Learning Enhancement Team

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STUDENT SUPPORT SERVICE

Model answers: Exponential Functions

Exponential Functions study guide

1. i) The base rate of growth b is equal to 3.

You can see this by noticing that $12b = 36$ in this system, dividing both sides by 12 gives $b = 3$.

ii) The starting number A is equal to 4.

You can see this by noticing when $t = 1$ then $Ab^1 = 12$. As $b = 3$, you can substitute this value into the formula and rearrange to get $A = 4$.

iii) The general exponential function that describes this system is $y = 4 \cdot 3^t$.
In this system, $C = 324$, $D = 2916$, $E = 8748$ and the value of the system when $t = 15$ is 57395628.

You can write down the general exponential function by putting in the values you found for A and b into the general formula for an exponential system $y = Ab^{kt}$. You know here that $k = 1$ as the system grows at base rate b once every time step.

You can see that at time $t = 4$, the system is C. So you can write

$$C = 4 \cdot 3^4 = 4 \cdot 81 = 324.$$

You can see that at time $t = 6$, the system is D. So you can write

$$D = 4 \cdot 3^6 = 4 \cdot 729 = 2916.$$

You can see that at time $t = 7$, the system is E. So you can write

$$E = 4 \cdot 3^7 = 4 \cdot 2187 = 8748.$$

You can see that at time $t = 15$, the value of the system is some number y. So you can write:

$y = 4 \cdot 3^{15} = 57395628$.

iv) It takes $t = 7.026$ minutes for the system to grow above 9000.

Remember the logarithmic transformation from the study guide:

$$\frac{y}{A} = b^{kt} \Leftrightarrow \log_b\left(\frac{y}{A}\right) = kt \quad .$$

You are given in this case that $y = 9000$. You can put this value in to the general exponential formula for this system to get

$$9000 = 4 \cdot 3^{t}$$

Using the logarithmic transformation gives:

$$\frac{9000}{4} = 3^{t} \Leftrightarrow \log_3\left(\frac{9000}{4}\right) = t \quad .$$

You can see here that *t* is equal to $\log_3\left(\frac{9000}{4}\right) = 7.026$ to 3 d.p.

2. i) The base rate of decay *b* is equal to 5.

You can see this by noticing that $\frac{80}{b} = 16$ in this system, and so you can rearrange the equation to get $b = 5$.

ii) The general exponential function that describes this system is $y = 10000 \cdot 5^{-t}$. In this system, $F = 2000$, $G = 400$, $H = 3.2$, $I = 0.64$, $J = 0.128$ and the value of the system when $t = 10$ is 0.001024.

You can write down the general exponential function by putting in the values you found for *A* and *b* into the general formula $y = Ab^{kt}$ for an exponential system. As the system is decaying once by the base rate at each time step, you know here that $k = -1$.

You can see that at time $t = 1$, the system is *F*. So you can write

$$F = 10000 \cdot 5^{-1} = \frac{10000}{5} = 2000$$

You can see that at time $t = 2$, the system is G. So you can write

$$G = 10000 \cdot 5^{-2} = \frac{10000}{25} = 400$$

You can see that at time $t = 5$, the system is H. So you can write

$$H = 10000 \cdot 5^{-5} = \frac{10000}{3125} = 3.2$$

You can see that at time $t = 6$, the system is I. So you can write

$$I = 10000 \cdot 5^{-6} = \frac{10000}{15625} = 0.64$$

You can see that at time $t = 7$, the system is J. So you can write

$$J = 10000 \cdot 5^{-7} = \frac{10000}{78125} = 0.128$$

You can see that at time $t = 10$, the value of the system is some number y. So you can write

$$y = 10000 \cdot 5^{-10} = \frac{10000}{9765625} = 0.001024.$$

iv) It takes $t = 8.584$ minutes for the system to decay below 0.01.

Remember the logarithmic transformation from the study guide:

$$\frac{y}{A} = b^{kt} \Leftrightarrow \log_b\left(\frac{y}{A}\right) = kt.$$

You are given in this case that $y = 0.01$. You can put this value in to the general exponential formula for this system to get

$$0.01 = 10000 \cdot 5^{-t}$$

Using the logarithmic transformation gives:

$$\frac{0.01}{10000} = 5^{-t} \Leftrightarrow \log_5\left(\frac{0.01}{10000}\right) = -t \quad .$$

You can see here that $-t = \log_5\left(\frac{0.01}{10000}\right) = -8.584$ to 3 d.p. and so $t = 8.584$ minutes.

3.

Material	N_t	N_0	t (s)	λ (s^{-1})	$t_{1/2}$ (s)
carbon-15	0.006	500	40	0.283	2.449
oxygen-15	45.359	500	400	0.006	122.240
beryllium-11	250	500	13.863	0.050	13.863
dubnium-261	159.271	500	44	0.026	26.660
nobelium-253	125	500	198.042	0.007	99.002

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Remember the two equations given in the worksheet:

$$N_t = N_0 e^{-\lambda t} \qquad (1)$$

$$t_{1/2} = \frac{\ln 2}{\lambda} \qquad (2)$$

You will need both to complete the table.

For carbon-15, you are given that $N_0 = 500$, $t = 40$ s and $\lambda = 0.283$ s^{-1}. You can put these values into (1) to get:

$$N_t = 500 \cdot e^{-(0.283 \times 40)} = 0.006$$

To work out $t_{1/2}$ you can put your value $\lambda = 0.283$ s^{-1} into (2) to get:

$$t_{1/2} = \frac{\ln 2}{0.283} = 2.449 \text{ s.}$$

For oxygen-15, you are given that $N_t = 45.359$, $t = 400$ s and $t_{1/2} = 100.240$ s. You can put your value of $t_{1/2}$ into (2) to get

$$122.240 = \frac{\ln 2}{\lambda}$$

You can rearrange this to get

$$\lambda = \frac{\ln 2}{122.240} = 0.006 \text{ s}^{-1}.$$

Now you know the value of λ, you can put this and your values for N_t and t into (1) to get:

$$45.539 = N_0 \cdot e^{-(0.006 \times 400)} \quad .$$

You can rearrange this to get:

$$N_0 = \frac{45.539}{e^{-(0.006 \times 400)}} = 500 \quad .$$

For beryllium-11, you are given that $N_0 = 500$, $N_t = 250$ and $\lambda = 0.050\,\text{s}^{-1}$. You can put these values into (1) to get:

$$250 = 500 \cdot e^{-(0.050 \times t)}$$

You can use the logarithmic transformation to get:

$$\frac{250}{500} = \cdot e^{-(0.050 \times t)} \Leftrightarrow \ln\left(\frac{250}{500}\right) = -0.050t$$

You can rearrange this to get

$$t = \frac{\ln\left(\frac{1}{2}\right)}{-0.050} = 13.863\,\text{s}.$$

To work out $t_{1/2}$ you can put your value $\lambda = 0.050\,\text{s}^{-1}$ into (2) to get:

$$t_{1/2} = \frac{\ln 2}{0.050} = 13.863\,\text{s}.$$

As t was the time taken to lose half of the number of atoms (as 250 is half of 500), $t = t_{1/2}$ in this case.

For dubnium-261, you are given that $N_0 = 500$, $N_t = 159.271$ and $t = 44\,\text{s}$. You can put these values into (1) to get:

$$159.271 = 500 \cdot e^{-44\lambda}$$

You can use the logarithmic transformation to get:

$$\frac{159.271}{500} = \cdot e^{-44\lambda} \Leftrightarrow \ln\left(\frac{159.271}{500}\right) = -44\lambda$$

You can rearrange this to get:

$$\lambda = -\frac{\ln\left(\frac{159.271}{500}\right)}{44} = 0.026$$

To work out $t_{1/2}$ you can put your value $\lambda = 0.026\,\text{s}^{-1}$ into (2) to get:

$$t_{1/2} = \frac{\ln 2}{0.026} = 26.660\ \text{s}.$$

For nobelium-253, you are given that $N_t = 125$, $t = 198.042$ s and $t_{1/2} 99.002$ s. You can put your value of $t_{1/2}$ into (2) to get

$$99.002 = \frac{\ln 2}{\lambda}$$

You can rearrange this to get

$$\lambda = \frac{\ln 2}{99.002} = 0.007\,\text{s}^{-1}.$$

Now you know the value of λ, you can put this and your values for N_t and t into (1) to get:

$$125 = N_0 \cdot e^{-(0.007 \times 198.042)} \quad .$$

You can rearrange this to get:

$$125 = N_0 = \frac{125}{e^{-(0.007 \times 198.042)}} = 500\ .$$