Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Calculus

Differentiating Basic Functions

This guide will show you how to identify and differentiate five basic functions.

Introduction

Many complicated functions can be broken down into combinations of these five elementary functions:

$$y = ax^n \qquad y = a\sin kx \qquad y = a\cos kx$$

$$y = ae^{kx} \qquad y = a\ln kx$$

The factsheet: *Five Basic Functions* gives more information about these functions and the study guide: *More Complicated Functions* shows you how to use these basic functions to make other, more complicated functions. Identifying how a more complicated function is created from these simpler functions allows you to choose a suitable rule to differentiate that function. A strategy for differentiating more complicated functions is then:

1. Identify how the complicated function can be broken down into more basic functions and choose an appropriate rule of differentiation:
   a) By addition or subtraction: use term-by-term differentiation (see study guide: *What is Differentiation?*)
   b) By composition: use the chain rule (see study guide: *The Chain Rule*).
   c) By multiplication: use the product rule (see study guide: *The Product Rule*).
   d) By division: use the quotient rule (see study guide: *The Quotient Rule*).

2. Differentiate the elementary functions as needed.

This study guide is designed to help you with the last part of this process. You can think of each of the basic functions of x above as a template with **numerical constants** (a and n in the first and a and k in the other four). If the function you are differentiating fits one of these templates then you can differentiate it using the relevant rule given in this study guide. This guide shows you how to match the function you need to differentiate with the relevant template by finding the correct values of the constants a, n or k.

This study guide is not concerned with how these results are derived but can be used to help in differentiating the functions using a given rule. (If you are interested, the results can be proved using rigorous differential methods – a **Learning Enhancement Tutor** will be happy to go through this with you.)

1. Differentiating $y = ax^n$

The study guide: *Differentiating using the Power Rule* covers this topic and it is recommended that you re-read this guide for help with using the following formulas, which together are known as the **power rule**. The main results of the power rule are:

If	$y = a$	then	$\frac{dy}{dx} = 0$
If	$y = ax$	then	$\frac{dy}{dx} = a$
If	$y = ax^n$	then	$\frac{dy}{dx} = anx^{n-1}$

2. Differentiating the trigonometric functions $y = a\sin kx$ and $y = a\cos kx$

The derivatives of the elementary sine and cosine functions are closely linked.

If	$y = a\sin kx$	then	$\frac{dy}{dx} = ak\cos kx$
If	$y = a\cos kx$	then	$\frac{dy}{dx} = -ak\sin kx$

Make special note of the minus sign in the derivative of cosine as this is often neglected by students who are learning calculus. As with differentiation of $y = ax^n$, the key to successfully finding the derivative is the identification of the constants *a* and *k*.

Example: Differentiate $y = 3\sin 2x$.

When attempting a problem like this, it is important to correctly identify the rule you need to use. Can you see that $y = 3\sin 2x$ fits the first rule in this section with $a = 3$ and $k = 2$? So the derivative is (using a dot to represent multiplication):

$$\frac{dy}{dx} = ak\cos kx = 3 \cdot 2 \cdot \cos(2 \cdot x) = 6\cos 2x \quad .$$

So the derivative of $y = 3\sin 2x$ is $\frac{dy}{dx} = 6\cos 2x$.

There are two special cases of these rules, when *a* and *k* are both equal to 1, which give the important results:

The derivative of	$y = \sin x$	is	$\frac{dy}{dx} = \cos x$
The derivative of	$y = \cos x$	is	$\frac{dy}{dx} = -\sin x$

3. Differentiating the exponential function $y = ae^{kx}$

If	$y = ae^{kx}$	then	$\frac{dy}{dx} = ake^{kx}$

The key to success in differentiating an exponential function of this type is identifying when a function fits this pattern **precisely** which means that you must correctly assign values to the constants *a* and *k*.

Example: Find the gradient of $y = \frac{e^{-2x}}{5}$.

Remember that the derivative and the gradient of a function are the same thing so the question is really asking you to find the derivative. The function which needs to be differentiated fits the pattern of the rule with $a = 1/5$ and $k = -2$ so the derivative is:

$$\frac{dy}{dx} = \frac{1}{5} \cdot (-2) \cdot e^{-2 \cdot x} = -\frac{2}{5} e^{-2x}$$

The case when $k = 1$ leads to the interesting result:

If	$y = ae^{x}$	then	$\frac{dy}{dx} = ae^{x}$

It is an important property of functions of the form $y = ae^{x}$ that they differentiate to give themselves.

4. Differentiating the natural logarithm function $y = a\ln kx$.

If	$y = a\ln kx$	then	$\frac{dy}{dx} = \frac{a}{x}$

The derivative of the function $y = a\ln kx$ is interesting as the constant *k* plays no part in the derivative. In other words, **the derivative is independent of *k***. You can use the laws of logarithms (see study guide: *The Laws of Logarithms*) to rewrite the function as:

$$y = a\ln kx = a(\ln kx) = a(\ln k + \ln x)$$
$$= a\ln k + a\ln x$$

As the first term $a\ln k$ is constant and as the derivative of a constant is zero (see section 1 of this guide), you only need to differentiate $y = a\ln x$ which has no k in it.

Example: Differentiate $y = -5\ln\left(\dfrac{3x}{7}\right)$.

In this case $k = 3/7$ but that does not matter since k plays no part in the derivative. When using the rule all you need to identify is a, in this case $a = -5$ and so the derivative of the function is:

$$\frac{dy}{dx} = -\frac{5}{x}$$

The case when $a = 1$ leads to the interesting and important result:

If	$y = \ln kx$	then $\dfrac{dy}{dx} = \dfrac{1}{x}$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761

Ask: ask.let@uea.ac.uk

Click: https://portal.uea.ac.uk/student-support-service/learning-enhancement

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