Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

*Steps into Trigonometry*

# The Sine Rule

***This guide introduces the sine rule and illustrates the specific situations in when and how it can be used to solve problems involving triangles which do not contain a right-angle.***

## Introduction

Problems involving triangles are extremely common in many mathematical subject areas including geometry, vectors, calculus and complex numbers. Many such problems involve right-angled triangles and can be solved using the trigonometric ratios (see study guide: *Solving Right-Angled Triangles*). You should be comfortable solving right-angled triangles before continuing with this guide.

It is also common for problems to involve triangles which do not contain a right-angle. To solve these types of triangles you need to use the more general mathematical rules of either the **sine rule** or the **cosine rule**. You cannot use the trigonometric ratios (or Pythagoras' theorem) in these cases.

The table below shows all the triangles problems that are possible and the method you can use to solve them.

| Type of Triangle | Known values | Method |
|---|---|---|
| Right-angled | Two sides | Trigonometric Ratios |
| Right-angled | One side and one angle | Trigonometric Ratios |
| Non right-angled | Two sides and non-included angle | Sine Rule |
| Non right-angled | One side and two angles | Sine Rule |
| Non right-angled | Three sides | Cosine Rule |
| Non right-angled | Two sides and included angle | Cosine Rule |
| Any | Three angles | Not Solvable |

The sine rule is given by:

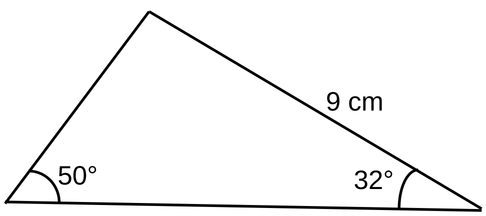
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

where $a$ and $b$ are lengths of the sides of a triangle, $A$ is the angle opposite the side of length $a$ and $B$ is the angle opposite the side of length $b$. This rule is often shown as a set of three equations but this is unnecessary as you will see in this guide's examples.

This study guide only offers help with using the sine rule. There is a separate guide for help with the cosine rule, see study guide: *The Cosine Rule*. To use the sine rule effectively should be comfortable with the manipulation of equations and inverse trigonometric functions, see study guides: *Rearranging Equations* and *Trigonometric Ratios: Sine, Cosine and Tangent* for help with these topics.

You can see from the table on the first page of this guide that you should use the sine rule to help you solve a triangle if you know the length of one side and two angles *or* if you have the length of two sides and the size of an angle which is not made by them (sometimes called a **non-included angle**). As with all questions concerning triangles it is a good idea to sketch the triangle you are investigating and label the quantities you know as part of answering the question.

*Example*: (*Finding the length of a side*) Solve the triangle:



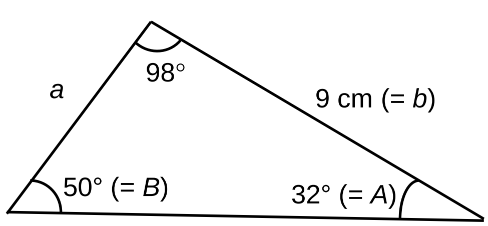

As you have **one side** (9 cm) and **two angles** (50° and 32°) you can use the **sine rule** by following this method:

(i) You can work out the size of the third angle as you know the other two.
(ii) Label the side you want to calculate as $a$.
(iii) Label the angle opposite this side as $A$ (**this is important**).
(iv) Label the side you know as $b$ and the angle opposite it as $B$ (**this is also important**).

As you know that the angles in a triangle add to 180°, the size of the unknown angle is:

$$180° - 50° - 32° = 98°$$

Now label your diagram as shown in the triangle below.



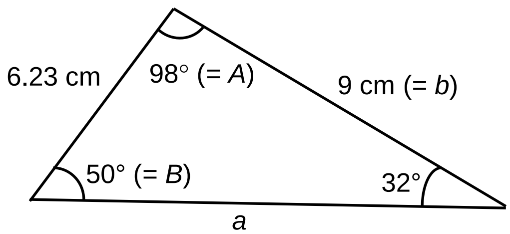

So $A = 32°$, $B = 50°$ and $b = 9\,\text{cm}$. Putting these values into the sine rule you get:

$$\frac{a}{\sin 32°} = \frac{9}{\sin 50°}$$

Rearranging for $a$ gives:

$$a = \frac{9 \sin 32°}{\sin 50°} = 6.23\,\text{cm to 2 decimal places.}$$

You can find the other missing side in a similar manner by labelling your triangle accordingly, see below.



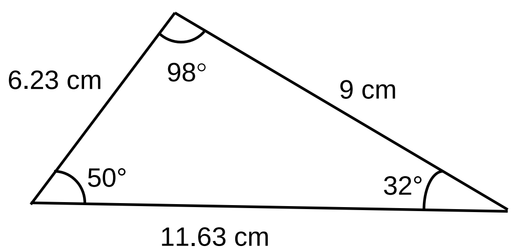

So this time $A = 98°$, $B = 50°$ and $b = 9$ cm. Putting these new values into the sine rule you get:

$$\frac{a}{\sin 98°} = \frac{9}{\sin 50°}$$

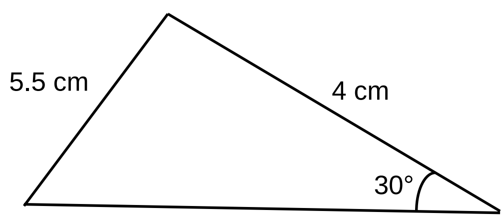
Rearranging for *a* gives:

$$a = \frac{9\sin 98°}{\sin 50°} = 11.63 \text{ cm to 2 decimal places.}$$

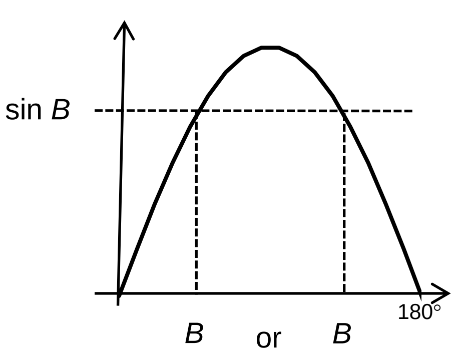
So the solved triangle is:




*Example: (Finding the size of an angle)* Solve the triangle:




As you have the length of two sides and the size of an angle which is not made by them you can use the sine rule to solve the triangle. However **finding angles using the sine rule requires extra care**. Triangles without a right-angle can contain an angle which is greater than 90°. Because of this, when you use the inverse sine function to find an angle you have the potential for two answers because of the shape of the sine graph:

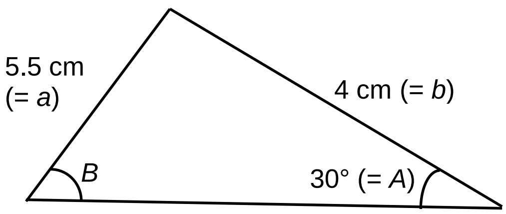



As you can see from the sketch to the left there are two possible values between 0° and 180° for the angle *B* whose sine is given by $\sin B$. This leads to the potential for two solutions when you use the sine rule to find angles. See study guide: *Solving Trigonometric Equations* for more details. Your calculator gives the value of *B* nearest to the vertical axis. You can find the other value for *B* by subtracting the first value from 180°.

If you are using the sine rule to find an angle you need to consider each answer in turn and decide whether they provide a reasonable answer. You may find that one of the answers is not possible. However you may find that both answers are possible and lead to two separate, but correct, answers. Either way you should follow the method:

(i) You know a side and its opposite angle, label the side *a* and the angle *A*.
(ii) Label the other side you know as *b*.
(iii) Label the angle opposite this side as *B* (**this is important**).
(iv) Calculate *B* and work out the second value by subtracting *B* from 180°.
(v) Test the validity of both angles.

So, for the example in question you get the triangle below.




As you can see $A = 30°$, $a = 5.5$ cm and $b = 4$ cm. Putting these values into the sine rule gives:

$$\frac{5.5}{\sin 30°} = \frac{4}{\sin B}$$

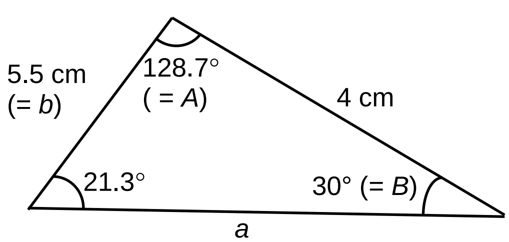
Rearranging for *B* you get two possible values:

$$B = \sin^{-1}(0.363) = 21.3° \qquad \text{or} \qquad B = 180° - 21.3° = 158.7°$$

You can check on your calculator that $\sin(21.3°) = \sin(158.7°) = 0.363$. As you know one angle in the triangle is 30°, the option of $B = 158.7°$ is too large as the angles would sum to over 180°. So $B = 21.3°$ and the other unknown angle is:

$$180° - 30° - 21.3° = 128.7°$$

You can now use the method in the first example in this guide to find the missing side:




So $\quad \dfrac{a}{\sin 128.7°} = \dfrac{5.5}{\sin 30°}$

Which gives, after rearranging, $a = 8.58$ cm.

*Example*: If one side of a triangle is 2.5 cm long and the angle opposite this side is 40°, what triangles can you draw if you know the length of a second side in the triangle is 2.9 cm?

If you follow the method above then you can work out that $a = 2.5$ cm, $A = 40°$ and

$b = 2.9$ cm. You can now substitute these values directly into the sine rule to find that:

$$\frac{2.9}{\sin B} = \frac{2.5}{\sin 40^\circ}$$

So, after rearranging for $B$:

$$B = \sin^{-1} 0.75 = 48.6^\circ \qquad \text{or} \qquad B = 180^\circ - 48.6^\circ = 131.4^\circ$$

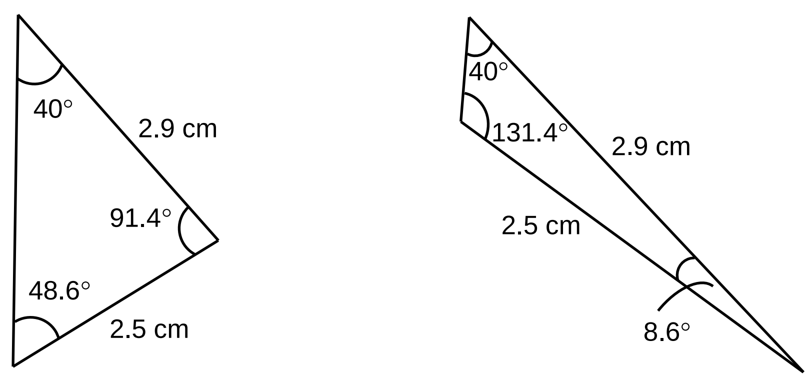
Taking the first value and using the sum of the angles of a triangle is $180^\circ$ gives the final angle as:

$$180^\circ - 40^\circ - 48.6^\circ = 91.4^\circ$$

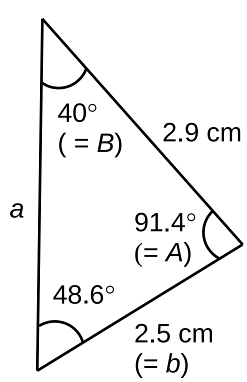
Taking the second value gives the final angle as:

$$180^\circ - 40^\circ - 131.4^\circ = 8.6^\circ$$

Which leads to two potential solutions, both of which are feasible:




Taking the triangle on the left , you know all three angles and one side and you can use the method in the first example in this guide to find the length of the missing side.



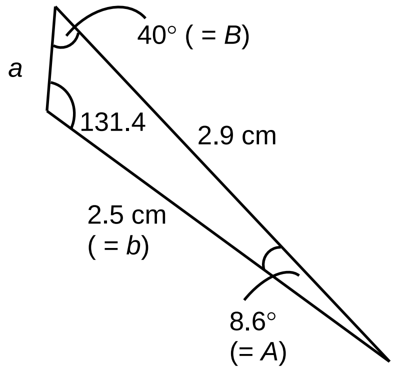

As $b = 2.5$ cm, $A = 91.4^\circ$ and $B = 40^\circ$, the sine rule gives:

$$\frac{a}{\sin 91.4^\circ} = \frac{2.5}{\sin 40^\circ}$$

And you can rearrange this equation to find that:

$$a = \frac{2.5 \sin 91.4^\circ}{\sin 40^\circ} = 3.89 \text{ cm to 2 d.p.}$$

Looking at the triangle on the right, you can use the same method to find the length of the missing side.



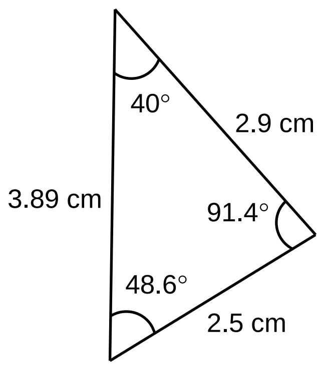

As $b = 2.5\,\text{cm}$, $A = 8.6°$ and $B = 40°$, the sine rule gives:

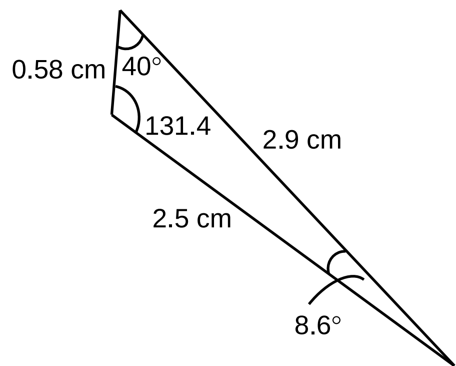
$$\frac{a}{\sin 8.6°} = \frac{2.5}{\sin 40°}$$

And you can rearrange this equation to find that:

$$a = \frac{2.5 \sin 8.6°}{\sin 40°} = 0.58\,\text{cm to 2 d.p.}$$

So there are two possible triangles which solve this question and they are (not to scale):




## Want to know more?

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