Learning Enhancement Team

Bridging Between Algebra and Calculus

Sketching a Graph

This guide discusses the skill of sketching a graph. It gives methods to help you understand how to sketch a graph from a given function.

Introduction

It is not unusual in mathematics to be asked to sketch a graph of a given function or functions. This often happens when a quick impression of the shape of the function is required rather than an accurate plot or computer representation. This regularly plays a part in calculus, for example sketching a curve with stationary points or finding a region of integration (see study guides: *Finding Stationary Points* and *Definite Integrals*). **It is important for you to know the difference between sketching and plotting graphs**. A **sketch** depicts the important parts of a graph, it does not have to be to scale although it still has to be labeled correctly and any lines or points need to be correctly positioned in relation to each other and the axes. In a **plot** on the other hand, you work out precise positions of the coordinates of the graph and plot them either using a computer or on graph paper (see study guide: *Plotting a Graph by Hand*)

There is no formalised way to sketch a graph but this guide offers some ideas and methods which you may want to follow. However, you should always start your sketch by drawing axes on a piece of paper. You should use a pencil and ruler and have an eraser to hand as sketches often need to be adjusted as they take shape. When you find out some information about your function you should mark it in roughly the right place on the paper and label it. Eventually your sketch will form and, when you are happy you have all the relevant information, you should draw the function.

It is also extremely useful to learn the shapes of basic functions, such as linear, quadratic, cubic, exponential, logarithmic and trigonometric (sine, cosine and tangent). You can also calculate a few points on the graph to help you. The best way to improve you graph sketching is to look at functions, attempt to sketch them and then use a computer or a plot to see what that function actually looks like. For more details about sketching straight lines you can read the study guide: *Sketching Straight Lines*.

Important parts of graphs and how to find them

There is a list of questions which you can ask yourself which help you to sketch the graph of a function. Each question is not always relevant to every function but they form a good starting point.

1.   Does the function cross the axes, if so where?

To find where a function crosses the *y*-axis (the so-called **y-intercept**) you should **set x equal to zero** in the function and calculate the corresponding value of *y*. This value is the *y*-intercept and you can mark it on your sketch. Most functions have a *y*-intercept and it is a good idea to find it first.

To find where a function crosses the *x*-axis (the so-called **roots of the function**) you should **set y equal to zero** in the function and calculate the value(s) of *x* which make this true by solving the resulting equation. For simple functions, such as straight lines and quadratic functions, solving the equation is relatively straightforward (see study guides: *What is a Straight Line?*, *Finding Equations of Straight Lines*, *Solving Quadratic Equations by Factorisation* and *Solving Quadratic Equations Using the Quadratic Formula*). Finding the roots of more complicated functions is not as straightforward. There are many mathematical methods dedicated to finding roots but they are beyond the remit of this guide. You can talk to a **Learning Enhancement Tutor** for more advice about finding roots.

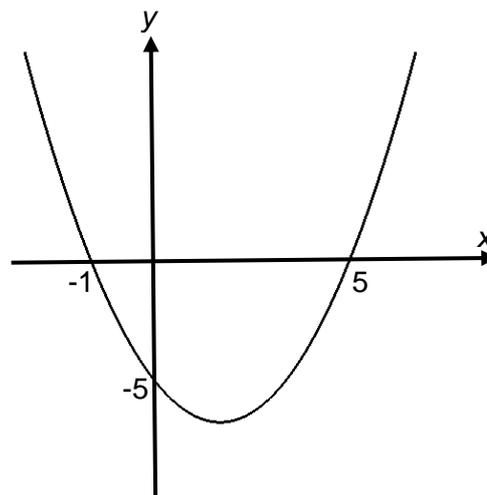*Example*:   Sketch the function $y = x^2 - 4x - 5$.

Set $x = 0$ in $y = x^2 - 4x - 5$ to find the *y*-intercept.

So as $y = 0^2 - 4 \cdot 0 - 5 = -5$, the *y*-intercept is -5.

Set $y = 0$ in $y = x^2 - 4x - 5$ and solve the resulting equation to find the roots. Factorising $0 = x^2 - 4x - 5$ gives $0 = (x - 5)(x + 1)$ and so $x = -1$ and $x = 5$. Which means the function crosses the *x*-axis at $x = -1$ and $x = 5$.

You can use these three points, along with the general shape of a quadratic function to produce the sketch on the right.

Sketch of the function $y = x^2 - 4x - 5$

2.   Are there any asymptotes, if so where are they?

Although there are four types of asymptotes: horizontal, vertical, oblique and curved, this guide is only concerned with the first two types as they are the most common. If you need advice about the other types of asymptotes you can talk to a **Learning Enhancement Tutor**.

(i)   **A horizontal asymptote** is a line parallel to the *x*-axis of the form $y = k$, which a function approaches, but never crosses, as *x* approaches $\pm\infty$. Here *k* is a constant. **Horizontal asymptotes occur at values of y which cannot be produced by the function, regardless of the value of x.** You should find out whether your function approaches the horizontal asymptote from above or below as *x* approaches $+\infty$ and

$-\infty$ and use this information in your sketch. You can use a brief table to help you.

(ii) **A vertical asymptote** is a line parallel to the *y*-axis of the form $x = k$, where the function approaches either $+\infty$ or $-\infty$ as *x* approaches *k*. Here *k* is a constant. **Vertical asymptotes occur when a value of *x* results in dividing by 0.** If you have vertical asymptotes you should consider what happens as you approach the value *k* from the left and from the right. Again you can use a table to find whether the function approaches either $+\infty$ or $-\infty$. Use this information in your sketch.

Reciprocal functions, functions where *x* appears below a dividing line, invariably contain asymptotes. If you are asked to sketch a reciprocal function then you should begin by finding its asymptotes and what happens as you approach them. Other functions which contain asymptotes are the tangent and exponential functions (see study guide: *Exponential Functions*).

Example: Sketch the function $y = \dfrac{3}{x+3} + 2$.

If $x = -3$ then you are dividing by 0 and so the line $x = -3$ is a vertical asymptote. You can show asymptotes on a sketch with a dotted line.
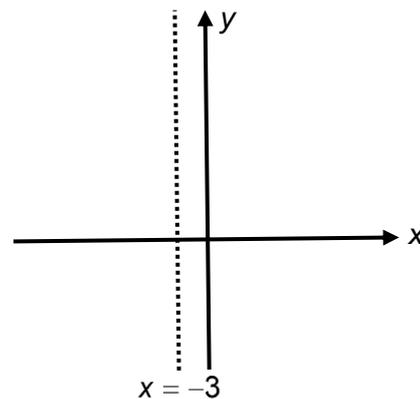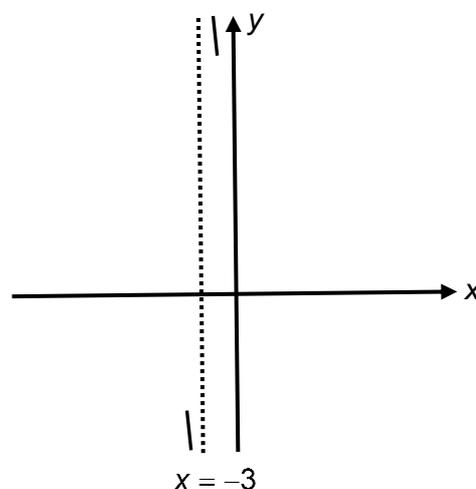

You should find out what happens as you approach the asymptote from the left and the right.

As you approach $x = -3$ from the left, the value of *y* gets large and negative as seen from the table below:

x	-4	-3.5	-3.1	-3.0001
y	-1	-4	-28	-29998

As you approach $x = -3$ from the right, the value of *y* gets large and positive as seen from the table below:

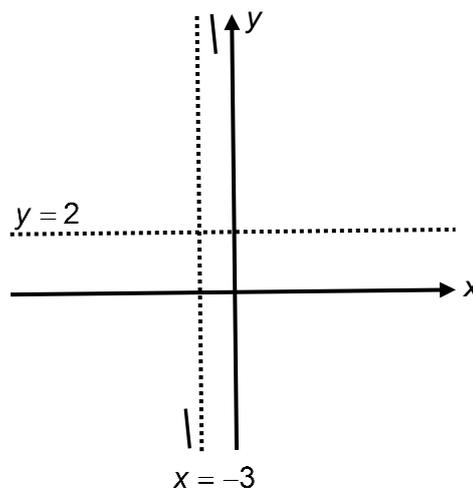| *x*	-2	-2.5	-2.9	-2.9999
y	5	8	32	30002

You can put this information on your sketch using short lines near to the asymptotes.

It is impossible for the $\dfrac{3}{x+3}$ part of the function to take the value 0.

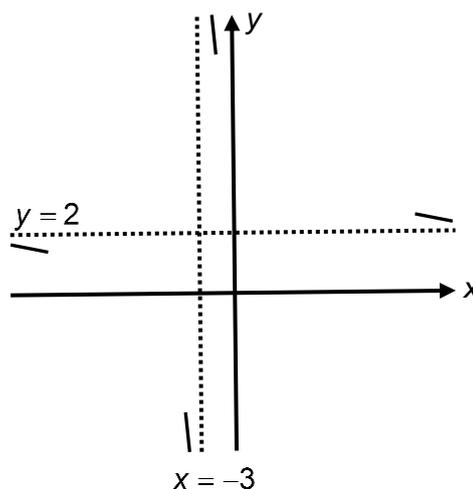Therefore it is impossible for *y* to equal 2 and so the line $y = 2$ is an asymptote.

You can now test what happens as you approach this new asymptote.

As *x* approaches $+\infty$ the function approaches the asymptote from above.

x	10	100	1000	100000
y	2.23	2.03	2.003	2.00003

As *x* approaches $-\infty$ the function approaches the asymptote from below.

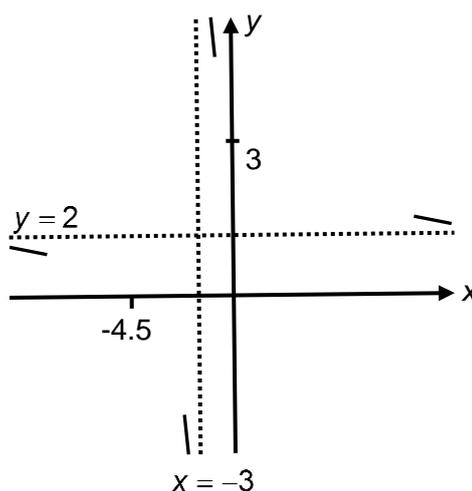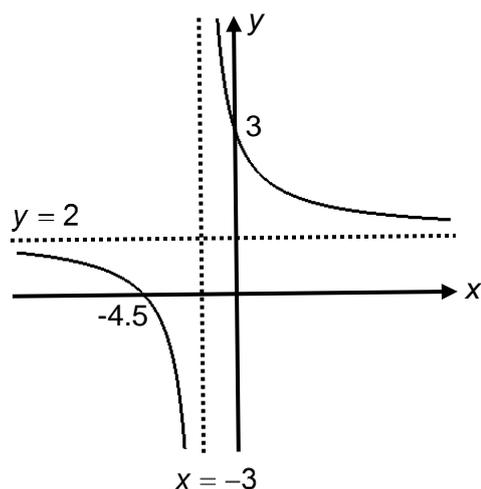| x	-10	-100	-1000	-100000
y	1.57	1.97	1.997	1.99997

Finally you can work out where the function crosses the axes.

When $x = 0$, $y = \dfrac{3}{3} + 2 = 1 + 2 = 3$, so the *y*-intercept is at $y = 3$.

When $y = 0$, $0 = \dfrac{3}{x+3} + 2$ which is solved to give a root of $x = -4.5$.

Combining all this information about the function together with the knowledge of the shape of a simple reciprocal function produces the sketch to the right.

$y = 2$

3

-4.5

x

$x = -3$

Sketch of the function $y = \dfrac{3}{x+3} + 2$

3.   Are there any stationary points, if so where are they and what is their nature?

It is important to gather as much information as possible before beginning to sketch a graph. Many functions have **stationary points**, points at which the graph of the function has a gradient of 0. There are three types of stationary points, **maxima, minima** and **points of inflection**. The position and nature of stationary points can be found by using differentiation (see study guides: *Stationary Points* and *Finding Stationary Points*) and can prove to be extremely useful when you are sketching a function.

Let's return to the first example in this guide.

Example: Sketch the function $y = x^2 - 4x - 5$.

You already know that the *y*-intercept is at $y = -5$ and the roots are $x = -1$ and $x = 5$ from work done in the first example.

However, you can improve your sketch by using differentiation to find the position and nature of any stationary points.

By differentiating the function $y = x^2 - 4x - 5$, you can see that:

$$\frac{dy}{dx} = 2x - 4 = 0 \text{ when } x = 2.$$

As there is only one solution to the equation, there is only one stationary point. Substituting $x = 2$ into the original function gives $y = -9$ and so the position of the stationary point is $(2, -9)$.
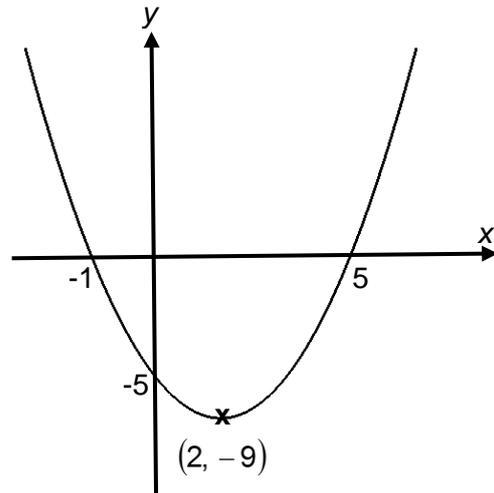
The second derivate of $y = x^2 - 4x - 5$ is:

$$\frac{d^2y}{dx^2} = 2$$

Which is positive and so the nature of the stationary point is a minimum.

You can add this information to your sketch. Notice that the sketch is not to scale but is correct **relative** to the other points.

Improved sketch of the function $y = x^2 - 4x - 5$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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