Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Worksheet: The Chain Rule

This worksheet has questions using **The Chain Rule**: the method of differentiating composite functions. Using the chain rule is a common in calculus problems. It is also one of the most frequently used rules in more advanced calculus techniques such as implicit and partial differentiation. Before attempting the questions below you should be able to differentiate elementary functions and understand what a composite function is.

Differentiating Basic Functions study guide

More Complicated Functions study guide

Chain Rule study guide

Model Answers to this Sheet

1.

Use the chain rule to differentiate the following composite functions with respect to x:

(a) $y=(3x+5)^8$ (use $u=3x+5$ and $y=u^8$)

(b) $y=9(8-5x^2)^3$ (use $u=8-5x^2$ and $y=9u^3$)

(c) $y=\dfrac{-\sqrt{9x^4-7}}{5}$ (use $u=9x^4-7$ and $y=-\frac{1}{5}u^{1/2}$)

(d) $y=\dfrac{3}{4-3x^2}$ (use $u=4-3x^2$ and $y=3u^{-1}$)

(e) $y=\dfrac{5e^{3x-5}}{7}$ (use $u=3x-5$ and $y=\frac{5}{7}e^u$)

(f) $y=-3\sin(4-3x^4)$ (use $u=4-3x^4$ and $y=-3\sin u$)

(g) $y=\dfrac{\ln(7-6x^3)}{5}$ (use $u=7-6x^3$ and $y=\frac{1}{5}\ln u$)

(h) $y=-e^{3x^5}$ (use $u=3x^5$ and $y=-e^u$)

(i) $y=e^{4x^3}+6x^2$ (use $u=4x^3$ in the first term)

(j) $y = \cos(1 - 2x^3) + \ln(x^5) + e^{-x^4}$ (use $u = 1 - 2x^3$ in the first term and $u = -x^4$ in the third term)

2. Find the derivatives of the following composite functions with respect to the variable indicated:

(a) $A = (4t^2 - 3t - 2)^4$ with respect to t

(b) $P = -5e^{-3t^2}$ with respect to t

(c) $r = \dfrac{3}{(\theta^3 + 10)^4}$ with respect to θ

(d) $v = \ln(3 - 7w^7)$ with respect to w

(e) $\theta = \sqrt{1 - r^2}$ with respect to r

3. Find the gradient of the following curves when $x = 2$:

(a) $y = 2(x^3 - 12)^3$ (b) $y = -\cos(2x^2)$

(c) $y = -9e^{5-4x^2}$ (d) $y = \ln(4x^2 - 9)$

(e) $y = 4x^3 - (3 - 8x)^5$ (f) $y = 4\sin(5x - 2)$

4. Find the equation of the tangent to the curve $y = (2x - 3)^4$ at the point $(1, 1)$.

5. Use the chain rule twice to differentiate the following functions with respect to x:

(a) $y = \sqrt{\sin(2 - 3x^2)}$ (b) $y = \dfrac{1}{\sqrt{\cos(3x - 5)}}$

(c) $y = \dfrac{3}{\sqrt[3]{\ln(x + 4)}}$ (d) $y = \dfrac{e^{\cos^2 x}}{2}$

(e) $y = 3\cos\left(2e^{x^3}\right)$ (f) $y = \left(2 - 3x^2 + e^{x^3}\right)^3$