Worksheet: Exponential Functions

This worksheet covers exponential functions, exponential growth and decay, and gives another example of exponentials in action. Please read the study guide: *Exponential Functions* before doing these questions.

Exponential Functions study guide

Model answers for this sheet

1. You are given the following exponentially growing system below, starting at some number A and growing at the base rate of b once every time step t, beginning at $t = 0$.

$$A, 12, 36, 108, C, 972, D, E, \ldots$$

i) What is the base rate of growth b?

ii) What is the starting number A?

iii) Using your answers to parts i) and ii), write down the general exponential function $y = Ab^t$ that describes this system. Use your answer to work out C, D, E and the value of the system when $t = 15$.

iv) Use the logarithmic transformation to find out how many minutes it takes for the system to grow over 9000.

2. You are given the following exponentially decaying system below, starting at 10000 and decaying at base rate of b once every time step t, beginning at $t = 0$.

$$10000, F, G, 80, 16, H, I, J, \ldots$$

i) What is the base rate of decay b?

ii) Using you answer to part i). write down the general exponential function $y = Ab^t$ that describes this system. Use your answer to work out F, G, H, I, J, and the value of the system when $t = 10$.

iii) Use the logarithmic transformation to find out how many minutes it takes for the system to decay below 0.01.

3. This question investigates another example of exponentials in action. Many isotopes of elements such as uranium emit radiation. This process is called **radioactive decay**, and is modelled by exponential systems like the ones in this worksheet. The exponential system for the radioactive decay of an element is given by

$$N_t = N_0 e^{-\lambda t}$$

where N_t is the number of atoms left at time t, N_0 is the starting number of atoms, and λ is the **decay constant**, which varies from element to element. The time taken for a radioactive element to lose one half of the starting number of atoms is called the **half-life** $t_{1/2}$, and is given by the equation:

$$t_{1/2} = \frac{\ln 2}{\lambda},$$

where ln is the **natural logarithm**. Using this information together with the use of the logarithmic transformation where needed, complete the following table. You should give your answers to 3 decimal places. Time is given in seconds (s).

Isotope	N_t	N_0	t (s)	λ (s^{-1})	$t_{1/2}$ (s)
carbon-15		500	40	0.283	
oxygen-15	45.359		400		122.240
beryllium-11	250	500		0.050	
dubnium-261	159.271	500	44		
nobelium-253	125		198.042		99.002

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