Learning Enhancement Team

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Model Answers: Differentiating $y = ax^n$

Differentiating using the Power Rule study guide

You can do all the questions on this sheet using the rule:

If	$y = ax^n$
then	$\frac{dy}{dx} = anx^{n-1}$

You just need to identify the *a* and the *n.*

1.

a) $\frac{dy}{dx} = 9x^2$

The question $y = 3x^3$ fits the pattern $y = ax^n$ with $a = 3$ and $n = 3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 3 \cdot 3 \cdot x^{3-1} = 9x^2$$

b) $\frac{dy}{dx} = 3x^2$

The question $y = x^3$ fits the pattern $y = ax^n$ with $a = 1$ and $n = 3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 1 \cdot 3 \cdot x^{3-1} = 3x^2$$

c) $\frac{dy}{dx} = -4x$

The question $y = -2x^2$ fits the pattern $y = ax^n$ with $a = -2$ and $n = 2$ and so:

$$\frac{dy}{dx} = anx^{n-1} = -2 \cdot 2 \cdot x^{2-1} = -4x$$

d) $\frac{dy}{dx} = 9.9x^8$

The question $y = 1.1x^9$ fits the pattern $y = ax^n$ with $a = 1.1$ and $n = 9$ and so:

$$\frac{dy}{dx} = anx^{n-1} = (1.1)\cdot 9 \cdot x^{9-1} = 9.9x^8$$

e) $\frac{dy}{dx} = 0$

The question $y = 7$ fits the pattern $y = ax^n$ with $a = 7$ and $n = 0$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 7 \cdot 0 \cdot x^{-1} = 0$$

But you might also notice that, because y is a constant, $\frac{dy}{dx} = 0$.

f) $\frac{dy}{dx} = 0$

As in the previous question, y is a constant and so $\frac{dy}{dx} = 0$.

g) $\frac{dy}{dx} = 0$

As in the previous question, y is a constant and so $\frac{dy}{dx} = 0$.

h) $\frac{dy}{dx} = 0$

As in the previous question, y is a constant and so $\frac{dy}{dx} = 0$.

i) $\frac{dy}{dx} = 2$

The question $y = 2x$ fits the pattern $y = ax^n$ with $a = 2$ and $n = 1$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 2 \cdot 1 \cdot x^{1-1} = 2x^0 = 2 \cdot 1 = 2$$

But you might also notice that this is in the form $y = mx + c$ and so it is a straight line with gradient $m = \frac{dy}{dx} = 2$ (see study guide: *What is a Straight Line?*).

j) $\frac{dy}{dx} = -1$

You must differentiate each term (4 and $-x$) separately and then add the results.

The number 4 is a constant and so differentiates to 0 and $-x$ differentiates to -1 and so adding them gives -1.

But you might also notice that this is in the form $y = mx + c$ and so it is a straight line with gradient $m = \frac{dy}{dx} = -1$.

k) $\frac{dy}{dx} = \frac{2}{3}x^2$

The question $y = \frac{2}{9}x^3$ fits the pattern $y = ax^n$ with $a = \frac{2}{9}$ and $n = 3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = \frac{2}{9} \cdot 3 \cdot x^{3-1} = \frac{6}{9}x^2 = \frac{2}{3}x^2$$

l) $\frac{dy}{dx} = 2 - 12x^2$

You must differentiate each term (-3, $2x$ and $-4x^3$) separately and then add the results together.

The number -3 is a constant and so differentiates to 0, $2x$ differentiates to 2 and $-4x^3$ differentiates to $-12x^2$. Adding these gives the answer.

2. The gradient of a curve is given by $\frac{dy}{dx}$ and so, to find the gradient at $x = 1$, you need to substitute $x = 1$ into your answers for question 1.

a) 9

From question 1, $\frac{dy}{dx} = 9x^2$. If you substitute $x = 1$ into this you get $\frac{dy}{dx} = 9 \cdot 1^2 = 9$.

b) 3

From question 1, $\frac{dy}{dx} = 3x^2$. If you substitute $x = 1$ into this you get $\frac{dy}{dx} = 3 \cdot 1^2 = 3$.

c) -4

From question 1, $\frac{dy}{dx} = -4x$. If you substitute $x = 1$ into this you get $\frac{dy}{dx} = -4 \cdot 1 = -4$.

d) 9.9

From question 1, $\frac{dy}{dx} = 9.9x^8$. If you substitute $x = 1$ into this you get $\frac{dy}{dx} = 9.9 \cdot 1^2 = 9.9$

e) 0

From question 1, $\frac{dy}{dx} = 0$. So the gradient is 0 for any *x*.

f) 0

From question 1, $\frac{dy}{dx} = 0$. So the gradient is 0 for any *x*.

g) 0

From question 1, $\frac{dy}{dx} = 0$. So the gradient is 0 for any *x*.

h) 0

From question 1, $\frac{dy}{dx} = 0$. So the gradient is 0 for any *x*.

i) 2

From question 1, $\frac{dy}{dx} = 2$. So the gradient is 2 for any *x*.

j) -1

From question 1, $\frac{dy}{dx} = -1$. So the gradient is -1 for any *x*.

k) $\frac{2}{3}$

From question 1, $\frac{dy}{dx} = \frac{2}{3}x^2$. If you substitute $x = 1$ into this you get $\frac{dy}{dx} = \frac{2}{3} \cdot 1^2 = \frac{2}{3}$.

l) -10

From question 1, $\frac{dy}{dx} = 2 - 12x^2$. If you substitute $x = 1$ into this you get:

$$\frac{dy}{dx} = 2 - 12 \cdot 1^2 = 2 - 12 = -10$$

3. In these questions you may need to manipulate the equations in order to get them to fit the pattern $y = ax^n$. You will need to be familiar with the laws of indices. See the study guide: *Laws of Indices* if you need help with this. You should also express your answer in the form of the question, so for example, if the question uses roots of reciprocals, you should give your answer in that form.

a) $\frac{dy}{dx} = \frac{3}{2}x^2 = \frac{3x^2}{2}$

You can rewrite the question $y = \frac{x^3}{2}$ as $y = \frac{1}{2}x^3$ and then it fits the pattern $y = ax^n$ with $a = \frac{1}{2}$ and $n = 3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = \frac{1}{2} \cdot 3 \cdot x^{3-1} = \frac{3}{2}x^2$$

b) $\frac{dy}{dx} = -\frac{1}{3}$

You can rewrite the question $y = \frac{4-x}{3}$ as $y = \frac{4}{3} - \frac{1}{3}x$. So then the constant $\frac{4}{3}$ differentiates to 0 and $-\frac{1}{3}x$ fits the pattern ax^n with $a = -\frac{1}{3}$ and $n = 1$ and so:

$$\frac{dy}{dx} = -\frac{1}{3}.$$

c) $\frac{dy}{dx} = -2x^{-2}$

The function $y = 2x^{-1}$ fits the pattern $y = ax^n$ with $a = 2$ and $n = -1$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 2 \cdot -1 \cdot x^{-1-1} = -2x^{-2}$$

d) $\frac{dy}{dx} = -2x^{-2} = -\frac{2}{x^2}$

You can rewrite the question $y = \frac{2}{x}$ as $y = 2x^{-1}$ and then it is the same as the previous question.

e) $\frac{dy}{dx} = -15x^{-4} = -\frac{15}{x^4}$

You can rewrite the question $y = \frac{5}{x^3}$ as $y = 5x^{-3}$ and then it fits the pattern $y = ax^n$ with $a = 5$ and $n = -3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 5 \cdot -3 \cdot x^{-3-1} = -15x^{-4}$$

f) $\frac{dy}{dx} = \frac{3}{5}x^2$

The function $y = \frac{1}{5}x^3$ fits the pattern $y = ax^n$ with $a = \frac{1}{5}$ and $n = 3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = \frac{1}{5} \cdot 3 \cdot x^{3-1} = \frac{3}{5}x^2$$

g) $\frac{dy}{dx} = \frac{3}{5}x^2 = \frac{3x^2}{5}$

You can rewrite the question $y = \frac{x^3}{5}$ as $y = \frac{1}{5}x^3$ and then it is the same as the previous question.

h) $\frac{dy}{dx} = -\frac{3}{5}x^{-4} = -\frac{3}{5x^4}$

You can rewrite the question $y = \frac{1}{5x^3}$ as $y = \frac{1}{5}x^{-3}$ and then it fits the pattern $y = ax^n$ with $a = \frac{1}{5}$ and $n = -3$ and so:

$$\frac{dy}{dx} = anx^{n-1} = \frac{1}{5} \cdot -3 \cdot x^{-3-1} = -\frac{3}{5}x^{-4}$$

i) $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$

The function fits the pattern $y=ax^n$ with $a=1$ and $n=\frac{1}{2}$ and so

$$\frac{dy}{dx}=anx^{n-1}=1\cdot\frac{1}{2}\cdot x^{\frac{1}{2}-1}=\frac{1}{2}x^{-\frac{1}{2}}$$

j) $\frac{dy}{dx}=\frac{1}{2}x^{-\frac{1}{2}}=\frac{1}{2\sqrt{x}}$

You can rewrite the question $y=\sqrt{x}$ as $y=x^{\frac{1}{2}}$ and so it is the same as the previous question.

k) $\frac{dy}{dx}=\frac{1}{21}x^{-\frac{2}{3}}=\frac{1}{21\sqrt[3]{x^2}}$

You can rewrite the question $y=\frac{\sqrt[3]{x}}{7}$ as $y=\frac{1}{7}x^{\frac{1}{3}}$ and then it fits the pattern $y=ax^n$ with $a=\frac{1}{7}$ and $n=\frac{1}{3}$ and so:

$$\frac{dy}{dx}=anx^{n-1}=\frac{1}{7}\cdot\frac{1}{3}\cdot x^{\frac{1}{3}-1}=\frac{1}{21}x^{-\frac{2}{3}}$$

l) $\frac{dy}{dx}=-\frac{1}{15}x^{-\frac{4}{5}}=-\frac{1}{15\sqrt[5]{x^4}}$

You can rewrite the question $y=\frac{4-\sqrt[5]{x}}{3}$ as $y=\frac{4}{3}-\frac{1}{3}x^{\frac{1}{5}}$. The first term, $\frac{4}{3}$ is a constant and so differentiates to 0. The second term and then it fits the pattern $y=ax^n$ with $a=-\frac{1}{3}$ and $n=\frac{1}{5}$ and so:

$$\frac{dy}{dx}=anx^{n-1}=-\frac{1}{3}\cdot\frac{1}{5}\cdot x^{\frac{1}{5}-1}=-\frac{1}{15}x^{-\frac{4}{5}}$$

m) $\frac{dy}{dx}=-x^{-\frac{3}{2}}=-\frac{1}{\sqrt{x^3}}$

You can rewrite the question $y=\frac{2}{\sqrt{x}}$ as $y=2x^{-\frac{1}{2}}$ and then it fits the pattern $y=ax^n$ with $a=2$ and $n=-\frac{1}{2}$ and so:

$$\frac{dy}{dx}=anx^{n-1}=2\cdot-\frac{1}{2}\cdot x^{-\frac{1}{2}-1}=-x^{-\frac{3}{2}}$$

n) $\frac{dy}{dx}=-x^{-\frac{8}{7}}=-\frac{1}{\sqrt[7]{x^8}}$

You can rewrite the question $y=\frac{7}{\sqrt[7]{x}}$ as $y=7x^{-\frac{1}{7}}$ and then it fits the pattern $y=ax^n$ with $a=7$ and $n=-\frac{1}{7}$ and so:

$$\frac{dy}{dx} = anx^{n-1} = 7 \cdot -\frac{1}{7} \cdot x^{-\frac{1}{7}-1} = -x^{-\frac{8}{7}}$$

o) $\frac{dy}{dx} = 0$

Adding the two fractions gives you 0 which is a constant and so differentiates to 0.

4. Differentiation does not just apply to *x*'s and *y*'s. You must be able to differentiate many different formulae and use the correct terms. In order to do this you must know which symbols are constants and which ones are variables.

a) $\frac{dE}{dv} = mv$

The equation fits the pattern $y = ax^n$ with the variables $y = E$ and $x = v$, the constant $a = \frac{1}{2}m$ and the power $n = 2$ and so:

$$\frac{dE}{dv} = \frac{1}{2}m \cdot 2v^{2-1} = mv$$

b) $\frac{dE}{dP} = -\frac{Q}{P^2}$

The equation $E = \frac{Q}{P}$ can be rewritten as $E = QP^{-1}$ which fits the pattern $y = ax^n$ with the variables $y = E$ and $x = P$, the constant $a = Q$ and the power $n = -1$ and so:

$$\frac{dE}{dP} = Q \cdot (-1) \cdot P^{-1-1} = -\frac{Q}{P^2}$$

c) $\frac{dE}{dQ} = \frac{1}{P}$

The equation $E = \frac{Q}{P}$ fits the pattern $y = ax^n$ with the variables $y = E$ and $x = Q$, the constant $a = \frac{1}{P}$ and the power $n = 1$ and so it is a straight line equation with $m = \frac{dE}{dQ} = \frac{1}{P}$.

d) $\frac{dE}{dm} = c^2$

The equation $E = mc^2$ fits the pattern $y = ax^n$ with the variables $y = E$ and $x = m$, the constant $a = c^2$ and the power $n = 1$ and so it is a straight line equation with gradient c^2.

e) $\frac{ds}{dt} = u + at$

You can use term-by-term differentiation for $s = ut + \frac{1}{2}at^2$. The first term fits the pattern $y = ax^n$ with the variables $y = s$ and $x = t$, the constant $a = u$ and the power $n = 1$ and so it is a straight line equation with gradient u. The second term fits the pattern $y = ax^n$ with the variables $y = s$ and $x = t$, the constant $a = \frac{1}{2}a$ and the power $n = 2$ and so it differentiates to give at. Adding these two results gives the overall derivative.

f) $\frac{d(v^2)}{ds} = 2a$

You can use term-by-term differentiation for $v^2 = u^2 + 2as$. The first term is constant with respect to s and so has a derivative of 0. The second term fits the pattern $y = ax^n$ with the variables $y = v^2$ and $x = s$, the constant $a = 2a$ and the power $n = 1$ and so it is a straight line equation with gradient $2a$.

g) $\frac{dy}{dx} = m$

This is an equation of a straight line and so has a gradient, and therefore derivative of m.