

Worksheet: Basics of Relations

This worksheet covers the basics of relations, including what a relation is, and reflexive, symmetric, antisymmetric and transitive relations. You could read the factsheet: [Sets](#), for help with the language and symbols of **sets**.

Model answers for
this sheet



Basics of Relations
study guide



1. You are given the following sets:

$$A = \{\text{triangle, square, circle, cross}\}$$

$$B = \{\text{pink, blue, red, green}\}$$

$$C = \{\text{up, down, left, right}\}$$

Decide whether each of the sets (1)–(4) are relations. If not, explain why not.

- a) (1) from A to B , with (1) = $\{(\text{triangle, green}), (\text{circle, red}), (\text{cross, blue}), (\text{square, pink})\}$.
- b) (2) from C to A , with (2) = $\{(\text{up, square}), (\text{left, cross}), (\text{pink, blue})\}$.
- c) (3) from B to C , with (3) = $\{(\text{up, green}), (\text{down, blue}), (\text{left, pink}), (\text{red, right})\}$.
- d) (4) from A to C , with (4) = $\{(\text{triangle, left})\}$.

2. You are given a set $D = \{a, b, c, d\}$, and a list of relations (5)–(12) on D . For each relation, decide whether or not that relation is reflexive, symmetric, antisymmetric or transitive. Explain your decisions clearly.

$$(5) = \{(a, b), (b, a)\}$$

$$(6) = \{(b, c), (c, d), (b, d)\}$$

$$(7) = \{(a, a), (a, b), (b, c), (a, c), (b, b), (c, d), (b, d)\}$$

$$(8) = \{(a, a), (b, b), (a, c), (c, a), (c, d), (c, c), (d, d)\}$$

$$(9) = \{(a, a), (b, b), (a, b), (b, a), (c, d), (d, c), (c, c)\}$$

$$(10) = \{(a, b), (b, c), (c, d), (a, c), (b, d)\}$$

$$(11) = \{(a, a), (b, b), (c, c), (c, d)\}$$

$$(12) = D \times D \text{ (the universal relation)}$$

3. Remember that a relation E is called an **equivalence relation** if E is reflexive, transitive and symmetric.

Another kind of relation P is a **partial order**. You can say that P is a partial order if P is reflexive, transitive and antisymmetric.

For each relation (13)–(18) below, decide whether or not that relation is reflexive, symmetric, antisymmetric or transitive.

Write down which of these are equivalence relations, partial orders or neither. Are there any which are both? Explain all of your decisions clearly. NQZ

- a) $(13) \subseteq \mathbb{R} \times \mathbb{R}$, where $(13) = \{(a, b) \in \mathbb{R} \times \mathbb{R} : a - b = 0\}$
- b) $(14) \subseteq \mathbb{N} \times \mathbb{N}$, where $(14) = \{(a, b) \in \mathbb{N} \times \mathbb{N} : a \text{ divides } b\}$
- c) $(15) \subseteq \mathbb{Z} \times \mathbb{Z}$, where $(15) = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : ab \neq 0\}$
- d) $(16) \subseteq P(S) \times P(S)$, where S is any set and $(16) = \{(A, B) \in P(S) \times P(S) : A \subseteq B\}$
- e) $(17) \subseteq P(S) \times P(S)$, where S is any set and $(17) = \{(A, B) \in P(S) \times P(S) : |A| = |B|\}$
- f) $(18) \subseteq (\mathbb{N} \times \mathbb{N}) \times (\mathbb{N} \times \mathbb{N})$, where
- $$(18) = \{(a, b), (c, d) \in (\mathbb{N} \times \mathbb{N}) \times (\mathbb{N} \times \mathbb{N}) : a \leq b \text{ and } c \leq d\}$$



This worksheet is one of a series on mathematics produced by the Learning Enhancement Team with funding from the UEA Alumni Fund. Scan the QR-code with a smartphone app for [more resources](#).



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