Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model Answers: Basics of Logarithms

Basics of Logarithms study guide

1. The logarithmic transformation is:

$$y = a^x \Leftrightarrow \log_a y = x$$

note that the arrow works both ways. So going from left-to-right helps you convert an exponential equation to a logarithm and vice-versa.

(a) $8 = 2^3 \Leftrightarrow \log_2 8 = 3$

(b) $10^6 = 1000000 \Leftrightarrow \log_{10} 1000000 = 6$

(c) $64 = 8^2 \Leftrightarrow \log_8 64 = 2$

(d) $0.04 = 5^{-2} \Leftrightarrow \log_5 0.04 = -2$

(e) $\log_3 81 = 4 \Leftrightarrow 3^4 = 81$

(f) $\log_{10} 0.001 = -3 \Leftrightarrow 10^{-3} = 0.001$

(g) $0 = \log_7 1 \Leftrightarrow 7^0 = 1$

(h) $1 = \log_7 7 \Leftrightarrow 7^1 = 7$

2.

(a) $9 = 3^x \Leftrightarrow \log_3 9 = x$

This could help you find *x* as $\log_3 9$ can be found using a calculator and gives $x = 2$. You could also deduce the answer as $3^2 = 9$.

(b) $x = 9^3 \Leftrightarrow \log_9 x = 3$

This does not help you find *x*, you can find it directly as $9^3 = 729$.

(c) $3 = 9^x \Leftrightarrow \log_9 3 = x$

This could help you find *x* as $\log_9 3$ can be found using a calculator and gives $x = 1/2$. You could also deduce the answer as $9^{1/2} = 3$.

(d) $9 = x^3 \Leftrightarrow \log_x 9 = 3$

This does not help you find *x* as you can find it by calculating the cube root of 9.

(e) $x = 3^9 \Leftrightarrow \log_3 x = 9$

This does not help you find *x* as you can find it directly as $3^9 = 19683$.

(f) $3 = x^9 \Leftrightarrow \log_x 3 = 9$

This does not help you find *x* as you can find it by calculating the ninth root of 3.

(g) $x = 3^0 \Leftrightarrow \log_3 x = 0$

This does not help you find *x* as you can find it directly as $3^0 = 1$.

(h) $1 = x^0 \Leftrightarrow \log_x 1 = 0$

This is a well-known identity in logarithms, that the logarithm base *x* of 1 is 0.

(i) $3 = 3^x \Leftrightarrow \log_3 3 = x$

This is an example of a well-known identity in logarithms, that the logarithm in some base of that number is 1, mathematically this is written as $\log_a a = 1$. You could also deduce the answer as $3^1 = 3$.

(j) $x^3 = \frac{1}{27} \Leftrightarrow 3 = \log_x \frac{1}{27}$

This does not help you find *x* as you can find it by calculating the cube root of $1/27$ which is $1/3$.

(k) $\frac{1}{27} = 3^x \Leftrightarrow \log_3 \frac{1}{27} = x$

This could help you find *x* as $\log_9(1/27)$ can be found using a calculator and gives $x = -3$. You could also deduce the answer as $1/27 = 3^{-3}$.

(l) $3^x = -3 \Leftrightarrow x = \log_3(-3)$

You should notice that there is no power of 3 which gives a negative number. Logarithms of negative numbers can be found but they are outside the remit of this guide. If you need to understand the logarithms of negative numbers then you can talk to a **Learning Enhancement Tutor**.

3.

(a) $x = 0.613$ to 3 d.p.

Use the logarithmic transformation to give $6^x = 3 \Leftrightarrow x = \log_6 3 = 0.613$ to 3 d.p.

(b) $x = 1.301$ to 3 d.p.

Use the logarithmic transformation to give $10^x = 20 \Leftrightarrow x = \log_{10} 20 = 1.301$ to 3 d.p.

(c) $x = 0.693$ to 3 d.p.

Use the logarithmic transformation to give $3^{4x} = 21 \Leftrightarrow 4x = \log_3 21$, then divide by 4 to give $x = 0.693$ to 3 d.p.

(d) $x = 0.317$ to 3 d.p.

Use the logarithmic transformation to give $e^{7x} = 9.2 \Leftrightarrow 7x = \ln 9.2$, then divide by 7 to give $x = 0.317$ to 3 d.p. Notice that you use natural logarithm (ln) in this question as the base is *e*.

(e) $t = 0.356$ to 3 d.p.

Use the logarithmic transformation to give $7^{t+1} = 14 \Leftrightarrow t+1 = \log_7 14$, then subtract 1 give $t = 0.356$ to 3 d.p.

(f) $x = -1.322$ to 3 d.p.

Use the logarithmic transformation to give $2^{3-x} = 20 \Leftrightarrow 3 - x = \log_2 20$, then add x and subtract $\log_2 20$ to give $x = -1.322$ to 3 d.p.

(g) $x = \pm 1.745$ to 3 d.p.

Use the logarithmic transformation to give $e^{x^2} = 21 \Leftrightarrow x^2 = \ln 21$, then take the square root to give $x = \pm 1.745$ to 3 d.p. Notice that you use natural logarithm (ln) in this question as the base is *e*.

(h) $t = 1.215$ to 3 d.p.

Firstly open the brackets using the laws of indices to give $(5^t)(5^{t-1}) = 5^{2t-1}$ and then Use the logarithmic transformation to give $5^{2t-1} = 10 \Leftrightarrow 2t - 1 = \log_5 10$, then add 1 and divide by 2 to give $t = 1.215$ to 3 d.p.

4. The solution to the equations are 10^2, 10^{20}, 10^{200} and 10^{2000} respectively. These numbers get very big, very quickly. However the corresponding logarithms only increase by a factor of 10 each time. This reveals a very useful property of logarithms – that they can compress a wide range of numbers into a much smaller range. This is why you often see logarithms used in the analysis of date as you can show very small and very large numbers on the same scale with greater accuracy.

5. Solve the following equations without using a calculator.

(a) $x = 2$

At first, this looks like a difficult question but when you have a natural logarithm equal to another such as, $\ln(x+4) = \ln 6$, it follows that $x + 4 = 6$, i.e. the arguments of the logarithms are equal. So subtracting 4 from gives $x = 2$.

(b) $x = 1.5$

You can approach this question in the same manner as 5(a). As you have a logarithm equal to another of the same base, such as $\log_{10}(3 - x) = \log_{10} x$, it follows that $3 - x = x$, i.e. the arguments of the logarithms are equal. So adding *x* and then dividing by 2 gives $x = 1.5$

(c) $x = 1.5$

You can approach this question in the same manner as 5(a). As you have a natural logarithm equal to another, such as $\ln(3 - x) = \ln x$, it follows that $3 - x = x$, i.e. the arguments of the logarithms are equal. So adding *x* and then dividing by 2 gives $x = 1.5$

(d) $x = -2$ and $x = -1$

You can approach this question in the same manner as 5(a) and also remembering that $\ln 1 = 0$. So, after using this you have a natural logarithm equal to another,

$\ln(x^2+3x+3)=\ln 1$, it follows that $x^2+3x+3=1$, i.e. the arguments of the logarithms are equal. Subtracting 1 from each side gives the quadratic equation $x^2+3x+2=0$. Which can be factorised to give $(x+2)(x+1)=0$ which has solutions $x=-2$ and $x=-1$.

(e) $x=7$

As logarithm and exponential operations of the same base are inverses of each other, the natural logarithm, which is base *e*, cancels out the *e* and you are left with the power *x*. So $x=7$.

(f) $x=3$

As in 5(e), logarithm and exponential operations of the same base are inverses of each other. Therefore the logarithm base 10 cancels out the 10 and you are left with the power $x+2$. So $x+2=5$ and, after subtracting 2, $x=3$.

(g) $x=2$

At first, this looks like a difficult question but when you have an exponential function equal to another of the same base, such as $e^{x+4}=e^6$, it follows that the powers must be equal and so $x+4=6$. So subtracting 4 from gives $x=2$.

(h) $x=1.5$

You can approach this question in the same way as 5(g). When you have an exponential function equal to another of the same base, such as $10^{3-x}=10^x$, it follows that the powers must be equal and so $3-x=x$. So adding x and then dividing by 2 gives $x=1.5$

(i) $x=1.5$

This is the same question as the previous one but here the base is *e* rather than 10. This does not affect the analysis and so the powers must be equal giving $3-x=x$. So adding *x* and then dividing by 2 gives $x=1.5$

(j) $x=6$

Using the laws of indices on the left-hand side gives $(e^3)^2=e^6$ and so $e^6=e^x$. You can now approach this question in the same way as 5(g) and it follows that the powers must be equal and so $x=6$.

(k) $x = -1$

Using the laws of indices on the left-hand side gives $10^{3x}10^4 = 10^{3x+4}$ and on the right-hand side gives $10^{2x}10^3 = 10^{2x+3}$ and so $10^{3x+4} = 10^{2x+3}$. You can now approach this question in the same way as 5(g) and it follows that the powers must be equal and so $3x + 4 = 2x + 3$. Subtracting $2x$ and 4 from both sides gives $x = -1$.

(l) $x = 2$

Using the laws of indices on the left-hand side gives e^{x-2} and on the right-hand side gives e^{4-2x} and so $e^{x-2} = e^{4-2x}$. You can now approach this question in the same way as 5(g) and it follows that the powers must be equal and so $x - 2 = 4 - 2x$. Adding $2x$ and 4 to both sides gives $3x = 6$. Finally dividing by 3 gives $x = 2$.