Learning Enhancement Team

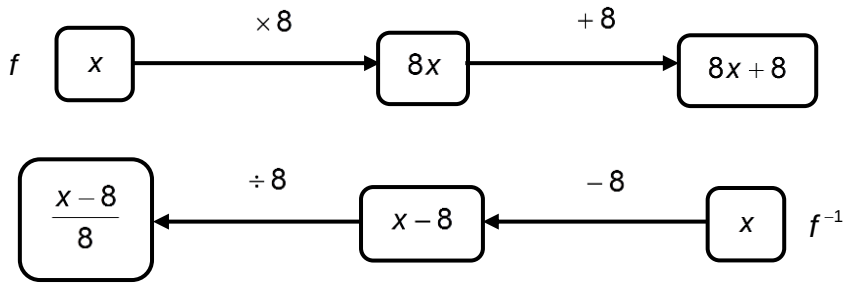
University of East Anglia
STUDENT SUPPORT SERVICE

Model answers: Inverse Functions and Graphs

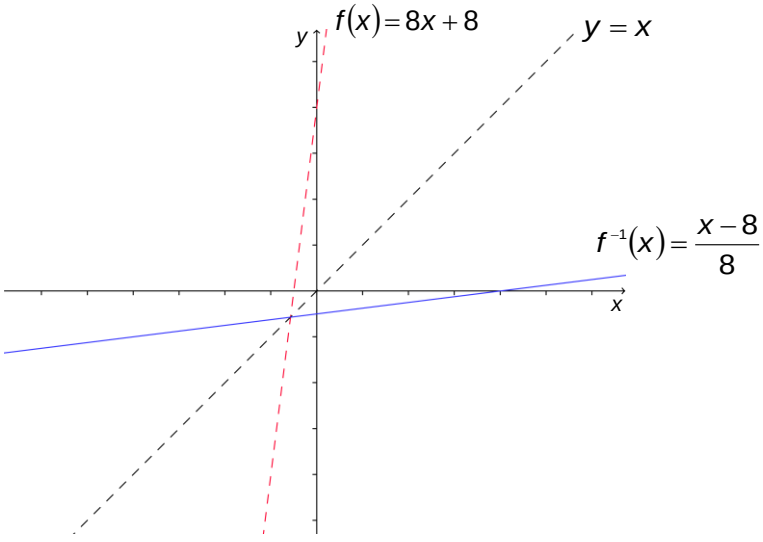
Inverse Functions and Graphs study guide

1. i) $f(x) = 8x + 8$ is a bijection as it has a unique output for every input.

You can use the flow chart method to work out $f^{-1}(x)$:



So $f^{-1}(x) = \frac{x-8}{8}$. Remember that the graph of the inverse function is the graph of the function reflected in the line $y = x$. You can see a sketch below.



ii) The function $f(x) = x^2 + 8$ is not a bijection.

You can see that:

$$f(1) = 1^2 + 8 = 9$$

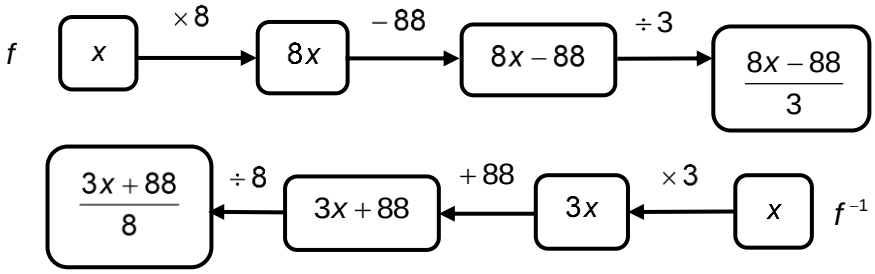
and that:

$$f(-1) = (-1)^2 + 8 = 9\,.$$

The output 9 is paired with two inputs 1 and -1; so the function is not a bijection.

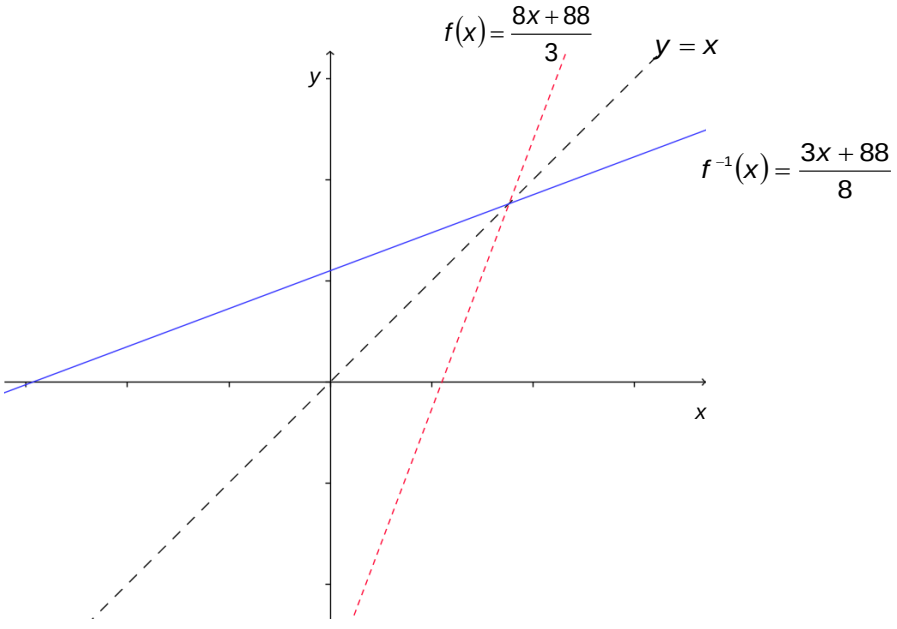
iii) $f(x) = \dfrac{8x - 88}{3}$ is a bijection as it has a unique output for every input.

You can use the flow chart method to work out $f^{-1}(x)$:



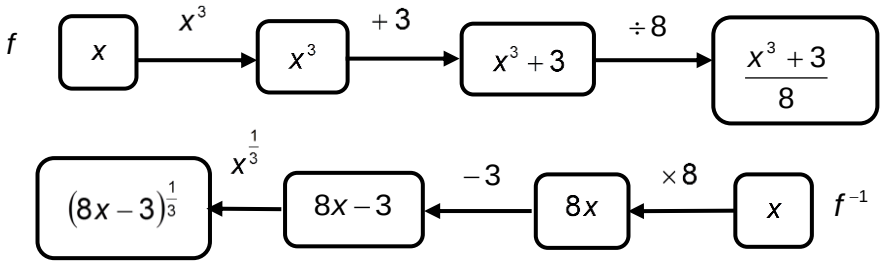
So $f^{-1}(x) = \dfrac{3x + 88}{8}$.

Remember that the graph of the inverse function is the graph of the function reflected in the line $y = x$. You can see a sketch below.



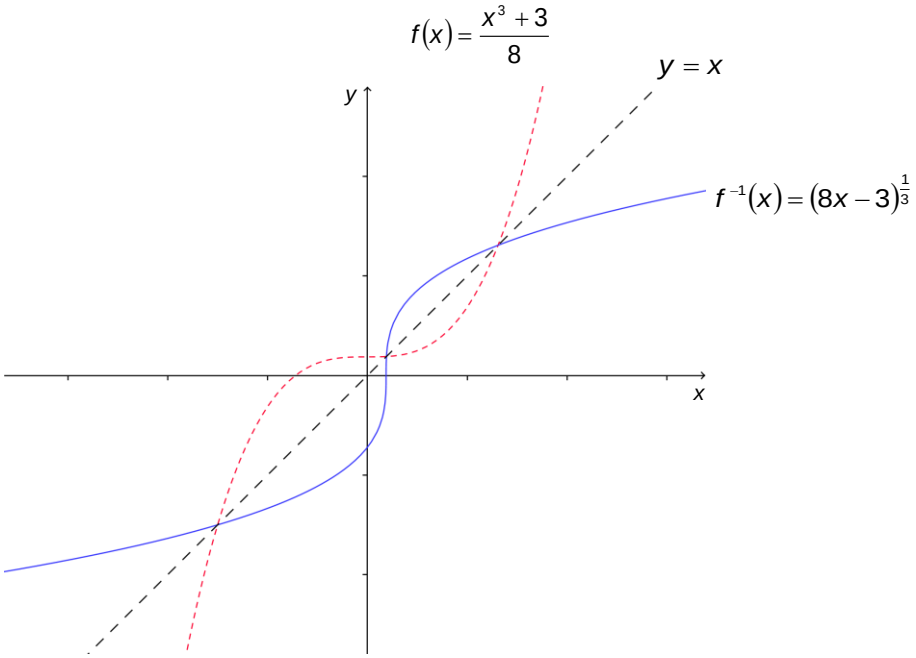
iv) $f(x)=\dfrac{x^3+3}{8}$ is a bijection as it has a unique output for every input.

You can use the flow chart method to work out $f^{-1}(x)$:



So $f^{-1}(x)=(8x-3)^{\frac{1}{3}}$.

Remember that the graph of the inverse function is the graph of the function reflected in the line $y=x$. You can see a sketch below.



v) The function $f(x)=\dfrac{8x^2+8x}{8}$ is not a bijection.

You can see that:

$$f(-1)=\frac{8\cdot(-1)^2+8\cdot(-1)}{8}=0$$

and that:

$$f(0) = \frac{8 \cdot (0)^2 + 8 \cdot (0)}{8} = 0.$$

The output 0 is paired with two inputs 0 and -1; so the function is not a bijection.

vi) The function $f(x) = 8x^4$ is not a bijection.

You can see that:

$$f(-1) = 8 \cdot (-1)^4 = 8$$

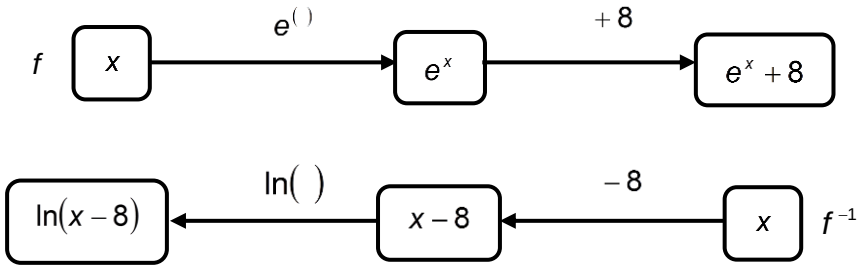
and that

$$f(1) = 8 \cdot (1)^4 = 8.$$

The output 8 is paired with two inputs 1 and -1 and so the function is not a bijection.

vii) $f(x) = e^x + 8$ is a bijection as it has a unique output for every input.

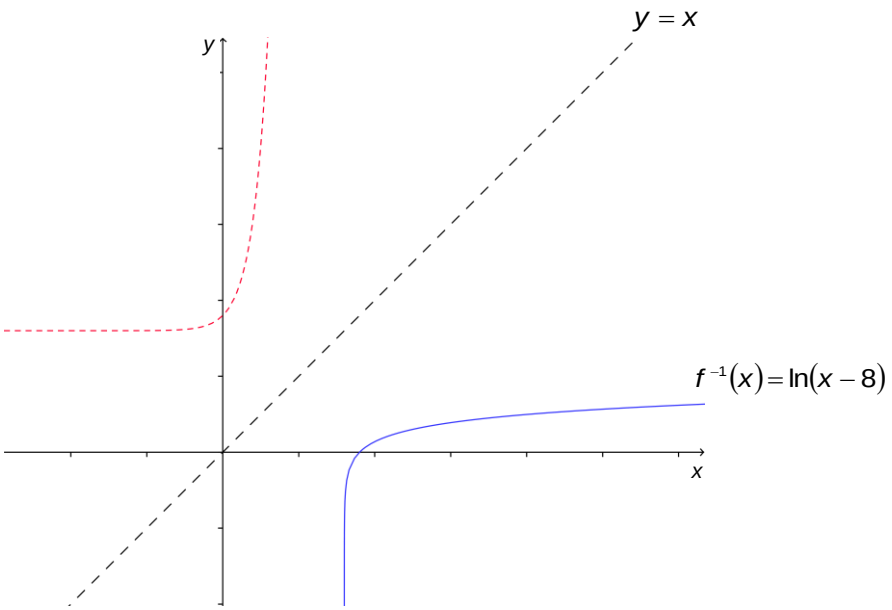
You can use the flow chart method to work out $f^{-1}(x)$:



So $f^{-1}(x) = \ln(x-8)$.

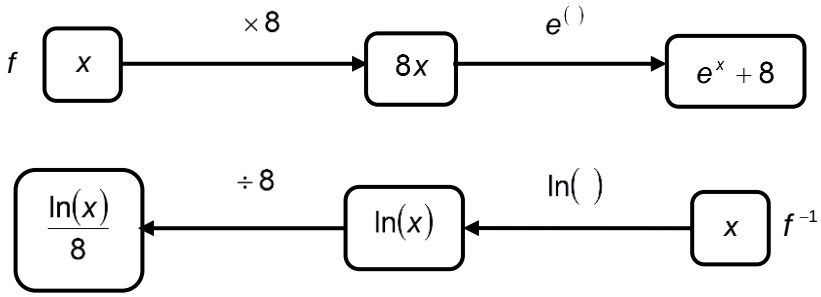
Remember that the graph of the inverse function is the graph of the function reflected in the line $y = x$. You can see a sketch below.

$$f(x) = e^x + 8$$



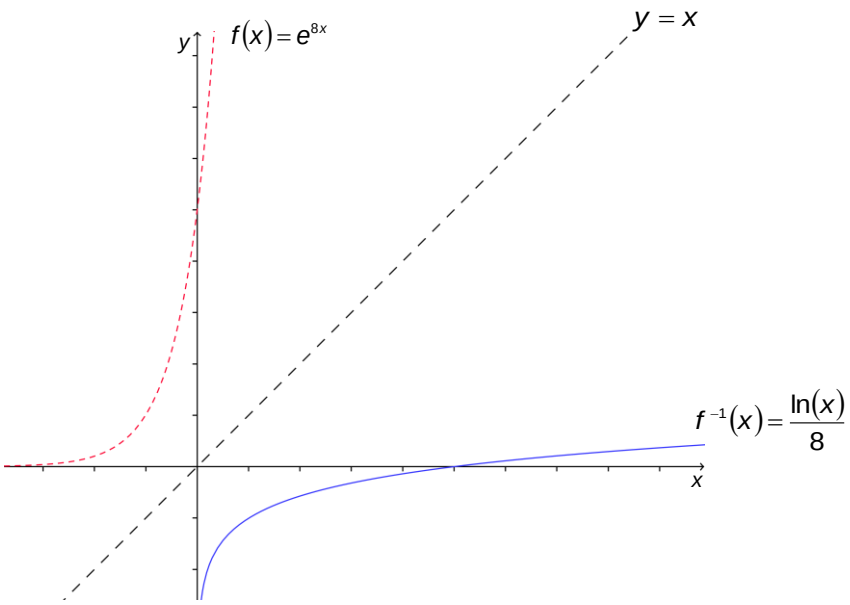
vii) $f(x) = e^{8x}$ is a bijection as it has a unique output for every input.

You can use the flow chart method to work out $f^{-1}(x)$:



So $f^{-1}(x) = \ln(x-8)$.

Remember that the graph of the inverse function is the graph of the function reflected in the line $y = x$. You can see a sketch below.



ix) The function $f(x)=(8x+8)^2$ is not a bijection.

You can see that:

$$f(-2)=(8\cdot(-2)+8)^2=(-8)^2=64$$

and that

$$f(0)=(8\cdot(0)+8)^2=(8)^2=64\,.$$

The output 64 is paired with two inputs 0 and -2; so the function is not a bijection.

x) The function $f(x)=x^8$ is not a bijection.

You can see that:

$$f(-1)=(-1)^8=1$$

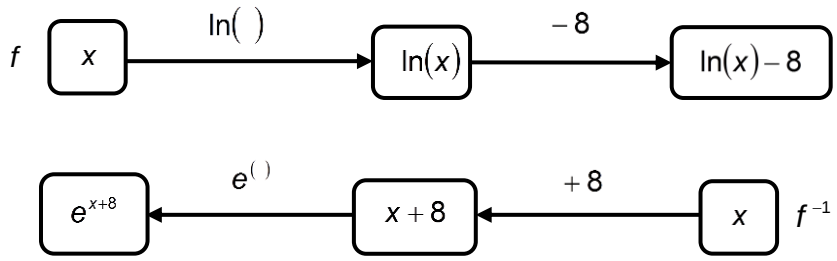
and that

$$f(1)=(1)^8=1.$$

The output 1 is paired with two inputs 1 and -1; so the function is not a bijection.

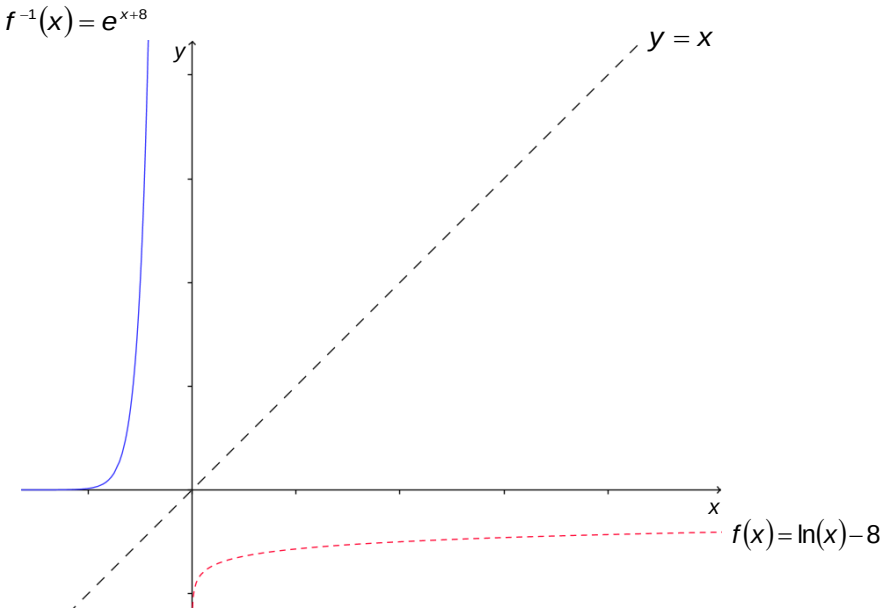
xi) $f(x) = \ln(x) - 8$ is a bijection as it has a unique output for every input.

You can use the flow chart method to work out $f^{-1}(x)$:



So $f^{-1}(x) = e^{x+8}$.

Remember that the graph of the inverse function is the graph of the function reflected in the line $y = x$. You can see a sketch below.



xii) The function $f(x) = 8$ is not a bijection.

You can see that:

$f(0) = 8$

$f(3) = 8$

$f(-75) = 8$

$f(88\pi) = 8$

and so on.

You can notice that the output 8 is paired with all inputs; so the function is not a bijection.