Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Model answers: Simultaneous Equations

Simultaneous equations study guide

There are several methods for solving simultaneous equations. The one used in these model solutions is described in the study guide: *Simultaneous Equations*. It's acceptable to use a different method if you get the right answers.

You must be able to rearrange equations in order to solve simultaneous equations. If you are not proficient in this then please read the study guide: *Rearranging Equations* for help.

1.

a) $x = 1$ and $y = 5$

You have two equations with *y* as the subject and so the right hand sides of these equations can be made to equal each other:

$$2x + 3 = x + 4$$

This can be solved for *x* to give:

$$x = 1$$

and this is substituted into either of the original equations to give:

$$y = 5$$

The answer can be checked by substituting these values of *x* and *y* back into the original equations.

b) $x = 5$ and $y = 14$

The two equations can be rearranged to make *y* the subject so that $y = 4 + 2x$ and $y = 3x - 1$ and so the right hand sides of these equations can be made to equal each other:

$$4 + 2x = 3x - 1$$

This can be solved for x to give $x = 5$ and this is substituted into either of the original equations to give $y = 14$. The answer can be checked by substituting these values of x and y back into the original equations.

c) $x = 5$ and $y = 14$

The two equations can be rearranged to make y the subject so that $y = 4 + 2x$ and $y = 3x - 1$. These are the same as in question b) and so the answers are the same. The answer can be checked by substituting these values of x and y back into the original equations.

d) $x = 14$ and $y = 5$

The two equations already have x as the subject and so the right hand sides of these equations can be made to equal each other:

$$2y + 4 = 3y - 1$$

This can be solved for y to give $y = 5$ and this is substituted into either of the original equations to give $x = 14$. The answer can be checked by substituting these values of x and y back into the original equations. These equations are essentially the same as in b) and c) but with x and y swapped.

e) $\alpha = 1$ and $\beta = 3$

The two equations can be rearranged to make α the subject so that $\alpha = 7 - 2\beta$ and $\alpha = 4 - \beta$ and so the right hand sides of these equations can be made to equal each other:

$$7 - 2\beta = 4 - \beta$$

This can be solved for β to give $\beta = 3$ and this is substituted into either of the original equations to give $\alpha = 1$. The answer can be checked by substituting these values of α and β back into the original equations.

f) $x = 1$ and $y = 3$

The two equations can be rearranged to make y the subject so that:

$$y = \frac{7 + 2x}{3} \quad \text{and} \quad y = \frac{11 - 5x}{2}$$

and so the right hand sides of these equations can be made to equal each other:

$$\frac{7+2x}{3}=\frac{11-5x}{2}$$

This can be solved for x to give $x=1$ and this is substituted into either of the original equations to give $y=3$. The answer can be checked by substituting these values of x and y back into the original equations.

g) $x=10$ and $y=20$

The two equations can be rearranged to make y the subject so that $y=x+10$ and $y=40-2x$ and so the right hand sides of these equations can be made to equal each other:

$$x+10=40-2x$$

This can be solved for x to give $x=10$ and this is substituted into either of the original equations to give $y=20$. The answer can be checked by substituting these values of x and y back into the original equations.

h) $P=2.11$ and $Q=-0.81$

The two equations can be rearranged to make P the subject so that:

$$P=\frac{5+4.3Q}{0.72} \quad \text{and} \quad P=\frac{7-4.91Q}{5.21}$$

The question uses two decimal places and so the answer should also be to two decimal places. The working should therefore be done to three decimal places or else rounding errors may occur. By doing the divisions in the fractions above you get:

$$P=6.944+5.972Q \quad \text{and} \quad P=1.343-0.942Q$$

and so the right hand sides of these equations can be made to equal each other:

$$6.944+5.972Q=1.343-0.942Q$$

This can be solved for Q to give $Q=-0.81$ and this is substituted into either of the original equations to give $P=2.11$. The answer can be checked by substituting these values of P and Q back into the original equations.

i) $A=\frac{1}{2}$ and $B=-\frac{1}{2}$

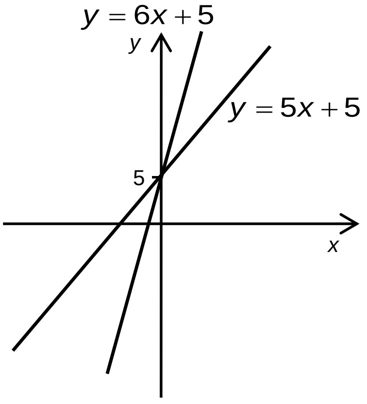
The two equations can be rearranged to make A the subject so that $A=-B$ and $A=1+B$ and so the right hand sides of these equations can be made to equal each other:

$$-B=1+B$$

This can be solved for B to give $B = -1/2$ and this is substituted into either of the original equations to give $A = 1/2$. The answer can be checked by substituting these values of A and B back into the original equations.

2) In this question you will explore the relationship between simultaneous equations and the intersection of straight lines on graphs. See the study guide *Sketching Straight Lines* for help with this.

a)

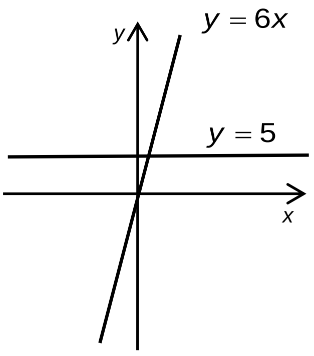


$x = 0$
$y = 5$

The y-intercept is 5 for both lines. One of them has gradient 5 and the other has a steeper gradient 6 and so the graph is as above.

From the graph you can see that the lines meet at the point $(0,5)$.

b)

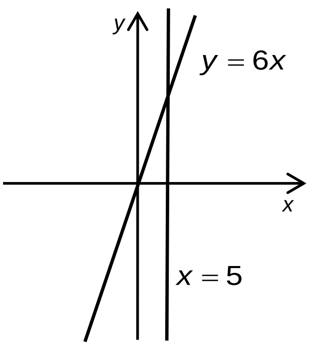


$x = \frac{5}{6}$
$y = 5$

The graph of $y = 6x$ goes through the origin and has gradient 6. The graph of $y = 5$ is flat and so the graph is as to the left.

From the graph, the lines meet when $y = 5$. Substituting this into $y = 6x$ gives $x = 5/6$.

c)



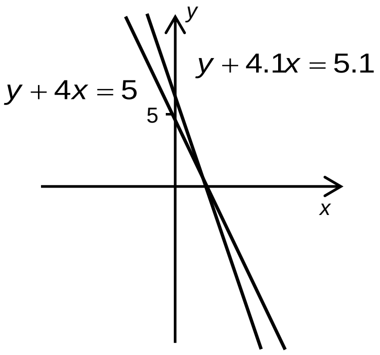
$x = 5$
$y = 30$

The graph of $y = 6x$ goes through the origin and has gradient 6. The graph of $x = 5$ is vertical and so the graph is to the left.

From the graph, the lines meet when $x = 5$. Substituting this into $y = 6x$ gives $y = 30$.

d) $x = 1$

$y = 1$



The equations can be rearranged to give:

$$y = 5.1 - 4.1x$$

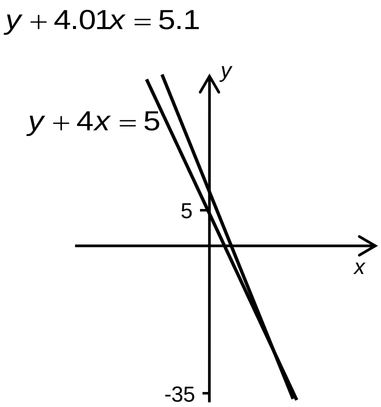
$$y = 5 - 4x$$

and so the first equation has a *y*-intercept at 5.1 and a gradient of -4.1. The second equation has a very similar *y*-intercept and gradient to the first equation and so the graph is as shown.

Now that *y* is the subject of both equations, the right hand sides must be equal and so $5.1 - 4.1x = 5 - 4x$ which can be solved to give $x = 1$ and this is substituted into either original equation to give $y = 1$.

e) $x = 10$

$y = -35$



The equations can be rearranged to give:

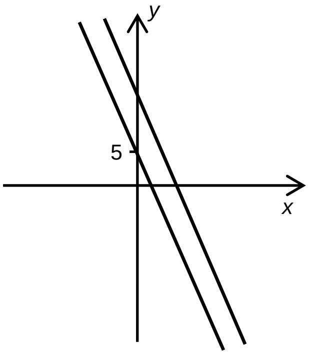
$$y = 5.1 - 4.01x$$

$$y = 5 - 4x$$

and so the graph is very similar to the graph in the previous question.

Now that *y* is the subject of both equations, the right hand sides must be equal and so $5.1 - 4.01x = 5 - 4x$ which can be solved to give $x = 10$ and this is substituted into either original equation to give $y = -35$. You can see that, even though this question is very similar to the previous one, the answers are very different.

f) $y+4x=5.1$
$y+4x=5$



No Solution

The equations can be rearranged to give:

$$y = 5.1 - 4x$$

$$y = 5 - 4x$$

and so the graph is very similar to the graph in the previous question but now the lines are parallel and so do not intersect. Therefore there is no solution to the simultaneous equations.

This can also be seen by making the right hand sides of both equations equal which leads to a nonsensical equation.

3)

a) $a=2$

The equations can be rearranged to give:

$$y = 3x - 7 \quad \text{and} \quad y = \frac{6x+3}{a} = \frac{6x}{a} + \frac{3}{a}$$

These two lines are parallel when the gradients are the same, when $a=2$

Alternatively, you can try to solve them:

$$3x - 7 = \frac{6x+3}{a}$$

and so

$$3ax - 7a = 6x + 3$$
$$(3a-6)x = 7a + 3$$

This can always be solved for x unless $a=2$. If $a=2$ the equation is nonsensical.

b) $b = \frac{3}{2}$

The equations can be rearranged to give:

$$y = 3x + b \quad \text{and} \quad y = \frac{6x+3}{2} = 3x + \frac{3}{2}$$

Now that y is the subject of both equations, the right hand sides must be equal and so

$$3x + b = 3x + \frac{3}{2}$$

and so $b = 3/2$. If $b = 3/2$ then the two equations are the same. They describe the two lines which are the same and therefore intersect each other everywhere on the line and so the equations have infinite solutions.

4) One number is 3 and the other is 5.

Let the two numbers be x and y then:

“If I add them together I get 8” $\Rightarrow x + y = 8$

“If I multiply one of them by 4 and the other by 2 then add I get 22” $\Rightarrow 4x + 2y = 22$

These two simultaneous equations can be solved to give $x = 3$ and $y = 5$.

5) 5 chickens and 3 pigs.

Let the number of pigs be p and the number of chickens be c then:

“There’s eight heads” $\Rightarrow c + p = 8$

“There’s twenty-two legs” $\Rightarrow 4p + 2c = 22$

These are the same simultaneous equations as in question 4 and so can be solved to give $p = 3$ and $c = 5$.

6) 3 cakes and 5 sandwiches.

Let the number of cakes be c and the number of sandwiches be s then:

“Eight people come to my café …” $\Rightarrow c + s = 8$

“I have made twenty-two pounds” $\Rightarrow 4c + 2s = 22$

These are the same simultaneous equations as in question 4 and so can be solved to give $c = 3$ and $s = 5$.

7) The coordinate is $(3,5)$.

This question uses techniques described in the study guide: *Finding Equations of Straight Lines* so read that if you are not sure about this. The first line had gradient -2 and so its equation can be written as:

$$y = -2x + c$$

It also goes through the point $(0,11)$ you can substitute these values into the equation above to find $c = 11$.

The second line has gradient given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5-10}{3-(-2)} = \frac{-5}{5} = -1$$

and so its equation can be written as $y = -x + c$. You can substitute either point $(-2,10)$ or $(3,5)$ into this equation to find $c = 8$. The two equations are then:

$$y = -2x + 11 \text{ and } y = -x + 8$$

These are essentially the same simultaneous equations as in question 4 and so can be solved to give $x = 3$ and $y = 5$.