University of East Anglia
STUDENT SUPPORT SERVICE

Learning Enhancement Team

Model Answers: The Chain Rule

Chain Rule study guide

To help you to use the chain rule effectively you should have a good understanding of composite functions and how to differentiate basics functions. You can read the study guides: *Differentiating Basic Functions* and *More Complicated Functions* to help you with this.

1.

(a) As $y = (3x+5)^8$ is a composite function, if you use $u = 3x+5$ as a substitution you can re-write the function as $y = u^8$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = u^8$ then $\frac{dy}{du} = 8u^7$ and if $u = 3x+5$ then $\frac{du}{dx} = 3$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = (8u^7) \times (3) = 24u^7 = 24(3x+5)^7 .$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 3x+5$) to replace u.

(b) As $y = 9(8-5x^2)^3$ is a composite function, if you use $u = 8-5x^2$ as a substitution you can re-write the function as $y = 9u^3$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = 9u^3$ then $\frac{dy}{du} = 27u^2$ and if $u = 8-5x^2$ then $\frac{du}{dx} = -10x$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = 27u^2 \times (-10x) = -270xu^2 = -270x(8-5x^2)^2.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 8-5x^2$) to replace u.

(c) As $y = \frac{-\sqrt{9x^4-7}}{5}$ is a composite function, if you use $u = 9x^4-7$ as a substitution you can re-write the function as $y = -\frac{1}{5}u^{1/2}$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = -\frac{1}{5}u^{1/2}$ then $\frac{dy}{du} = -\frac{1}{10}u^{-1/2}$ and if $u = 9x^4-7$ then $\frac{du}{dx} = 36x^3$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = \left(-\tfrac{1}{10}u^{-1/2}\right) \times 36x^3 = -\tfrac{18}{5}x^3u^{-1/2} = -\tfrac{18}{5}x^3(9x^4-7)^{-1/2} = -\frac{18x^3}{5\sqrt{9x^4-7}}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 9x^4-7$) to replace u.

(d) As $y = \dfrac{3}{4-3x^2}$ is a composite function, if you use $u = 4-3x^2$ as a substitution you can re-write the function as $y = 3u^{-1}$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = 3u^{-1}$ then $\dfrac{dy}{du} = -3u^{-2}$ and if $u = 4-3x^2$ then $\dfrac{du}{dx} = -6x$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = \left(-3u^{-2}\right)\times\left(-6x\right) = 18xu^{-2} = 18x\left(4-3x^2\right)^{-2} = \frac{18x}{\left(4-3x^2\right)^2}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 4-3x^2$) to replace *u*.

(e) As $y = \dfrac{5e^{3x-5}}{7}$ is a composite function, if you use $u = 3x-5$ as a substitution you can re-write the function as $y = \frac{5}{7}e^u$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = \frac{5}{7}e^u$ then $\dfrac{dy}{du} = \frac{5}{7}e^u$ and if $u = 3x-5$ then $\dfrac{du}{dx} = 3$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = \tfrac{5}{7}e^u \times 3 = \tfrac{15}{7}e^u = \tfrac{15}{7}e^{3x-5}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 3x-5$) to replace *u*.

(f) As $y = -3\sin\left(4-3x^4\right)$ is a composite function, if you use $u = 4-3x^4$ as a substitution you can re-write the function as $y = -3\sin u$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = -3\sin u$ then $\frac{dy}{du} = -3\cos u$ and if $u = 4-3x^4$ then $\frac{du}{dx} = -12x^3$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = \left(-3\cos u\right) \times \left(-12x^3\right) = 36x^3\cos u = 36x^3\cos\left(4-3x^4\right).$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 4-3x^4$) to replace u.

(g) As $y = \frac{\ln\left(7-6x^3\right)}{5}$ is a composite function, if you use $u = 7-6x^3$ as a substitution you can re-write the function as $y = \frac{1}{5}\ln u$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = \frac{1}{5}\ln u$ then $\frac{dy}{du} = \frac{1}{5u}$ and if $u = 7-6x^3$ then $\frac{du}{dx} = -18x^2$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = \frac{1}{5u} \times \left(-18x^2\right) = -\frac{18x^2}{5u} = -\frac{18x^2}{5\left(7-6x^3\right)}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 7-6x^3$) to replace u.

(h) As $y = -e^{3x^5}$ is a composite function, if you use $u = 3x^5$ as a substitution you can re-write the function as $y = -e^u$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = -e^u$ then $\frac{dy}{du} = -e^u$ and if $u = 3x^5$ then $\frac{du}{dx} = 15x^4$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = \left(-e^u\right) \times 15x^4 = -15x^4 e^u = -15x^4 e^{3x^5}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 3x^5$) to replace u.

(i) As the first term in $y = e^{4x^3} + 6x^2$ is a composite function, you need to use the chain rule to differentiate it. The second term can be differentiated using the power rule. Taking the first term first, if you use $u = 4x^3$ as a substitution you can re-write the term as e^u. The derivative is then:

$$\frac{dy}{dx} = \left(e^u \times 12x^2\right) + 12x = 12x^2 e^u + 12x = 12x\left(xe^{4x^3} + 1\right)$$

As the derivative of e^u is e^u and $\frac{du}{dx} = 12x^2$.

(j) To differentiate $y = \cos\left(1 - 2x^3\right) + \ln\left(x^5\right) + e^{-x^4}$ you should treat each term separately and then combine them to find the answer. So, one term at a time:

Term 1 As $y = \cos\left(1 - 2x^3\right)$ is a composite function, if you use $u = 1 - 2x^3$ as a substitution you can re-write the functions as $y = \cos u$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = \cos u$ then $\frac{dy}{du} = -\sin u$ and if $u = 1 - 2x^3$ then $\frac{du}{dx} = -6x^2$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = (-\sin u) \times \left(-6x^2\right) = 6x^2 \sin u = 6x^2 \sin\left(1 - 2x^3\right).$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 1 - 2x^3$) to replace *u*.

Term 2 You can use the laws of logarithms to show that $y = \ln\left(x^5\right) = 5\ln x$ which is a basic function and can be differentiated to give:

$$\frac{dy}{dx} = \frac{5}{x}$$

Term 3 As $y = e^{-x^4}$ is a composite function, if you use $u = -x^4$ as a substitution you can re-write the functions as $y = e^u$. To perform the differentiation you use the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

If $y = e^u$ then $\frac{dy}{du} = e^u$ and if $u = -x^4$ then $\frac{du}{dx} = -4x^3$. Using these results in the chain rule you get:

$$\frac{dy}{dx} = e^u \times \left(-4x^3\right) = -4x^3 e^u = -4x^3 e^{-x^4}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = -x^4$) to replace *u*.

Bringing the results together the derivative of $y = \cos\left(1 - 2x^3\right) + \ln\left(x^5\right) + e^{-x^4}$ is:

$$\frac{dy}{dx} = 6x^2 \sin\left(1 - 2x^3\right) + \frac{5}{x} - 4x^3 e^{-x^4}$$

2.

(a) As $A = (4t^2 - 3t - 2)^4$ is a composite function, if you use $u = 4t^2 - 3t - 2$ as a substitution you can re-write the function as $A = u^4$. To perform the differentiation you use the chain rule which states:

$$\frac{dA}{dt} = \frac{dA}{du} \times \frac{du}{dt}$$

If $A = u^4$ then $\frac{dA}{du} = 4u^3$ and if $u = 4t^2 - 3t - 2$ then $\frac{du}{dt} = 8t - 3$. Using these results in the chain rule you get:

$$\frac{dA}{dt} = 4u^3 \times (8t - 3) = 4(8t - 3)u^3 = 4(8t - 3)(4t^2 - 3t - 2)^3.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 4t^2 - 3t - 2$) to replace u.

(b) As $P = -5e^{-3t^2}$ is a composite function, if you use $u = -3t^2$ as a substitution you can re-write the function as $P = -5e^u$. To perform the differentiation you use the chain rule which states:

$$\frac{dP}{dt} = \frac{dP}{du} \times \frac{du}{dt}$$

If $P = -5e^u$ then $\frac{dP}{du} = -5e^u$ and if $u = -3t^2$ then $\frac{du}{dt} = -6t$. Using these results in the chain rule you get:

$$\frac{dP}{dt} = (-5e^u) \times (-6t) = 30te^u = 30te^{-3t^2}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = -3t^2$) to replace u.

(c) As $r = \frac{3}{(\theta^3 + 10)^4}$ is a composite function, if you use $u = \theta^3 + 10$ as a substitution you can re-write the function as $r = 3u^{-4}$. To perform the differentiation you use the chain rule which states:

$$\frac{dr}{d\theta} = \frac{dr}{du} \times \frac{du}{d\theta}$$

If $r = 3u^{-4}$ then $\frac{dr}{du} = -12u^{-5}$ and if $u = \theta^3 + 10$ then $\frac{du}{d\theta} = 3\theta^2$. Using these results in the chain rule you get:

$$\frac{dr}{d\theta} = \left(-12u^{-5}\right) \times 3\theta^2 = -36u^{-5}\theta^2 = -\frac{36\theta^2}{(\theta^3 + 10)^5}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = \theta^3 + 10$) to replace *u*.

(d) As $v = \ln(3 - 7w^7)$ is a composite function, if you use $u = 3 - 7w^7$ as a substitution you can re-write the function as $v = \ln u$. To perform the differentiation you use the chain rule which states:

$$\frac{dv}{dw} = \frac{dv}{du} \times \frac{du}{dw}$$

If $v = \ln u$ then $\frac{dv}{du} = \frac{1}{u}$ and if $u = 3 - 7w^7$ then $\frac{du}{dw} = -49w^6$. Using these results in the chain rule you get:

$$\frac{dv}{dw} = \frac{1}{u} \times \left(-49w^6\right) = -\frac{49w^6}{u} = -\frac{49w^6}{3 - 7w^7}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 3 - 7w^7$) to replace *u*.

(e) As $\theta = \sqrt{1-r^2}$ is a composite function, if you use $u = 1-r^2$ as a substitution you can re-write the function as $\theta = u^{1/2}$. To perform the differentiation you use the chain rule which states:

$$\frac{d\theta}{dr} = \frac{d\theta}{du} \times \frac{du}{dr}$$

If $\theta = u^{1/2}$ then $\frac{d\theta}{du} = \frac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{u}}$ and if $u = 1-r^2$ then $\frac{du}{dr} = -2r$. Using these results in the chain rule you get:

$$\frac{d\theta}{dr} = \frac{1}{2\sqrt{u}} \times (-2r) = -\frac{r}{\sqrt{u}} = -\frac{r}{\sqrt{1-r^2}} \quad .$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitution (in this case $u = 1-r^2$) to replace u.

3.

(a) Remember that the gradient of a function is the same as its derivative and so to find the value of the gradient you first must differentiate the function. As $y = 2(x^3 - 12)^3$ is a composite function, if you use $u = x^3 - 12$ as a substitution you can re-write the function as $y = 2u^3$. Using the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \qquad \text{and} \qquad \frac{dy}{du} = 6u^2 \qquad \text{and} \qquad \frac{du}{dx} = 3x^2$$

gives $$\frac{dy}{dx} = 6u^2 \times 3x^2 = 18x^2u^2 = 18x^2(x^3 - 12)^2 .$$

To find the gradient you need to substitute $x = 2$ into the derivative. This is written as:

$$\left.\frac{dy}{dx}\right|_{x=2} = 18 \cdot 2^2(2^3 - 12)^2 = 1152$$

Which implies that the gradient of this function is steeply uphill at $x = 2$.

(b) Remember that the gradient of a function is the same as its derivative and so to find the value of the gradient you first must differentiate the function. As $y = -\cos(2x^2)$ is a composite function, if you use $u = 2x^2$ as a substitution you can re-write the function as $y = -\cos u$. Using the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \qquad \text{and} \qquad \frac{dy}{du} = \sin u \qquad \text{and} \qquad \frac{du}{dx} = 4x$$

gives $$\frac{dy}{dx} = (\sin u) \times (4x) = 4x \sin u = 4x \sin\left(2x^2\right).$$

To find the gradient you need to substitute $x = 2$ into the derivative. Remember to use **radians** mode on you calculator when performing calculations involving trigonometric functions and calculus. So:

$$\left.\frac{dy}{dx}\right|_{x=2} = 4 \cdot 2 \sin\left(2^3\right) = 7.915 \text{ to 3 d.p.}$$

Which implies that the gradient of this function is uphill at $x = 2$.

(c) Remember that the gradient of a function is the same as its derivative and so to find the value of the gradient you first must differentiate the function. As $y = -9e^{5-4x^2}$ is a composite function, if you use $u = 5 - 4x^2$ as a substitution you can re-write the function as $y = -9e^u$. Using the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \qquad \text{and} \qquad \frac{dy}{du} = -9e^u \qquad \text{and} \qquad \frac{du}{dx} = -8x$$

gives $$\frac{dy}{dx} = \left(-9e^u\right) \times (-8x) = 72xe^u = 72xe^{\left(5-4x^2\right)}.$$

To find the gradient you need to substitute $x = 2$ into the derivative. So:

$$\left.\frac{dy}{dx}\right|_{x=2} = 72 \cdot 2 \cdot e^{5-4\cdot 2^2} = 2.405 \times 10^{-3} \text{ to 3 d.p.}$$

Which implies that the gradient of this function is almost flat at $x = 2$.

(d) Remember that the gradient of a function is the same as its derivative and so to find the value of the gradient you first must differentiate the function. As $y = \ln\left(4x^2 - 9\right)$ is a composite function, if you use $u = 4x^2 - 9$ as a substitution you can re-write the function as $y = \ln u$. Using the chain rule which states:

$$\frac{dy}{dx}=\frac{dy}{du}\times\frac{du}{dx} \qquad \text{and} \qquad \frac{dy}{du}=\frac{1}{u} \qquad \text{and} \qquad \frac{du}{dx}=8x$$

gives $$\frac{dy}{dx}=\left(\frac{1}{u}\right)\times(8x)=\frac{8x}{u}=\frac{8x}{4x^2-9}.$$

To find the gradient you need to substitute $x=2$ into the derivative. So:

$$\left.\frac{dy}{dx}\right|_{x=2}=\frac{8\cdot 2}{4\cdot 4-9}=\frac{16}{7}$$

Which implies that the gradient of this function is uphill at $x=2$.

(e) Remember that the gradient of a function is the same as its derivative and so to find the value of the gradient you first must differentiate the function. The first term in $y=4x^3-(3-8x)^5$ can be differentiated using the power rule whereas the second term is a composite function.

Term 1 The derivative of $4x^3$ is $12x^2$

Term 2 Use $u=3-8x$ and then differentiate u^5. Using the chain rule which states:

$$\frac{d}{dx}(3-8x)^5=\frac{d}{du}\left(u^5\right)\times\frac{du}{dx} \qquad \text{and} \qquad \frac{d}{du}\left(u^5\right)=5u^4 \quad \text{and} \qquad \frac{du}{dx}=-8$$

gives $$\frac{d}{dx}(3-8x)^5=5u^4\times(-8)=-40(3-8x)^4.$$

Combining the answers to term one and two gives:

$$\frac{dy}{dx}=12x^2+40(3-8x)^4$$

To find the gradient you need to substitute $x=2$ into the derivative. So:

$$\left.\frac{dy}{dx}\right|_{x=2}=12\cdot 4+40(3-16)^4=1142488$$

Which implies that the gradient of this function is steeply uphill at $x=2$.

(f) Remember that the gradient of a function is the same as its derivative and so to find the value of the gradient you first must differentiate the function. As $y=4\sin(5x-2)$

is a composite function, if you use $u = 5x - 2$ as a substitution you can re-write the function as $y = 4\sin u$. Using the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \quad \text{and} \quad \frac{dy}{du} = 4\cos u \quad \text{and} \quad \frac{du}{dx} = 5$$

gives $$\frac{dy}{dx} = 4\cos u \times (5) = 20\cos u = 20\cos(5x - 2).$$

To find the gradient you need to substitute $x = 2$ into the derivative. As this is a trigonometric function remember to use radians. So:

$$\left.\frac{dy}{dx}\right|_{x=2} = 20\cos(8) = -2.91 \text{ to 2 d.p.}$$

Which implies that the gradient of this function is downhill at $x = 2$.

4. To find the equation of the tangent you should you use the general equation of a straight line $y = mx + c$ and find the values of *m* and *c* (see study guides: *What is a Straight Line?* and *Finding Equations of Straight Lines*). The key to answering this question is that you should make the connection between *m* and the derivative of $y = (2x - 3)^4$. As $y = (2x - 3)^4$ is a composite function if you use $u = 2x - 3$ as a substitution you can re-write the function as $y = u^4$. Using the chain rule which states:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} \quad \text{and} \quad \frac{dy}{du} = 4u^3 \quad \text{and} \quad \frac{du}{dx} = 2$$

gives $$\frac{dy}{dx} = 4u^3 \times 2 = 8u^3 = 8(2x - 3)^3.$$

To find the gradient *m* at the point $(1, 1)$ you need to substitute $x = 1$ into the derivative. So:

$$\left.\frac{dy}{dx}\right|_{x=1} = 8(2 - 3)^3 = -8$$

Which implies that the gradient of the tangent is $m = -8$.
To find *c* you substitute the values of *x* and *y* at the point $(1, 1)$ to give:

$1 = -8 + c$ therefore $c = 9$.

And the equation of the tangent line is $y = -8x + 9$

5.

(a) $y = \sqrt{\sin(2-3x^2)}$ is a more complicated composite function than those previously discussed, you need to make two substitutions to allow you to the chain rule. Using $u = 2-3x^2$ and $v = \sin u$ as substitutions means you can re-write the function as $y = v^{1/2}$. To perform the differentiation you use an extended version of the chain rule:

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

If $y = v^{1/2}$ then $\frac{dy}{dv} = \frac{1}{2v^{1/2}}$, if $v = \sin u$ then $\frac{dv}{du} = \cos u$ and if $u = 2-3x^2$ then $\frac{du}{dx} = -6x$.

Using these results in the chain rule you get:

$$\frac{dy}{dx} = \left(\frac{1}{2v^{1/2}}\right) \times (\cos u) \times (-6x) = -\frac{3x\cos u}{v^{1/2}} = -\frac{3x\cos(2-3x^2)}{\sqrt{\sin(2-3x^2)}}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitutions (in this case $u = 2-3x^2$ and $v = \sin u$) to replace *u* and *v*.

(b) $y = \frac{1}{\sqrt{\cos(3x-5)}}$ is a more complicated composite function than those previously discussed, you need to make two substitutions to allow you to the chain rule. Using $u = 3x-5$ and $v = \cos u$ as substitutions means you can re-write the function as $y = v^{-1/2}$. To perform the differentiation you use an extended version of the chain rule:

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

If $y = v^{-1/2}$ then $\frac{dy}{dv} = -\frac{1}{2v^{3/2}}$, if $v = \cos u$ then $\frac{dv}{du} = -\sin u$ and if $u = 3x-5$ then $\frac{du}{dx} = 3$.

Using these results in the chain rule you get:

$$\frac{dy}{dx}=\left(-\frac{1}{2v^{3/2}}\right)\times(-\sin u)\times(3)=\frac{3\sin u}{2v^{3/2}}=-\frac{3\sin(3x-5)}{\sqrt{\cos^3(3x-5)}}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitutions (in this case $u=3x-5$ and $v=\cos u$) to replace u and v.

(c) $y=\dfrac{3}{\sqrt[3]{\ln(x+4)}}$ is a more complicated composite function than those previously discussed, you need to make two substitutions to allow you to the chain rule. Using $u=x+4$ and $v=\ln u$ as substitutions means you can re-write the function as $y=3v^{-1/3}$. To perform the differentiation you use an extended version of the chain rule:

$$\frac{dy}{dx}=\frac{dy}{dv}\times\frac{dv}{du}\times\frac{du}{dx}$$

If $y=3v^{-1/3}$ then $\dfrac{dy}{dv}=-\dfrac{1}{v^{4/3}}$, if $v=\ln u$ then $\dfrac{dv}{du}=\dfrac{1}{u}$ and if $u=x+4$ then $\dfrac{du}{dx}=1$.

Using these results in the chain rule you get:

$$\frac{dy}{dx}=\left(-\frac{1}{v^{4/3}}\right)\times\left(\frac{1}{u}\right)\times(1)=-\frac{1}{uv^{4/3}}=-\frac{1}{(x+4)\sqrt[3]{(\ln(x+4))^4}}$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitutions (in this case $u=x+4$ and $v=\ln u$) to replace u and v.

(d) As $y=\dfrac{e^{\cos^2 x}}{2}$ is a more complicated composite function than those previously discussed, you need to make two substitutions to allow you to the chain rule. Using $u=\cos x$ and $v=u^2$ as substitutions means you can re-write the function as $y=\frac{1}{2}e^v$. To perform the differentiation you use an extended version of the chain rule:

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

If $y = \tfrac{1}{2}e^v$ then $\frac{dy}{dv} = \tfrac{1}{2}e^v$, if $v = u^2$ then $\frac{dv}{du} = 2u$ and if $u = \cos x$ then $\frac{du}{dx} = -\sin x$.

Using these results in the chain rule you get:

$$\frac{dy}{dx} = \left(\tfrac{1}{2}e^v\right) \times (2u) \times (-\sin x) = -ue^v \sin x = -\cos x \sin x e^{\cos^2 x}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitutions (in this case $u = \cos x$ and $v = u^2$) to replace *u* and *v*.

(e) As $y = 3\cos\left(2e^{x^3}\right)$ is a more complicated composite function than those previously discussed, you need to make two substitutions to allow you to the chain rule. Using $u = x^3$ and $v = 2e^u$ as substitutions means you can re-write the function as $y = 3\cos v$. To perform the differentiation you use an extended version of the chain rule:

$$\frac{dy}{dx} = \frac{dy}{dv} \times \frac{dv}{du} \times \frac{du}{dx}$$

If $y = 3\cos v$ then $\frac{dy}{dv} = -3\sin v$, if $v = 2e^u$ then $\frac{dv}{du} = 2e^u$ and if $u = x^3$ then $\frac{du}{dx} = 3x^2$.

Using these results in the chain rule you get:

$$\frac{dy}{dx} = (-3\sin v) \times \left(2e^u\right) \times \left(3x^2\right) = -18x^2 \sin v e^u = -18x^2 \sin\left(2e^{x^3}\right)e^{x^3}.$$

You can use brackets to help separate the terms which need to be multiplied and then Snalphabet to perform the multiplication. When you have performed the multiplication and simplified where you can you should use the original substitutions (in this case $u = x^3$ and $v = 2e^u$) to replace *u* and *v*.

(f) $y = \left(2 - 3x^2 + e^{x^3}\right)^3$ is a more complicated composite function than those previously discussed, you need to make two substitutions to allow you to the chain rule. Firstly using $u = 2 - 3x^2 + e^{x^3}$ you can re-write the function as $y = u^3$. To perform the differentiation you use the chain rule:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

The derivative of y with respect to u is straightforward $\frac{dy}{du} = 3u^2$.

To perform the derivative of u you need to use the chain rule again to differentiate the final term e^{x^3} . If $v = x^3$ then:

$$\frac{d}{dx}\left(e^{x^3}\right) = \frac{d}{dv}\left(e^v\right) \times \frac{dv}{dx} = \left(e^v\right) \times \left(3x^2\right) = 3x^2 e^{x^3}$$

So, using this result, along with the power rule:

$$\frac{du}{dx} = -6x + 3x^2 e^{x^3}$$

So:

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \left(3u^2\right) \times \left(-6x + 3x^2 e^{x^3}\right) = 3\left(2 - 3x^2 + e^{x^3}\right)^2 \left(-6x + 3x^2 e^{x^3}\right)$$