Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Model answers: Functions

Functions study guide

1. You can describe the people as a set, let's call it P for people (you can call it what you like). Written mathematically this set is:

$$P = \{\text{A, B, D, E}\}$$

Where the initials of the people are used instead of their full names for simplicity.

You can also describe the stops on the train as a set, let's call it S for stops (you can also call this what you like but you should avoid using the same letter as other sets in a question). Written mathematically this set is:

$$S = \{\text{I, C, M, L}\}$$

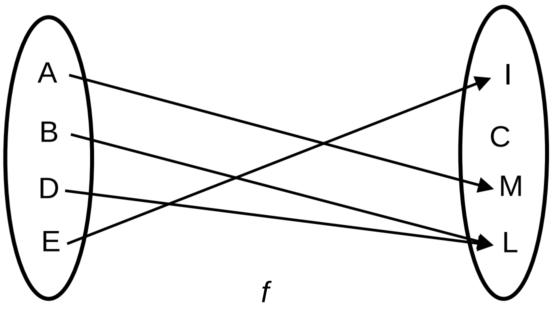
Where the initials of the stops are used instead of their full names for simplicity.

You can now describe a function, call it f for function, which connects an individual to the stop where they get off the train. You can say that f connects a person to a stop. You can write this mathematically as:

$$f : P \to S$$

Where the people form the input of the function and the stops form the output. The domain of a function is the set of its inputs. In this case the domain of f is P. The co-domain is the set of train stops, S. The range of the function is those stops which are used by the people on the train. Here the range is the set $\{\text{I, M, L}\}$. You should note that, for every function, the range is always a subset of the co-domain.

You could represent this function in the following diagram

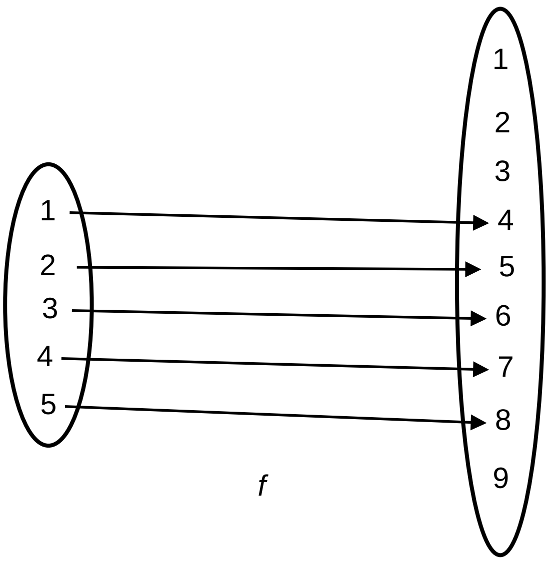


2. For this question the sets are explicitly given by:

$A = \{1,2,3,4,5\}$

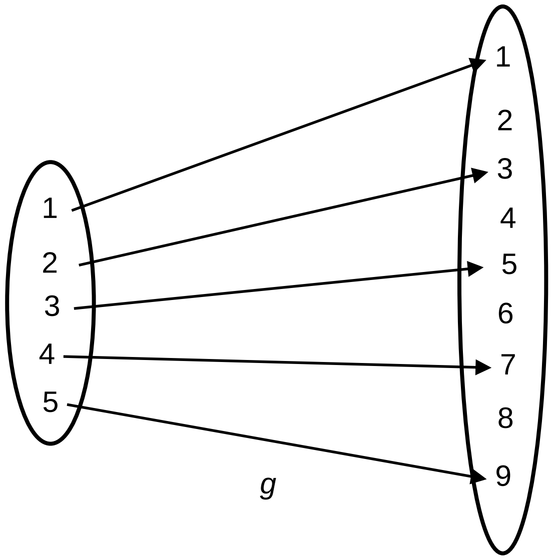
$B = \{1,2,3,4,5,6,7,8,9\}$

(i) The function *f* says take the input *x* and then add three to it. As the set *A* acts as the input to this function, to find the output (each of which has to be a part of the set *B*) you need to add three to each element of set *A*. You can represent this pictorially as:



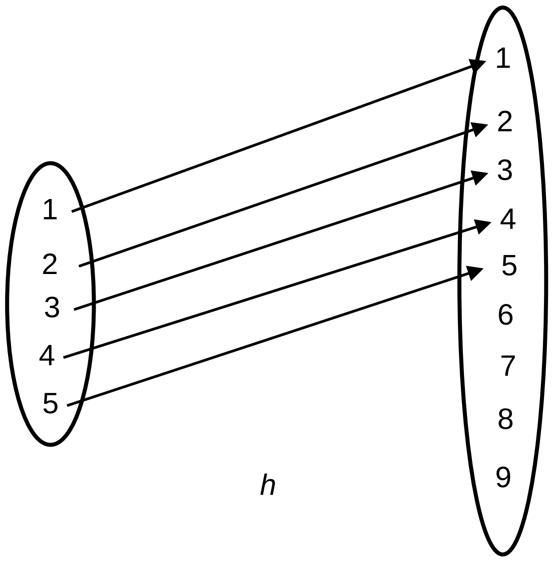
The domain of *f* is the set *A*, the co-domain of *f* is the set *B* and the range of *f* is the set $\{4,5,6,7,8\}$.

(ii) The function *g* says take the input *x* double it and then subtract 1. As the set *A* acts as the input to this function, to find the output (each of which has to be a part of the set *B*) you need to double and then subtract one from each element of set *A*. You can represent this pictorially as:



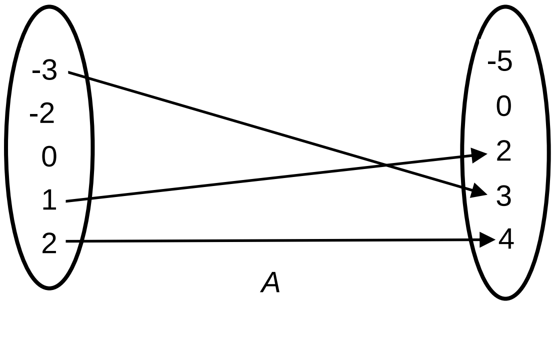
The domain of *g* is the set *A*, the co-domain of *g* is the set *B* and the range of *g* is the set $\{1,3,5,7,9\}$.

(iii) The function *h* says take the input *x* and map it to itself. As the set *A* acts as the input to this function, the output (each of which has to be a part of the set *B*) is the same as each element of set *A*. You can represent this pictorially as:

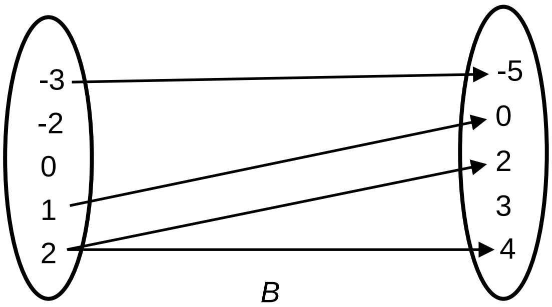


The domain of *h* is the set *A*, the co-domain of *h* is the set *B* and the range of *h* is the set $\{1,2,3,4,5\}$.

3.

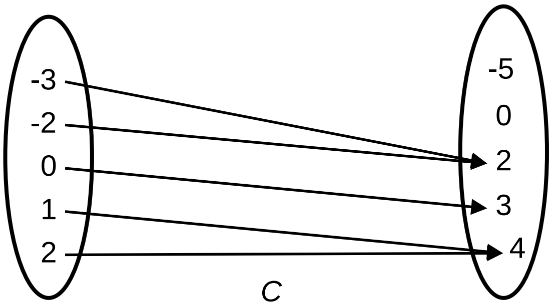


Not a function because there are some elements in the domain (-2 and 0) which are not connected to the co-domain.



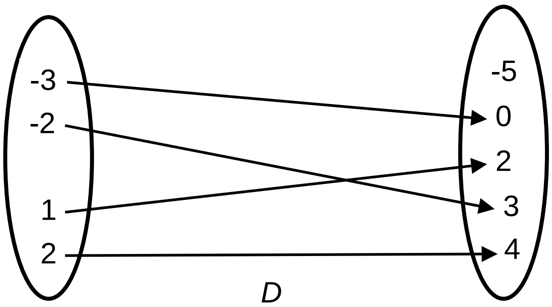
B

Not a function because there are some elements in the domain (-2 and 0) which are not connected to the co-domain and one element (2) is connected to 2 elements in the co-domain.



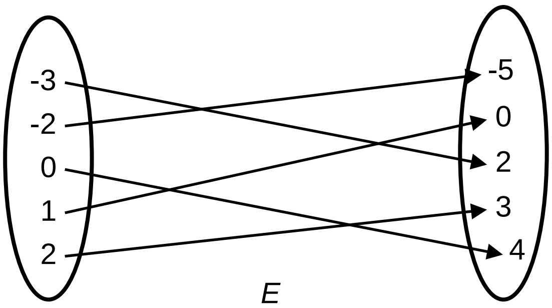
C

This a function because every element in the domain is connected to only one element in the co-domain. The domain is the set $\{-3,-2,0,1,2\}$, the co-domain is the set $\{-5,0,2,3,4\}$ and the range is $\{2,3,4\}$.



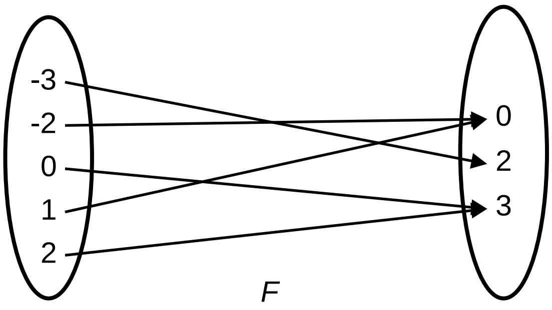
D

This a function because every element in the domain is connected to only one element in the co-domain. The domain is the set $\{-3,-2,1,2\}$, the co-domain is the set $\{-5,0,2,3,4\}$ and the range is $\{0,2,3,4\}$.



E

This a function because every element in the domain is connected to only one element in the co-domain. The domain is the set $\{-3,-2,0,1,2\}$, the co-domain and the range are both the set $\{-5,0,2,3,4\}$.



This a function because every element in the domain is connected to only one element in the co-domain. The domain is the set $\{-3, -2, 0, 1, 2\}$, the co-domain and the range are both the set $\{0, 2, 3\}$.

4. An injective function has each element in the range connected to only one element in the domain. So the functions in 2(i), 2(ii) and 2(iii) and 3*D* and 3*E* are injective.

A surjective function is one in which the co-domain and the range are the same. So 3*E* and 3*F* are surjective.

The functions in question 1 and 3*C* are neither injective nor surjective.

A bijective function is one which is both injective and surjective so only 3*E* is a bijective function.

Only bijective functions are invertible (they have an inverse) and so only 3*E* is invertible.