University of East Anglia
STUDENT SUPPORT SERVICE

Learning Enhancement Team

Model Answers: More Complicated Functions

More Complicated Functions study guide

The answers to the questions on this worksheet use the functions $f(x)=3x-1$, $g(x)=3-2x^2$ and $h(x)=e^x$.

(a) $f(x)+g(x)=3x-1+3-2x^2=3x+2-2x^2$

You need to add together $f(x)$ and $g(x)$ and then combine like terms

(b) $h(x)-f(x)=e^x-(3x-1)=e^x-3x+1$

You need to subtract $f(x)$ from $h(x)$. You can use brackets to help avoid mistakes with the minus sign which are very common in this type of question.

(c) $f(x)g(x)=(3x-1)(3-2x^2)=9x+2x^2-6x^3-3$

You need to multiply $f(x)$ and $g(x)$. Brackets can help and the study guide: *Opening Brackets* can provide you with a method if you are unsure how the answer has been achieved.

(d) $\dfrac{h(x)}{g(x)}=\dfrac{e^x}{3-2x^2}$

You need to divide $h(x)$ by $g(x)$.

2. This question uses composition to make new functions.

(a) $f(g(x))=8-6x^2$

Here the function $g(x)$ acts as the input to $f(x)$. So this calculation involves replacing the x in $3x-1$ with $3-2x^2$ so:

$$\begin{aligned} f(3-2x^2) &= 3(3-2x^2)-1 \\ &= 9-6x^2-1 \\ &= 8-6x^2 \end{aligned}$$

(b) $g(f(x)) = 7 - 18x^2 + 12x$

Here the function $f(x)$ acts as the input to $g(x)$. So this calculation involves replacing the x in $3 - 2x^2$ with $3x - 1$. Remember that it is only the x that is squared and not the -2 so:

$$\begin{aligned} g(3x-1) &= 3 - 2(3x-1)^2 \\ &= 9 - 2(9x^2 - 6x + 1) \\ &= 9 - 18x^2 + 12x - 2 \\ &= 7 - 18x^2 + 12x \end{aligned}$$

(c) $f(h(x)) = 3e^x - 1$

Here the function $h(x)$ acts as the input to $f(x)$. So this calculation involves replacing the x in $3x - 1$ with e^x so:

$$\begin{aligned} f(e^x) &= 3(e^x) - 1 \\ &= 3e^x - 1 \end{aligned}$$

(d) $h(f(x)) = e^{3x-1}$

Here the function $f(x)$ acts as the input to $h(x)$. So this calculation involves replacing the x in e^x with $3x - 1$ so:

$$h(3x-1) = e^{3x-1}$$

(e) $g(h(x)) = 3 - 2e^{2x}$

Here the function $h(x)$ acts as the input to $g(x)$. So this calculation involves replacing the x in $3 - 2x^2$ with e^x. Remember that it is only the x that is squared and not the -2 so:

$$\begin{aligned} g(e^x) &= 3 - 2(e^x)^2 \\ &= 3 - 2e^{2x} \end{aligned}$$

This calculation uses the law of indices $(a^m)^n = a^{mn}$, see study guide: *Laws of Indices*.

(f) $h(g(x)) = e^{3-2x^2}$

Here the function $g(x)$ acts as the input to $h(x)$. So this calculation involves replacing the x in e^x with $3 - 2x^2$ so:

$$h(3 - 2x^2) = e^{3-2x^2}$$

(g) $f(g(h(x))) = 8 - 6e^{2x}$

This is a bit more complicated as the composition is made from all three functions. You read the from inside out and so $h(x)$ acts as the input to $g(x)$ and the result of this acts as the input to $f(x)$. From question 2e, $g(h(x)) = 3 - 2e^{2x}$ and this acts as the input to $f(x)$ so you replace the x in $3x - 1$ with $3 - 2e^{2x}$:

$$\begin{aligned} f(3 - 2e^{2x}) &= 3(3 - 2e^{2x}) - 1 \\ &= 9 - 6e^{2x} - 1 \\ &= 8 - 6e^{2x} \end{aligned}$$

(h) $h(g(f(x))) = e^{7 - 18x^2 + 12x}$

This is also a composition made from all three functions, but this time in a different order. Again you read the from inside out and so $f(x)$ acts as the input to $g(x)$ and the result of this acts as the input to $h(x)$. From question 2b, $g(f(x)) = 7 - 18x^2 + 12x$ and this acts as the input to $h(x)$ so you replace the x in e^x with $7 - 18x^2 + 12x$:

$$h(7 - 18x^2 + 12x) = e^{7 - 18x^2 + 12x}$$

3.

(a) $f(3) + g(-2) = 3$

As $f(3) = 3 \cdot 3 - 1 = 8$ and $g(-2) = 3 - 2 \cdot (-2)^2 = 3 - 8 = -5$:

$$f(3) + g(-2) = 8 + -5 = 3$$

(b) $h(0) - f(4) = -10$

As $h(0) = e^0 = 1$ and $f(4) = 3 \cdot 4 - 1 = 11$:

$$h(0) - f(4) = 1 - 11 = -10$$

(c) $f(2)g(2) = -25$

As $f(2) = 3 \cdot 2 - 1 = 5$ and $g(2) = 3 - 2 \cdot (2)^2 = 3 - 8 = -5$:

$$f(2)g(2) = 5 \cdot (-5) = -25$$

(d) $\dfrac{h(t)}{g(2t)} = \dfrac{e^t}{3 - 8t^2}$

As $h(t) = e^t$ and $g(2t) = 3 - 2 \cdot (2t)^2 = 3 - 8t^2$

$$\frac{h(t)}{g(2t)} = \frac{e^t}{3 - 8t^2}$$

(e) $f(g(5)) = -142$

Using the answer to 2a, $f(g(x)) = 8 - 6x^2$ with 5 as the input you have:

$f(g(5)) = 8 - 6(5)^2 = 8 - 6 \cdot 25 = -142$

(f) $g(f(5)) = -383$

Using the answer to 2b, $g(f(x)) = 7 - 18x^2 + 12x$ with 5 as the input you have:

$g(f(5)) = 7 - 18(5)^2 + 12 \cdot 5 = 7 - 450 + 60 = -383$

(g) $f(h(-1)) = 0.104$ to 3 d.p.

Using the answer to 2c, $f(h(x)) = 3e^x - 1$ with -1 as the input you have:

$f(h(-1)) = 3e^{-1} - 1 = 0.104$ to 3 d.p.

(h) $f(h(t)) = 3e^t - 1$

Using the answer to 2c, $f(h(x)) = 3e^x - 1$ with t as the input you have:

$f(h(t)) = 3e^t - 1$

(i) $h(f(t^2)) = e^{3t^2 - 1}$

Using the answer to 2d, $h(f(x)) = e^{3x-1}$ with t^2 as the input you have:

$h(f(t^2)) = e^{3t^2 - 1}$

(j) $g(h(x+1)) = 3 - 2e^{2(x+1)}$

Using the answer to 2e, $g(h(x)) = 3 - 2e^{2x}$ with $x + 1$ as the input you have:

$g(h(x+1)) = 3 - 2e^{2(x+1)}$

(k) $f(g(h(6))) = -976520.749$ to 3 d.p.

Using the answer to 2g, $f(g(h(x))) = 8 - 6e^{2x}$ with 6 as the input you have:

$f(g(h(6))) = 8 - 6e^{2 \cdot 6} = 8 - 6e^{12} = -976520.749$ to 3 d.p.

(l) $h(g(f(1-x))) = e^{1+24x-18x^2}$

Using the answer to 2h, $h(g(f(x))) = e^{7-18x^2+12x}$ with $1 - x$ as the input you have:

$h(g(f(1-x))) = e^{7-18(1-x)^2+12(1-x)}$

As $(1-x)^2 = x^2 - 2x + 1$ you can expand the brackets in the power of e to give:

$$\begin{aligned} 7 - 18(1-x)^2 + 12(1-x) &= 7 - 18(x^2 - 2x + 1) + 12 - 12x \\ &= 7 - 18x^2 + 36x - 18 + 12 - 12x \\ &= 1 + 24x - 18x^2 \end{aligned}$$

4.

(a) $\sin(3x^2 - 4)$

$3x^2 - 4$ is a basic polynomial function which can act as an input to the sine function so if $f(x) = 3x^2 - 4$ and $g(x) = \sin(x)$ then $g(f(x)) = \sin(3x^2 - 4)$.

(b) $\cos(\ln(x))$

$\ln(x)$ is the natural logarithm function which can act as an input to the cosine function so if $f(x) = \ln(x)$ and $g(x) = \cos(x)$ then $g(f(x)) = \cos(\ln(x))$.

(c) $\sqrt{x-1}$

$x - 1$ is a basic polynomial function which can act as an input to the square root function so if $f(x) = x - 1$ and $g(x) = \sqrt{x}$ then $g(f(x)) = \sqrt{x-1}$.

(d) $e^{\sqrt{4t-9}}$

$4t - 9$ is a basic polynomial function which can act as an input to the square root function, the composition of *f* and *g* can then act as the input to the exponential function. So if $f(t) = 4t - 9$, $g(x) = \sqrt{x}$ and $h(x) = e^x$ then $h(g(f(t))) = e^{\sqrt{4t-9}}$.