Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model answers: Finding Equations of Straight Lines

Finding Equations of Straight Lines study guide

1. The straight line with gradient 3 which passes through the point $(4,2)$ is $y=3x-10$.

Because the gradient $m=3$, the equation of this line is $y=3x+c$. The coordinate $(4,2)$ is the same as saying $x=4$ and $y=2$. So, since the straight line passes through this point, the equation $y=3x+c$ will be true for these values of *x* and *y*. In other words $x=4$ and $y=2$ must satisfy this equation and so, after substituting these values into $y=3x+c$ you get:

$$2=3\cdot 4+c$$

Solving this equation for *c* by subtracting 12 from each side gives $c=-10$. You can use this value in $y=3x+c$ to write the equation of the line with gradient $m=3$ which goes through the point $(4,2)$ as:

$$y=3x-10$$

2. The straight line with gradient -5 which passes through the point $(4,2)$ is $y=-5x+22$.

Because the gradient $m=-5$, the equation of this line is $y=-5x+c$. The coordinate $(4,2)$ is the same as saying $x=4$ and $y=2$. So, since the straight line passes through this point, the equation $y=-5x+c$ will be true for these values of *x* and *y*. In other words $x=4$ and $y=2$ must satisfy this equation and so, after substituting these values into $y=-5x+c$ you get:

$$2=-5\cdot 4+c$$

Solving this equation for *c* by adding 20 to each side gives $c = 22$. You can use this value in $y = -5x + c$ to write the equation of the line with gradient $m = -5$ which goes through the point $(4,2)$ as:

$$y = -5x + 22$$

3. The straight line which passes through $(4,2)$ and $(5,3)$ is $y = x - 2$.

Given two points you can *always* calculate the gradient *m* of the straight line that passes through them. The first point is $(4,2)$ which is the same as saying $x_1 = 4$ and $y_1 = 2$. The second point is $(5,3)$ giving $x_2 = 5$ and $y_2 = 3$. So the change in *y* is $\Delta y = y_2 - y_1 = 1$ and the change in *x* is $\Delta x = x_2 - x_1 = 1$. So, using the formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1}{1} = 1$$

Note that the **slope of the graph is uphill as the gradient is positive**. Substituting the value of *m* into the linear model $y = mx + c$ gives you $y = x + c$.

Now that the gradient has been found, it only remains to evaluate the *y*-intercept *c*. This is the same situation as in questions 1 and 2, instead of knowing just one point on the line, you know two, $(4,2)$ and $(5,3)$. Either one can be substituted into the equation to find *c.* Using $(4,2)$ gives $x = 4$ and $y = 2$. After substituting these values into $y = x + c$ you get:

$$2 = 1 \cdot 4 + c \qquad \text{giving} \qquad c = 2 - 4 = -2.$$

Using $(5,3)$ gives $x = 5$ and $y = 3$. After substituting these values into $y = x + c$ you get:

$$3 = 1 \cdot 5 + c \qquad \text{giving} \qquad c = 3 - 5 = -2.$$

Which is the same result. So either way, the equation of the straight line which passes through the points $(4,2)$ and $(5,3)$ is $y = x - 2$.

4. The straight line which passes through $(4,2)$ and $(-8,-12)$ is $y = \frac{7}{6}x - \frac{8}{3}$

Given two points you can *always* calculate the gradient *m* of the straight line that passes through them. The first point is $(4,2)$ which is the same as saying $x_1 = 4$ and $y_1 = 2$. The second point is $(-8,-12)$ giving $x_2 = -8$ and $y_2 = -12$. So the change in *y* is

$\Delta y = y_2 - y_1 = -14$ and the change in x is $\Delta x = x_2 - x_1 = -12$. So, using the formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-14}{-12} = \frac{7}{6}$$

Note that the **slope of the graph is uphill as the gradient is positive**. Substituting the value of m into the linear model $y = mx + c$ gives you $y = \frac{7}{6}x + c$.

Now that the gradient has been found, it only remains to evaluate the y-intercept c. This is the same situation as in questions 1 and 2, instead of knowing just one point on the line, you know two, $(4,2)$ and $(-8,-12)$. Either one can be substituted into the equation to find *c.* Using $(4,2)$ gives $x = 4$ and $y = 2$.

After substituting these values into $y = \frac{7}{6}x + c$ you get:

$$2 = \frac{7}{6} \cdot 4 + c \qquad \text{giving} \qquad c = 2 - \frac{14}{3} = -\frac{8}{3}.$$

Using $(-8,-12)$ gives $x = -8$ and $y = -12$.

After substituting these values into $y = \frac{7}{6}x + c$ you get:

$$-12 = \frac{7}{6} \cdot (-8) + c \qquad \text{giving} \qquad c = -12 + \frac{24}{3} = -\frac{8}{3}.$$

Which is the same result. So either way, the equation of the straight line which passes through the points $(4,2)$ and $(-8,-12)$ is $y = \frac{7}{6}x - \frac{8}{3}$.

5. The straight line which passes through $(4,2)$ and the origin is $y = \frac{1}{2}x$.

Given two points you can *always* calculate the gradient m of the straight line that passes through them. The first point is $(4,2)$ which is the same as saying $x_1 = 4$ and $y_1 = 2$. The second point, the origin, is $(0,0)$ giving $x_2 = y_2 = 0$. So the change in y is $\Delta y = y_2 - y_1 = -2$ and the change in x is $\Delta x = x_2 - x_1 = -4$. So, using the formula:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2}{-4} = \frac{1}{2}$$

Note that the **slope of the graph is uphill as the gradient is positive**. Substituting the value of *m* into the linear model $y = mx + c$ gives you $y = \frac{1}{2}x + c$.

Now that the gradient has been found, it only remains to evaluate the *y*-intercept *c*. This is the same situation as in questions 1 and 2, instead of knowing just one point on the line, you know two, $(4,2)$ and $(0,0)$. Either one can be substituted into the equation to find *c.* Using $(4,2)$ gives $x = 4$ and $y = 2$.

After substituting these values into $y = \frac{1}{2}x + c$ you get:

$$2 = \frac{1}{2} \cdot 4 + c \qquad \text{giving} \qquad c = 2 - 0 = 0.$$

Using $(0,0)$ gives $x = y = 0$.

After substituting these values into $y = \frac{1}{2}x + c$ you get:

$$0 = \frac{1}{2} \cdot 0 + c \qquad \text{giving} \qquad c = 0.$$

Which is the same result. So either way, the equation of the straight line which passes through the points $(4,2)$ and $(0,0)$ is $y = \frac{1}{2}x$.

6. The straight line which passes through $(5,3)$ and $(-8,-12)$ is $y = \frac{15}{13}x - \frac{36}{13}$.

Given two points you can *always* calculate the gradient *m* of the straight line that passes through them. The first point is $(-8,-12)$ which is the same as saying $x_1 = -8$ and $y_1 = -12$. The second point is $(5,3)$ giving $x_2 = 5$ and $y_2 = 3$. So the change in *y* is $\Delta y = y_2 - y_1 = 15$ and the change in *x* is $\Delta x = x_2 - x_1 = 13$. So, using the formula above:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15}{13}$$

Note that the **slope of the graph is uphill as the gradient is positive**. Substituting the value of *m* into the linear model $y = mx + c$ gives you $y = \frac{15}{13}x + c$.

Now that the gradient has been found, it only remains to evaluate the *y*-intercept *c*. As in the previous question, either point can be substituted into the equation to find *c.* Using $(5,3)$

gives $x = 5$ and $y = 3$. After substituting these values into $y = \frac{15}{13}x + c$ you get:

$$3 = \frac{15}{13} \cdot 5 + c \qquad \text{giving} \qquad c = 3 - \frac{75}{13} = -\frac{36}{13}.$$

Using $(-8, -12)$ gives $x = -8$ and $y = -12$. After substituting these values into $y = \frac{15}{13}x + c$ you get:

$$-12 = \frac{15}{13} \cdot (-8) + c \qquad \text{giving} \qquad c = -12 + \frac{120}{13} = -\frac{36}{13}.$$

Which is the same result. So either way, the equation of the straight line which passes through the points $(5, 3)$ and $(-8, -12)$ is $y = \frac{15}{13}x - \frac{36}{13}$.

7. The horizontal line which passes through the point $(4, 2)$ is $y = 2$.

Any horizontal line has a gradient of zero and so $y = 0 \cdot x + c = c$ and so you only need to know the *y*-intercept. Here, again as the line is horizontal, any point on it has the same *y*-coordinate. In this case $y = 2$ which is also the equation of the straight line.

8. The vertical line which passes through the point $(4, 2)$ is $x = 4$.

Vertical lines have infinite gradient and take the form $x = a$ where *a* is the *x*-coordinate. For $(4, 2)$ the *x*-coordinate is 4 and so the equation of the line is $x = 4$.

9. The straight line which passes through $(4, 2)$ and is parallel to $y = 5x + 4$ is $y = 5x - 18$.

Parallel lines have the same gradient and so here $m = 5$ as the gradient of $y = 5x + 4$ is 5. So the equation of this line is $y = 5x + c$. The coordinate $(4, 2)$ is the same as saying $x = 4$ and $y = 2$. So, since the straight line passes through this point, the equation $y = 5x + c$ will be true for these values of *x* and *y*. In other words $x = 4$ and $y = 2$ must satisfy this equation and so, after substituting these values into $y = 5x + c$ you get:

$$2 = 5 \cdot 4 + c$$

Solving this equation for *c* by subtracting 20 from each side gives $c = -18$. You can use this value in $y = 5x + c$ to write the equation of the line with gradient $m = 5$ which goes through the point $(4,2)$ as:

$$y = 5x - 18$$

10. The straight line which passes through $(4,2)$ and is perpendicular to $y = 5x + 4$ is $y = -\frac{x}{5} + \frac{14}{5}$.

Lines that are perpendicular are at right-angles to one another. If the gradient of a line is known then the gradient of **any** line perpendicular (at right angles) to that line is found by using the formula:

$$m_1 m_2 = -1$$

where m_1 is the gradient of the first line and m_2 is the gradient of the perpendicular line. Here the gradient m_1 of the first line $y = 5x + 4$ is 5 and so the gradient m_2 of any line perpendicular to this line is given by:

$$5m_2 = -1$$

Dividing both sides by 5 gives $m_2 = -1/5$. Using this in the linear model means that the equation of the line you are trying to find is $y = -(x/5) + c$. Now the gradient is known and also a point $(4,2)$ has been given, you can use the technique from question 1 to find the equation of the straight line. You substitute $x = 4$ and $y = 2$ into $y = -(x/5) + c$ and solve the resulting equation for *c* to find:

$$2 = -\frac{4}{5} + c \qquad \text{and so} \qquad c = 2 + \frac{4}{5} = \frac{14}{5}$$

So the equation of the straight line which goes through the point $(4,2)$ and is perpendicular to the line $y = 5x + 4$ is:

$$y = -\frac{x}{5} + \frac{14}{5}$$

11. The straight line which passes through $(4,2)$ and is parallel to the *x*-axis is $y=2$.

A straight line which is parallel to the *x*-axis is horizontal and so this question is the same as question 7. Any horizontal line has a gradient of zero and so $y=0\cdot x+c=c$ and so you only need to know the *y*-intercept. Here, again as the line is horizontal, any point on it has the same *y*-coordinate. In this case $y=2$ which is also the equation of the straight line.

12. Straight line which passes through $(4,2)$ and is perpendicular to the *x*-axis.

A line perpendicular to the *x*–axis is at right-angles to it, i.e. it is vertical. This mean this question is the same as question 8. Vertical lines have infinite gradient and take the form $x=a$ where *a* is the *x*-coordinate. For $(4,2)$ the *x*-coordinate is 4 and so the equation of the line is $x=4$.