrt{t}}\right) - \left(-\sqrt{t}\right)\cdot(-5\sin t)}{25\cos^2 t}$	$\frac{du}{dt} = -\frac{1}{2}t^{-1/2} = -\frac{1}{2\sqrt{t}}$
$= \frac{5(-\cos t + 2t\sin t)}{50\sqrt{t}\cos^2 t}$	$\frac{dv}{dt} = -5\sin t$
$= \frac{-\cos t + 2t\sin t}{10\sqrt{t}\cos^2 t}$	

So the derivative of $y = \frac{-\sqrt{t}}{5\cos t}$ is $\frac{dy}{dt} = \frac{-\cos t + 2t\sin t}{10\sqrt{t}\cos^2 t}$.

When $t = 2$, $\frac{dy}{dt} = \frac{-\cos 2 + 4\sin 2}{10\sqrt{2}\cos^2 2} = 1.66$ to 2 decimal places. (Remember to use radians when performing calculus calculations which involve trigonometric functions.)

(f) The derivative of $y = \frac{3}{e^x} = 3e^{-x}$ is $\frac{dy}{dx} = -3e^{-x} = -\frac{3}{e^x}$, and $\frac{dy}{dx} = -0.41$ when $x = 2$.

See study guide: *Differentiation of Basic Functions* for more details.

(g) The derivative of $y = \frac{\sin\theta}{\theta} = \theta^{-1}\sin\theta$ can be found by using the product rule.

So $\frac{dy}{d\theta} = -\theta^{-2}\sin\theta + \theta^{-1}\cos\theta = \frac{-\sin\theta + \theta\cos\theta}{\theta^2}$.

When $\theta = 2$, $\frac{dy}{d\theta} = -0.44$ to 2 d.p.

See study guide: *Product Rule* for more details.

(h) The derivative of $y = \frac{5}{6}\frac{(3x-5)}{e^x} = \frac{5}{6}(3x-5)e^{-x}$ can be found by using the product rule

So $\frac{dy}{dx} = \frac{15}{6}e^{-x} - \frac{5}{6}(3x-5)e^{-x} = \frac{5(8-3x)}{6e^x}$ and $\frac{dy}{dx} = 0.23$ when $x = 2$.

See study guide: *Product Rule* for more details.

(i) The derivative of $y = \frac{x^2-1}{x+1} = x-1$ is $\frac{dy}{dx} = 1$. When $x = 2$, $\frac{dy}{dx} = 1$

See study guide: *Differentiation of Basic Functions* for more details.

(j) For $y = \tan\theta$, $\frac{dy}{d\theta} = \frac{1}{\cos^2\theta}$ and $\frac{dy}{d\theta} = 5.77$ when $\theta = 2$.

$y = \tan\theta = \frac{\sin\theta}{\cos\theta}$ is suitable for the quotient rule with $u = \sin\theta$ and $v = \cos\theta$.

Answer Space	*Exploring Space*
$y = \tan\theta = \frac{\sin\theta}{\cos\theta}$	$u = \sin\theta$
$y = \frac{u}{v}$	$v = \cos\theta$
So as $\frac{dy}{d\theta} = \frac{v\frac{du}{d\theta} - u\frac{dv}{d\theta}}{v^2}$	$v^2 = \cos^2\theta$
$= \frac{(\cos\theta)\cdot(\cos\theta) - (\sin\theta)\cdot(-\sin\theta)}{\cos^2\theta}$	$\frac{du}{d\theta} = \cos\theta$
$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$	$\frac{dv}{d\theta} = -\sin\theta$

So the derivative of $y = \tan\theta$ is $\frac{dy}{d\theta} = \frac{1}{\cos^2\theta}$. Often you will see $\frac{1}{\cos^2\theta}$ written as $\sec^2\theta$.

When $\theta = 2$, $\frac{dy}{d\theta} = \frac{1}{\cos^2 2} = 5.77$ to 2 decimal places. (Remember to use radians when performing calculus calculations which involve trigonometric functions.)

(k) The derivative of $y = \dfrac{3\sin 2\theta}{4\sin 2\theta} = \dfrac{3}{4}$ is $\dfrac{dy}{d\theta} = 0$, and $\dfrac{dy}{d\theta} = 0$ when $\theta = 2$.

See study guide: *Differentiation of Basic Functions* for more details.

(l) The derivative of $y = \dfrac{x^3 - 7x + 3}{2} = \dfrac{x^3}{2} - \dfrac{7x}{2} + \dfrac{3}{2}$ can be found using the power rule.

So $\dfrac{dy}{dx} = \dfrac{3x^2}{2} - \dfrac{7}{2}$ and $\dfrac{dy}{dx} = \dfrac{5}{2}$ when $x = 2$.

See study guide: *Differentiation of Basic Functions* for more details.