

Bridging Between Algebra and Calculus

What is a Straight Line?

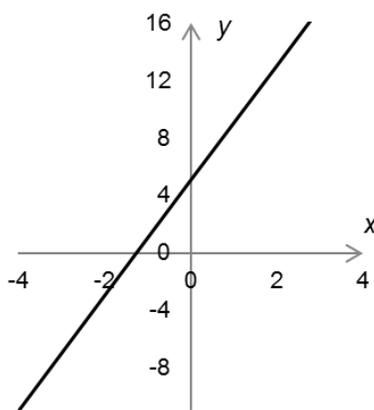
This guide discusses equations which describe straight line graphs and which are also known as linear equations. This guide will help you learn the meaning of the two important values in the equation of a straight line: the gradient and the y-intercept. It will also help you learn how to recognise equations of straight lines.

Introduction

The equation of a straight line (also known as a **linear equation**) is a fundamental mathematical equation. It occurs in all areas of science such as when you calculate gradients of tangents, convert units (e.g. Fahrenheit to Celsius), use the equations of motion in physics and fit straight lines to data in statistics (linear regression). In mathematics, harder (nonlinear) problems are often reduced to linear problems which can be analysed more easily. In all of these areas, and many more, one variable is related to another variable by multiplying it by a constant – just a number which does not change – and adding another, often different, constant. If the two variables are x and y and the constants are m and c , then this can be written as the equation:

$$y = mx + c$$

The graph of any equation which fits this pattern is a **straight line**.

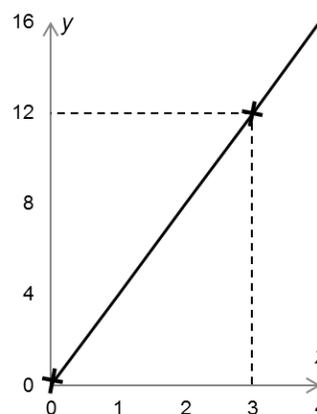


The figure on the left is the graph of $y = 4x + 5$. What does this mean? It means that every point on the line in the graph satisfies the equation $y = 4x + 5$. So the point with coordinates $(1, 9)$ is on the line because, if $x = 1$ then the equation states that $y = 4 \times 1 + 5 = 9$. Therefore $(1, 9)$ satisfies a straight line equation with $m = 4$ and $c = 5$. You can find out more about graphs in the study guide: [Plotting a Graph by Hand](#).

In this guide you will learn to interpret the constant m as the **gradient** of the line and c as the **y-intercept**. You will also learn how to recognise linear equations and you will be introduced to four special straight line equations. If you want to learn more you can also look at the study guides: [Finding Equations of Straight Lines](#) and [Sketching Straight Lines](#). (Note: in some disciplines such as economics, you may see the straight line equation written as $y = a + bx$ with a being the y-intercept and b being the gradient.)

Equation of a straight line through the origin

Look at the graph on the right. How can this graph be described using algebra? In this example, the point with coordinates $(0,0)$, the **origin**, is on the line (the cross at the bottom left of the graph). In other words the point where $x=0$ and $y=0$ is on the graph. Also the point $(3,12)$ is on the graph – the straight line will go through this point which is where $x=3$ and $y=12$ (the cross on the top right of the graph). In fact if you take any number (call it x) and multiply it by 4 (and call the result y) you will get a point on this graph. You could write that:



$$y = 4x$$

Which is in the form $y = mx + c$ with $m = 4$ and $c = 0$, in other words it is of the form:

$$y = mx$$

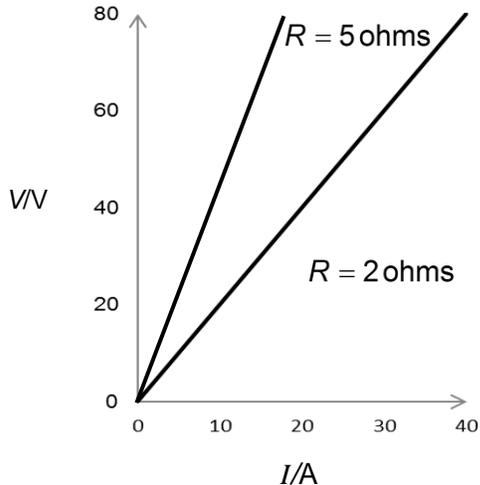
and this is the **equation of a straight line through the origin**.

Another way of looking at this is to imagine a steep mountain where, if you go one kilometre due east, then you will climb four kilometres up. In the language of mathematics you can say that the gradient of this mountain is 4.

For the equation $y = 4x$, the gradient of the graph is 4 and in the straight line equation this corresponds to the constant m . Importantly the gradient of the graph does not change, it is always 4. This is why the graph of $y = 4x$ is a straight line. By definition:

A straight line is a graph which has a constant gradient m .

Example: Interpret the graph of $V = IR$ (Ohm's law).



This is the equation used in electronics to relate voltage (V) and current (I) in a circuit with a **constant** resistance (R). Mathematically, this is a straight line and the gradient is given by the value of R . The graph shows lines for two different values of R (5 and 2 ohms). The line $V = 5I$, is steeper than the one for $V = 2I$, since the gradient is larger.

As the gradient m gets more positive, the slope of a graph gets steeper uphill.
As the gradient m gets more negative, the slope of a graph gets steeper downhill.

The general equation of a straight line

If the graph of $y = 4x$ (see previous section) was translated up by 5 then it would no longer go through the origin but it would go through the point $(0,5)$. Adding 5 to all the y values also moves them all up by 5. So the equation of this line would be $y = 4x + 5$ and this is the graph on the first page of this guide. This equation is of the form:

$$y = mx + c$$

In fact this is the general equation of all straight lines that do not necessarily go through the origin. In this equation m is the gradient, as you saw in the previous section, and c is the value of y when $x = 0$. If you substitute $x = 0$ into the general equation of a straight line you get:

$$y = m \times 0 + c = c$$

In other words, when $x = 0$, $y = c$. As the y -axis is the line on a graph where x always equals zero, the value c is known as the **y -intercept** because this is the value of y where the line **intercepts** ('goes through') the y -axis.

Any straight line that goes through the origin intercepts the y -axis at $y = c$ and so $c = 0$ for all lines that go through the origin. This means that, for lines that go through the origin, the equation of a straight line is $y = mx$, as stated in the previous section.

Identifying straight lines

In mathematics, equations represent graphs and more complicated equations have more complicated graphs. For example, the graph of $y = x^2$ is a **parabola** (see study guide: [Quadratic Functions](#)) and the graph of $y = \sin x$ is a **wave** (see factsheet: [Five Basic Functions](#)). In these graphs the gradient is not constant but changing: a parabola's gradient goes from negative to positive (or vice versa), a wave oscillates repeatedly. As you have seen, straight line graphs have a constant gradient.

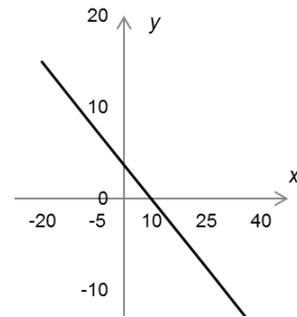
To see if an equation is that of a straight line (a linear equation) you need to find if it fits the pattern $y = mx + c$. Sometimes you may need to rearrange an equation and make y the subject to find this out. You should read the study guide: [Rearranging Equations](#) if you find this difficult.

Example: Does the equation $y = 5 - \frac{x}{2}$ represent a straight line?

This equation can be rewritten as:

$$y = -\frac{1}{2}x + 5$$

and so it *is* a straight line with $m = -1/2$ and $c = 5$ (see graph to the right). You can see that the slope is downhill as m is negative and the line passes through the point $(0, 5)$.



Example: Does the equation $y = 5 - \frac{2}{x}$ represent a straight line?

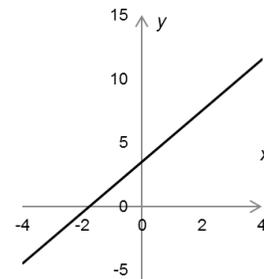
It does not matter how this is rearranged, this equation will never be in the form $y = mx + c$ and so no, this is not the equation of a straight line (in fact it is a **hyperbola**).

Example: Does the equation $4x - 2y + 7 = 0$ represent a straight line?

Rearranging the equation to make y the subject gives:

$$y = 2x + \frac{7}{2}$$

which fits the pattern of $y = mx + c$ with $m = 2$ and $c = 7/2$ and so the equation represents a straight line with gradient of 2 and y -intercept of $7/2$.



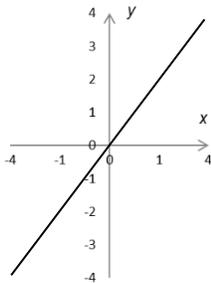
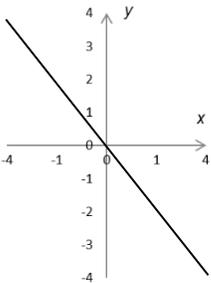
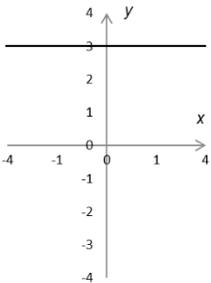
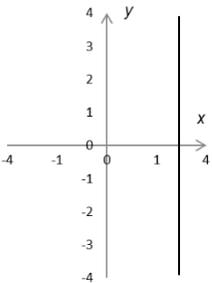
In the last example the equation is made up a number multiplied by x plus a number multiplied by y plus another number which are set equal to zero. In general such equations look like $ax + by + c = 0$. In the example above, $a = 4$, $b = -2$ and $c = 7$.

There are no indices on x or y , trigonometric functions or other functions in the equation, it is made up of just adding together numbers and multiples of x and y . These equations are called **linear equations**.

Linear equations can always be converted into the form $y = mx + c$ and so their graphs are straight lines.

Useful straight lines to remember

There are four very common straight line graphs that you should be able to recognise and draw.

Equation	$y = x$	$y = -x$	$y = \text{constant}$ Example: $y = 3$	$x = \text{constant}$ Example: $x = 3$
m	1	-1	0	Undefined
c	0	0	3	Undefined
Graph				

The graph of $y = x$ has a gradient $m = 1$, in other words as you go along 1 unit to the right you go up 1 unit. This also means that the graph is at 45° to the horizontal. The gradient is positive so the straight line goes upwards. It also passes through the origin so $c = 0$.

The graph of $y = -x$ has a gradient $m = -1$, in other words as you go along 1 unit to the right you go down 1 unit. This also means that the graph is at -45° to the horizontal.

The gradient is negative so the straight line goes downwards. It also passes through the origin so $c = 0$.

The graph of $y = 3$ (in the table this is an example of $y = \text{constant}$) passes through $(0, 3)$ and so has $c = 3$ and it has a gradient 0. If you go along 1 unit you do not go up or down. It is flat. If m is zero then the straight line is horizontal. A very important example of this type of straight line is the **x-axis** which is the line $y = 0$.

The graph of $x = \text{constant}$ (the example of $x = 3$ is given in the table) is unusual. It is a linear equation and its graph is a straight line but the gradient is not defined. You might think of the gradient as infinite but this is not quite right for subtle mathematical reasons. (You can talk to a [Learning Enhancement Tutor](#) if you would like to know more about this.) If you go along any distance at all you go down (or up) an infinite distance. Like falling off (or climbing) a vertical cliff! So it never crosses the y -axis and so has no y -intercept unless it is the y -axis which has an infinite number of intercepts. The **y-axis** is a very important example of this type of line and has the equation $x = 0$.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a [Learning Enhancement Tutor](#) in the [Student Support Service](#), as well as speaking to your lecturer or adviser.

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