Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model Answers: Differentiating Basic Functions

Differentiating Basic Functions study guide

The rules in the following table are used in these model answers. They are referred to throughout this sheet by their number in this table.

rule	function	derivative
1	k	0
2	ax	a
3	ax^n	anx^{n-1}
4	$a\sin kx$	$ak\cos kx$
5	$a\cos kx$	$-ak\sin kx$
6	ae^{kx}	ake^{kx}
7	$a\ln(kx)$	a/x

Also a good knowledge of the laws of indices (see study guide: *Laws of Indices*) is essential when manipulating the questions which use rule 3. More details regarding rules 1, 2 and 3 can be found in the study guide: *Differentiating using the Power Rule*.

1.

(a) $\frac{dy}{dx} = 0$

To differentiate $y = 7$, use rule 1 with $k = 7$. Rule 1 can be interpreted as "the derivative of a constant is zero".

(b) $\frac{dy}{dx} = 0$

To differentiate $y = \frac{1}{7}$, use rule 1 with $k = \frac{1}{7}$. Rule 1 can be interpreted as "the derivative of a constant is zero".

(c) $\frac{dy}{dx} = -7$

To differentiate $y = -7x$, use rule 2 with $a = -7$. Remember that the minus sign is part of the constant *a*.

(d) $\frac{dy}{dx} = \frac{1}{7}$

To differentiate $y = \frac{1}{7}x$, use rule 2 with $a = \frac{1}{7}$.

(e) $\frac{dy}{dx} = \frac{1}{7}$

To differentiate $y = \frac{x}{7}$ you should notice that the algebraic fraction can be rewritten as $y = \frac{1}{7}x$, which is the same as the previous question. So use rule 2 with $a = \frac{1}{7}$.

(f) $\frac{dy}{dx} = \frac{3}{7}$

To differentiate $y = \frac{3x}{7}$ you should notice that the algebraic fraction can be rewritten as $y = \frac{3}{7}x$. So use rule 2 with $a = \frac{3}{7}$.

(g) $\frac{dy}{dx} = -14x$

To differentiate $y = -7x^2$ use rule 3 with $a = -7$ and $n = 2$. So

$$\frac{dy}{dx} = (-7)\cdot 2 \cdot x^{2-1} = -14x^1 = -14x$$

(h) $\frac{dy}{dx} = 21x^2$

To differentiate $y = 7x^3$ use rule 3 with $a = 7$ and $n = 3$. So

$$\frac{dy}{dx} = 7 \cdot 3 \cdot x^{3-1} = 21x^2$$

(i) $\frac{dy}{dx} = 14x^{-3}$

To differentiate $y = -7x^{-2}$ use rule 3 with $a = -7$ and $n = -2$. So

$$\frac{dy}{dx} = (-7)\cdot(-2)\cdot x^{-2-1} = 14x^{-3}$$

(j) $\frac{dy}{dx} = \frac{14}{x^3}$

To differentiate $y = -\frac{7}{x^2}$ you should notice that the algebraic fraction can be rewritten as $y = -7x^{-2}$ which is the same as the previous question. So use rule 3 with $a = -7$ and $n = -2$:

$$\frac{dy}{dx} = (-7)\cdot(-2)\cdot x^{-2-1} = 14x^{-3} = \frac{14}{x^3}$$

Notice that the answers to both (i) and (j) are identical mathematically but have been written in different ways. The answers are given in these forms to reflect the way the question has been expressed. It is good practice in mathematics to give your answer in a form which mirrors that of the question.

(k) $\frac{dy}{dx} = \frac{14}{3x^3}$

To differentiate $y = -\frac{7}{3x^2}$ you should notice that the algebraic fraction can be rewritten as $y = -\frac{7}{3}x^{-2}$. So use rule 3 with $a = -\frac{7}{3}$ and $n = -2$:

$$\frac{dy}{dx} = \left(-\frac{7}{3}\right)\cdot(-2)\cdot x^{-2-1} = \frac{14}{3}x^{-3} = \frac{14}{3x^3}$$

(l) $\frac{dy}{dx} = -\frac{2}{7x^3}$

To differentiate $y = \frac{1}{7x^2}$ you should notice that the algebraic fraction can be rewritten as $y = \frac{1}{7}x^{-2}$. So use rule 3 with $a = \frac{1}{7}$ and $n = -2$:

$$\frac{dy}{dx} = \frac{1}{7}\cdot(-2)\cdot x^{-2-1} = -\frac{2}{7}x^{-3} = -\frac{2}{7x^3}$$

(m) $\frac{dy}{dx} = 294x^5$

To differentiate $y = (7x^3)^2$ you first need to open the brackets using the law of indices $(ab)^n = a^n b^n$ to get $y = 49x^6$ and so use rule 3 with $a = 49$ and $n = 6$:

$$\frac{dy}{dx} = 49\cdot 6\cdot x^{6-1} = 294x^5$$

(n) $\frac{dy}{dx} = 294x^5$

To differentiate $y = (-7x^3)^2$ you first need to open the brackets to get $y = 49x^6$ and so use rule 3 with $a = 49$ and $n = 6$:

$$\frac{dy}{dx} = 49 \cdot 6 \cdot x^{6-1} = 294x^5$$

(o) $\frac{dy}{dx} = \frac{1}{2\sqrt[2]{x}}$

To differentiate $y = \sqrt[2]{x}$ you first need rewrite the question as $y = x^{1/2}$. So use rule 3 with $a = 1$ and $n = 1/2$:

$$\frac{dy}{dx} = 1 \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt[2]{x}}$$

(p) $\frac{dy}{dx} = \frac{7}{2\sqrt[2]{x}}$

To differentiate $y = 7\sqrt[2]{x}$ you first need rewrite the question as $y = 7x^{1/2}$. So use rule 3 with $a = 7$ and $n = 1/2$:

$$\frac{dy}{dx} = 7 \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{7}{2}x^{-\frac{1}{2}} = \frac{7}{2x^{\frac{1}{2}}} = \frac{7}{2\sqrt[2]{x}}$$

(q) $\frac{dy}{dx} = \frac{1}{14\sqrt[2]{x}}$

To differentiate $y = \frac{\sqrt[2]{x}}{7}$ you first need rewrite the question as $y = \frac{1}{7}x^{1/2}$. So use rule 3 with $a = 1/7$ and $n = 1/2$:

$$\frac{dy}{dx} = \frac{1}{7} \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} = \frac{1}{14}x^{-\frac{1}{2}} = \frac{1}{14x^{\frac{1}{2}}} = \frac{1}{14\sqrt[2]{x}}$$

(r) $\frac{dy}{dx} = -\frac{7}{2\sqrt[2]{x^3}}$

To differentiate $y = \frac{7}{\sqrt[2]{x}}$ you first need rewrite the question as $y = 7x^{-1/2}$. So use rule 3 with $a = 7$ and $n = -1/2$:

$$\frac{dy}{dx} = 7 \cdot \left(-\frac{1}{2}\right) \cdot x^{-\frac{1}{2}-1} = -\frac{7}{2}x^{-\frac{3}{2}} = -\frac{7}{2x^{\frac{3}{2}}} = -\frac{7}{2\sqrt[2]{x^3}}$$

(s) $\frac{dy}{dx} = \frac{7}{2\sqrt[2]{x^3}}$

To differentiate $y = -\frac{7}{\sqrt[2]{x}}$ you first need rewrite the question as $y = -7x^{-1/2}$. So use rule 3 with $a = -7$ and $n = -1/2$:

$$\frac{dy}{dx} = (-7)\cdot\left(-\frac{1}{2}\right)\cdot x^{-\frac{1}{2}-1} = \frac{7}{2}x^{-\frac{3}{2}} = \frac{7}{2x^{\frac{3}{2}}} = \frac{7}{2\sqrt[2]{x^3}}$$

(t) $\frac{dy}{dx} = \frac{1}{14\sqrt[2]{x^3}}$

To differentiate $y = -\frac{1}{7\sqrt[2]{x}}$ you first need rewrite the question as $y = -\frac{1}{7}x^{-1/2}$. So use rule 3 with $a = -1/7$ and $n = -1/2$:

$$\frac{dy}{dx} = \left(-\frac{1}{7}\right)\cdot\left(-\frac{1}{2}\right)\cdot x^{-\frac{1}{2}-1} = \frac{1}{14}x^{-\frac{3}{2}} = \frac{1}{14x^{\frac{3}{2}}} = \frac{1}{14\sqrt[2]{x^3}}$$

2.

(a) $\frac{dy}{dx} = 7\cos x$

To differentiate $y = 7\sin x$ use rule 4 with $a = 7$ and $k = 1$:

$$\frac{dy}{dx} = 7\cdot 1\cdot \cos x = 7\cos x$$

(b) $\frac{dy}{dx} = -7\sin(7x)$

To differentiate $y = \cos(7x)$ use rule 5 with $a = 1$ and $k = 7$:

$$\frac{dy}{dx} = -1\cdot 7\cdot \sin(7x) = -7\sin(7x)$$

(c) $\frac{dy}{dx} = 21\cos(7x)$

To differentiate $y = 3\sin 7x$ use rule 4 with $a = 3$ and $k = 7$:

$$\frac{dy}{dx} = 3\cdot 7\cdot \cos(7x) = 21\cos(7x)$$

(d) $\frac{dy}{dx}=\frac{1}{7}\cos\left(\frac{x}{7}\right)$

To differentiate $y=\sin\left(\frac{x}{7}\right)$ use rule 4 with $a=1$ and $k=1/7$:

$$\frac{dy}{dx}=1\cdot\frac{1}{7}\cdot\cos\left(\frac{x}{7}\right)=\frac{1}{7}\cos\left(\frac{x}{7}\right)$$

(e) $\frac{dy}{dx}=\frac{\cos x}{7}$

To differentiate $y=\frac{\sin x}{7}$ use rule 4 with $a=1/7$ and $k=1$:

$$\frac{dy}{dx}=\frac{1}{7}\cdot 1\cdot\cos x=\frac{1}{7}\cos x=\frac{\cos x}{7}$$

(f) $\frac{dy}{dx}=7\sin(-x)$

To differentiate $y=7\cos(-x)$ use rule 5 with $a=7$ and $k=-1$:

$$\frac{dy}{dx}=-7\cdot(-1)\cdot\sin(-x)=7\sin(-x)$$

(g) $\frac{dy}{dx}=\frac{1}{7}\sin\left(\frac{x}{7}\right)$

To differentiate $y=-\cos\left(\frac{x}{7}\right)$ use rule 5 with $a=-1$ and $k=1/7$:

$$\frac{dy}{dx}=-(-1)\cdot\frac{1}{7}\cdot\sin\left(\frac{x}{7}\right)=\frac{1}{7}\sin\left(\frac{x}{7}\right)$$

(h) $\frac{dy}{dx}=\frac{-3\sin(-3x)}{7}$

To differentiate $y=\frac{-\cos(-3x)}{7}$ use rule 5 with $a=-1/7$ and $k=-3$:

$$\frac{dy}{dx}=-\left(-\frac{1}{7}\right)\cdot(-3)\cdot\sin(-3x)=-\frac{3}{7}\sin(-3x)$$

(i) $\frac{dy}{dx}=-\frac{3}{7}\sin\left(\frac{3x}{7}\right)$

To differentiate $y=\cos\left(\frac{3x}{7}\right)$ use rule 5 with $a=1$ and $k=3/7$:

$$\frac{dy}{dx}=-1\cdot\frac{3}{7}\cdot\sin\left(\frac{3x}{7}\right)=-\frac{3}{7}\sin\left(\frac{3x}{7}\right)$$

(j) $\frac{dy}{dx} = -\frac{1}{21}\cos\left(\frac{x}{7}\right)$

To differentiate $y = -\frac{\sin(x/7)}{3}$ use rule 4 with $a = -1/3$ and $k = 1/7$:

$$\frac{dy}{dx} = \left(-\frac{1}{3}\right)\cdot\frac{1}{7}\cdot\cos\left(\frac{x}{7}\right) = -\frac{1}{21}\cos\left(\frac{x}{7}\right)$$

(k) $\frac{dy}{dx} = \frac{-3\sin(x)}{7}$

To differentiate $y = \frac{3\cos x}{7}$ use rule 5 with $a = 3/7$ and $k = 1$:

$$\frac{dy}{dx} = -\frac{3}{7}\cdot 1\cdot \sin(x) = -\frac{3}{7}\sin(x)$$

(l) $\frac{dy}{dx} = 7e^{x}$

To differentiate $y = 7e^{x}$ use rule 6 with $a = 7$ and $k = 1$:

$$\frac{dy}{dx} = 7\cdot 1\cdot e^{x} = 7e^{x}$$

(m) $\frac{dy}{dx} = -7e^{-7x}$

To differentiate $y = e^{-7x}$ use rule 6 with $a = 1$ and $k = -7$:

$$\frac{dy}{dx} = 1\cdot(-7)\cdot e^{-7x} = -7e^{-7x}$$

(n) $\frac{dy}{dx} = -\frac{e^{-x}}{7}$

To differentiate $y = \frac{e^{-x}}{7}$ use rule 6 with $a = 1/7$ and $k = -1$:

$$\frac{dy}{dx} = \left(\frac{1}{7}\right)\cdot(-1)\cdot e^{-x} = -\frac{e^{-x}}{7}$$

(o) $\frac{dy}{dx} = 0$

To differentiate $y = e$, remember that e is just a number and so use rule 1 with $k = e$.

(p) $\frac{dy}{dx} = -\frac{1}{7e^x}$

To differentiate $y = \frac{1}{7e^x}$ you should realise that you can rewrite the question as $y = \frac{e^{-x}}{7}$. This the same as the previous question so use rule 6 with $a = 1/7$ and $k = -1$:

$$\frac{dy}{dx} = \left(\frac{1}{7}\right) \cdot (-1) \cdot e^{-x} = -\frac{e^{-x}}{7} = -\frac{1}{7e^x}$$

(q) $\frac{dy}{dx} = -\frac{21}{e^{3x}}$

To differentiate $y = \frac{7}{e^{3x}}$ you should realise that you can rewrite the question as $y = 7e^{-3x}$ so use rule 6 with $a = 7$ and $k = -3$:

$$\frac{dy}{dx} = 7 \cdot (-3) \cdot e^{-3x} = -21e^{-3x} = -\frac{21}{e^{3x}}$$

(r) $\frac{dy}{dx} = 2e^{2x}$

To differentiate $y = (e^x)^2$ you need to open the bracket using the law of indices $(x^a)^b = x^{ab}$ and so the question becomes $y = e^{2x}$. Therefore use rule 6 with $a = 1$ and $k = 2$:

$$\frac{dy}{dx} = 1 \cdot 2 \cdot e^{2x} = 2e^{2x}$$

(s) $\frac{dy}{dx} = -\frac{2}{e^{2x}}$

To differentiate $y = \left(\frac{1}{e^x}\right)^2$ you should realise that, when you use the laws of indices $\frac{1}{x^a} = x^{-a}$ and $(x^a)^b = x^{ab}$ to open the bracket, the question becomes $y = e^{-2x}$ so use rule 6 with $a = 1$ and $k = -2$:

$$\frac{dy}{dx} = 1 \cdot (-2) \cdot e^{-2x} = -2e^{-2x} = -\frac{2}{e^{2x}}$$

(t) $\frac{dy}{dx} = \frac{7}{x}$

To differentiate $y = 7\ln x$ use rule 7 with $a = 7$ and $k = 1$.

(u) $\frac{dy}{dx} = \frac{1}{x}$

To differentiate $y = \ln(7x)$ use rule 7 with $a = 1$ and $k = 7$. Note that only a plays a part in the answer.

(v) $\frac{dy}{dx} = 0$

To differentiate $y = \ln 7$, remember that $\ln 7$ is a number and so use rule 1 with $k = \ln 7$. Rule 1 can be interpreted as "the derivative of a constant is zero".

(w) $\frac{dy}{dx} = \frac{1}{7x}$

To differentiate $y = \frac{\ln x}{7}$ use rule 7 with $a = 1/7$ and $k = 1$.

(x) $\frac{dy}{dx} = \frac{1}{x}$

To differentiate $y = \ln\left(\frac{x}{7}\right)$ use rule 7 with $a = 1$ and $k = 1/7$. Note that only a plays a part in the answer.

3.

(a) $\frac{dy}{dx} = 14x + 1$

To differentiate the function $y = 7x^2 + x - 1/3$ use term-by-term differentiation. Use rule 3 for the first term with $a = 7$ and $n = 2$, use rule 2 for the second term with $a = 1$ and use rule 1 for the third term.

(b) $\frac{dy}{dx} = 3x^2 - 6x - 5$

To differentiate the function $y = x^3 - 3x^2 - 5x + 1$ use term-by-term differentiation. Use rule 3 for the first term with $a = 1$ and $n = 3$ and the second term with $a = -3$ and $n = 2$, use rule 2 for the third term with $a = -5$ and use rule 1 for the third term.

(c) $\frac{dy}{dx} = \cos x - \sin x$

To differentiate the function $y = \sin x + \cos x$ use term-by-term differentiation. Use rule 4 for the first term with $a = 1$ and $k = 1$ and use rule 5 for the second term with $a = 1$ and $k = 1$.

(d) $\frac{dy}{dx} = -7\sin x - 7\cos(7x)$

To differentiate the function $y = 7\cos x - \sin(7x)$ use term-by-term differentiation. Use rule 5 for the first term with $a = 7$ and $k = 1$ and use rule 4 for the second term with $a = 1$ and $k = 7$.

(e) $\frac{dy}{dx} = e^x + e^{-x}$

To differentiate the function $y = e^x - e^{-x} + 1/2$ use term-by-term differentiation. Use rule 6 for the first term with $a = 1$ and $k = 1$ and also for the second term with $a = -1$ and $k = -1$ then use rule 1 for the third term.

(f) $\frac{dy}{dx} = 0$

To differentiate $y = 7 - \ln 3$, remember that both 7 and $\ln 3$ are both numbers and so use rule 1 to differentiate both of the terms. As they are both constants the derivative of both terms is zero.

(g) $\frac{dy}{dx} = \frac{2}{7}x + \frac{2}{7} = \frac{2(x+1)}{7}$

To differentiate the function $y = \frac{x^2 + 2x + 2}{7}$ you should realise that you can express the function as three terms to give $y = \frac{1}{7}x^2 + \frac{2}{7}x + \frac{2}{7}$ and then use term-by-term differentiation. Use rule 3 for the first term with $a = 1/7$ and $n = 2$, use rule 2 for the second term with $a = 2/7$ and $n = 2$ and use rule 1 for the third term.

(h) $\frac{dy}{dx} = 2x + 2$

To differentiate the function $y = (x+1)^2$ you should open the brackets to give $y = x^2 + 2x + 1$ and then use term-by-term differentiation. Use rule 3 for the first term with $a = 1$ and $n = 2$, use rule 2 for the second term with $a = 2$ and $n = 2$ and use rule 1 for the third term.

(i) $\frac{dy}{dx} = 2e^{2x} - 2e^{-2x}$

To differentiate the function $y = (e^x - e^{-x})^2 + 1$ you should open the brackets to give $y = e^{2x} - 2 + e^{-2x} + 1 = e^{2x} + e^{-2x} - 1$ and the use term-by-term differentiation. Use rule 6 for the first term with $a = 1$ and $k = 2$ and also for the second term with $a = 1$ and $k = -2$. Use rule 1 for the third term.

(j) $\dfrac{dy}{dx} = -2\sin(2x) - 2\cos(2x)$

To differentiate the function $y = \cos(2x) - \sin(2x)$ use term-by-term differentiation. Use rule 5 for the first term with $a = 1$ and $k = 2$ and use rule 4 for the second term with $a = -1$ and $k = 2$.

(k) $y = \dfrac{2}{3}e^{2x} - \dfrac{2}{3}e^{-2x} = \dfrac{2\left(e^{2x} - e^{-2x}\right)}{3}$

To differentiate the function $y = \dfrac{e^{2x} + e^{-2x}}{3}$ you should realise that you can express the function as two terms to give $y = \dfrac{1}{3}e^{2x} + \dfrac{1}{3}e^{-2x}$ and then use term-by-term differentiation. Use rule 6 for the first term with $a = 1/3$ and $k = 2$ and also for the second term with $a = 1/3$ and $k = -2$.

(l) $y = -\dfrac{2}{x^2} + \dfrac{2}{x^3} = \dfrac{2(1-x)}{x^3}$

To differentiate the function $y = \dfrac{x^2 + 2x - 1}{x^2}$ you should realise that you can express the function as three terms to give $y = 1 + \dfrac{2}{x} - \dfrac{1}{x^2}$ and then use term-by-term differentiation. Use rule 1 for the first term and rule 3 for the second term with $a = 2$ and $n = -1$ and also for term 3 with $a = -1$ and $n = -2$.