Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Bridging Between Algebra and Calculus

Finding Equations of Straight Lines

This study guide describes how to find the equation of a straight line for various given situations in both mathematics and experiments.

Introduction

Many situations in mathematics, science and economics can be described by straight lines. In other words they can be described by the **linear model**:

$$y = mx + c$$

This equation is explained in more detail in the study guide: *What is a Straight Line?*. In the linear model the two **variables** x and y are related using two **numbers** m and c. Specifically ***m* represents the gradient of the straight line** and ***c* is the value where the line crosses the *y*-axis**. A common problem is then, how do you find m and c in different situations? For example, two coordinates can be used to find the equation of the straight line which passes through them. Alternatively, if you have experimental data which lies approximately on a straight line, it is not clear which straight line to use to best describe it. There are standard statistical methods to find the best straight line to describe such data. In this study guide you will learn how to find equations of straight lines for a variety of common situations.

1. When the gradient and a point are known

You may be given the gradient m of a straight line and the coordinates of a point which it goes through. Here, you only need to find c in the linear model. You can do this by substituting the coordinates you are given into the equation $y = mx + c$ and solving for c.

Example: Find the equation of the straight line with gradient -2 which goes through the point with coordinates $(2,1)$.

Because the gradient $m = -2$, the equation of this line is $y = -2x + c$. The coordinate $(2,1)$ is the same as saying $x = 2$ and $y = 1$. So, since the straight line passes through this point, the equation $y = -2x + c$ will be true for these values of *x* and *y*. In other words $x = 2$ and $y = 1$ must satisfy this equation and so, after substituting these values into $y = -2x + c$ you get:

$$1 = -2 \cdot 2 + c$$

Solving this equation for *c* by adding 4 to each side gives $c = 5$. You can use this value in $y = -2x + c$ to write the equation of the line with gradient $m = -2$ which goes through the point $(2,1)$ as:

$$y = -2x + 5$$

A common example of this kind of problem is finding the tangent to a curve at a certain point. In this case, the gradient of the tangent is found by using **differentiation** (see the study guide: *What is Differentiation?*) and the point can be used to find *c.*

2. When two points are known

Any two points define a straight line. Therefore, given any two points you can calculate the gradient *m* and the *y*-intercept *c* of the straight line that passes through them.

First, you calculate the gradient *m*. You can do this by using the definition that:

The gradient of a straight line is the change in *y* divided by the change in *x*

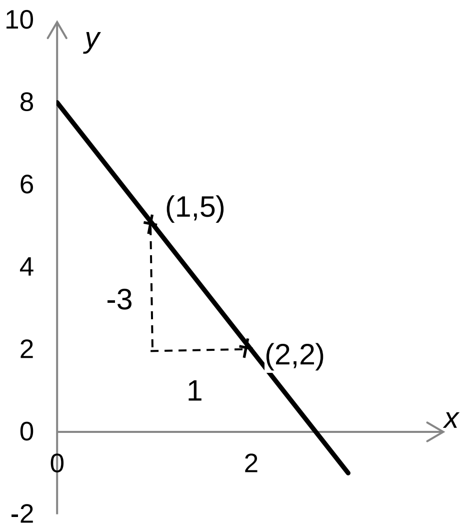
The study guide: *What is a Straight Line?* has a more in depth explanation of this. In this case, if you can define your two points as (x_1, y_1) and (x_2, y_2), the change in *y* is $y_2 - y_1$ which is sometimes written as Δy and the change in *x* is $x_2 - x_1$ which is sometimes written as Δx. In order to calculate *m*, you divide the change in *y* by the change in *x*:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

You may find it easier to remember the gradient as being **the rise over the run** where **the change in *y* is the rise** and **the change in *x* is the run**. You may also find that a rough sketch of the points and line is extremely useful.

Then you follow the procedure from section 1 to find *c* by using the gradient and either of the points you are given (you will get the same answer regardless of which point you choose).

Example: Find the equation of the straight line which passes through the points $(1,5)$ and $(2,2)$.



Given two points you can *always* calculate the gradient *m* of the straight line that passes through them. The first point is $(1,5)$ which is the same as saying $x_1 = 1$ and $y_1 = 5$. The second point is $(2,2)$ giving $x_2 = 2$ and $y_2 = 2$. So the change in *y* is $\Delta y = y_2 - y_1 = -3$ and the change in *x* is $\Delta x = x_2 - x_1 = 1$. So, using the formula above:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3}{1} = -3$$

Note that the **slope of the graph is downhill as the gradient is negative**. Substituting the value of *m* into the linear model $y = mx + c$ gives you $y = -3x + c$.

Now that the gradient has been found, it only remains to evaluate the *y*-intercept *c*. This is the same situation as in the previous section except that, instead of knowing just one point on the line, you know two, $(1,5)$ and $(2,2)$. Either one can be substituted into the equation to find *c.* Using $(1,5)$ gives $x = 1$ and $y = 5$. After substituting these values into $y = -3x + c$ you get:

$5 = -3 \cdot 1 + c$ giving $c = 5 + 3 = 8$.

Using $(2,2)$ gives $x = 2$ and $y = 2$. After substituting these values into $y = -3x + c$ you get:

$2 = -3 \cdot 2 + c$ giving $c = 2 + 6 = 8$.

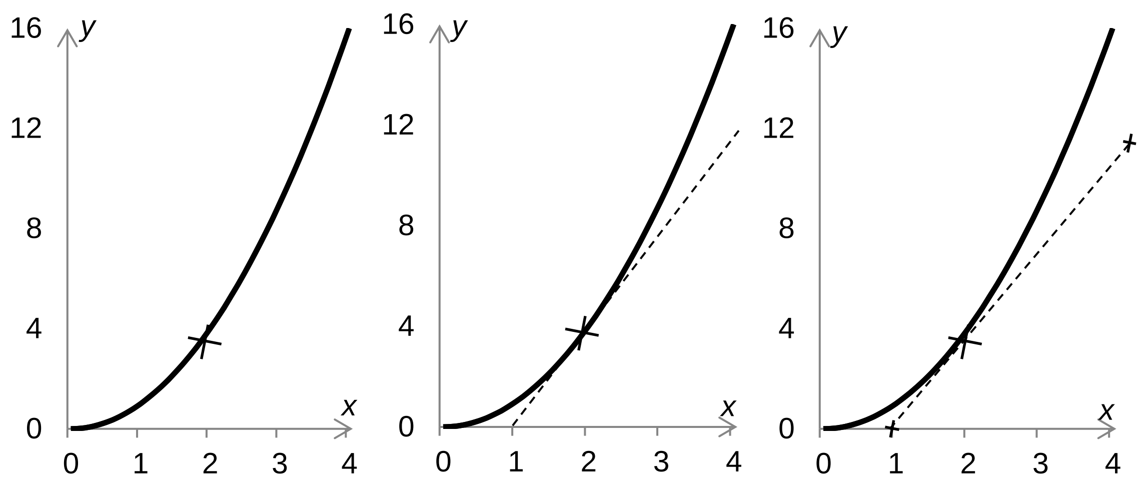
Which is the same result. So either way, the equation of the straight line which passes through the points $(1,5)$ and $(2,2)$ is $y = -3x + 8$.

3. When a curve and a point are known

Sometimes in science, you are required to find the gradient of a curve at a certain point. The curve is known, perhaps from experimental data, and the rate of change or gradient is required. The method is to draw a **tangent** at the point of interest and use the methods described in the previous section above to find its equation. **A tangent is a straight line which touches a curve at only one point and, at this point, has the same gradient as the curve.**

Example: Find the gradient of the tangent to the curve below at the point $(2,4)$.

You should first identify the point with a cross on the curve as in the first graph. Then place your ruler on that point and angle it so that it only touches the curve at one point and draw this line. This is the dotted line in the second graph. Then take any two points on this line such as $(1,0)$ and $(4,12)$ which are marked with crosses on the third graph.



Then, as in the previous section, the gradient is found by using the change in *y* over the change in *x*. Here the first point is $(1,0)$ and so $x_1 = 1$ and $y_1 = 0$. The second point is $(4,12)$ giving $x_2 = 4$ and $y_2 = 12$. Using these values, the change in *y* is $\Delta y = y_2 - y_1 = 12$ and the change in *x* is $\Delta x = x_2 - x_1 = 3$ so:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 0}{4 - 1} = \frac{12}{3} = 4$$

As gradient of the straight line is 4, the gradient of the curve at the point $(2,4)$ is also 4.

4. When a perpendicular line and a point are known

If the gradient of a line is known then the gradient of **any** line perpendicular (at right angles) to that line is found by using the formula:

$$m_1 m_2 = -1$$

where m_1 is the gradient of the first line and m_2 is the gradient of the perpendicular line.

Example: Find the equation of the straight line which goes through the point $(5,2)$ and is perpendicular to the line $y = 7 - 5x$.

The gradient m_1 of the first line is -5 and so the gradient m_2 of any line perpendicular to this line is given by:

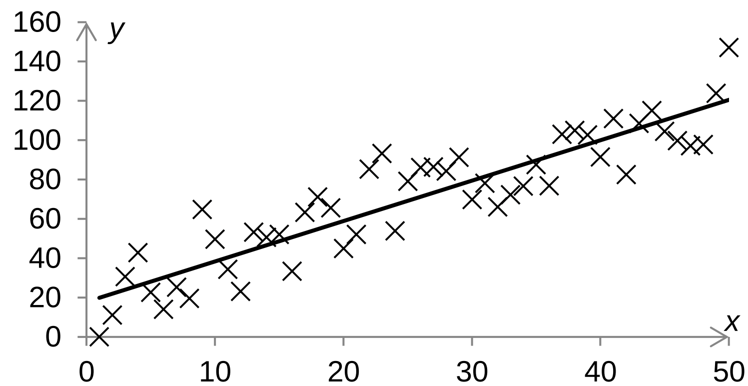
$$-5m_2 = -1$$

Dividing both sides by -5 gives $m_2 = 1/5$. Using this in the linear model means that the equation of the line you are trying to find is $y = (x/5) + c$. Now the gradient is known and also a point $(5,2)$ has been given, you can use the technique from section 1 to find the equation of the straight line. You substitute $x = 5$ and $y = 2$ into $y = (x/5) + c$ and solve the resulting equation for *c* to find $c = 3$. So the equation of the straight line which goes through the point $(5,2)$ and is perpendicular to the line $y = 7 - 5x$ is:

$$y = \frac{x}{5} + 3$$

5. Given experimental results or statistical data

Often the linear model is applied to experimental data. Here the dependent (measured) variable *y* and the independent (controlled) variable *x* are related by the equation $y = mx + c$ where *m* and *c* are constants that must be found. Often letters other than *x*, *y*, *m* and *c* are used but the same mathematics can be applied.



A simple method is to plot the data points on a graph and draw a line of best fit by hand, making sure that approximately half the points are above and half below the line. Then the equation of this line can be estimated by taking two points on it and *m* and *c* can be calculated using the methods described earlier in this guide.

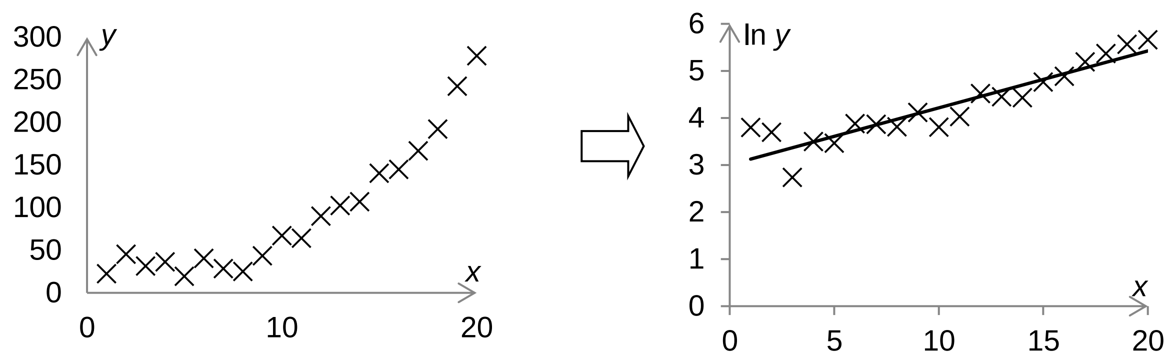
A more formal statistical procedure for accomplishing this is called **linear regression** and you can talk to a **Learning Enhancement Tutor** if you want to learn more.

6. Given exponential-like experimental data

You may suspect that your data is not linearly related and it would be preferable to fit an exponential curve to it (see the study guide: *Exponential Functions*):

$$y = Ae^{kx}$$

As with fitting a straight line, two numbers A and k must be found in order to fully describe the relationship between y and x. However, it is not so easy to draw and estimate these. In order to do this the data is transformed using logarithms and this procedure is described in the study guide: *The Laws of Logarithms.*



Once this transformation has been done the data will be approximately linear and the techniques described in the previous section can be applied.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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