Learning Enhancement Team

UEA University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Trigonometry

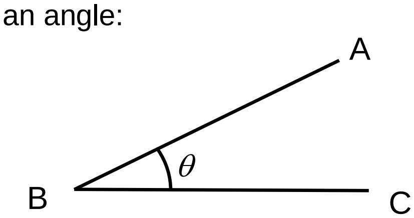
Angles

This guide introduces the definition of different types of angles and the different ways of measuring them. It also gives formulas to covert between degrees and radians.

Introduction

Angles are fundamental to geometry and having an understanding of angles can help you expand the way you look at mathematics. Making the connection between geometry and (the more commonly taught) algebra is important and often overlooked. Much of early mathematics was concerned with shapes and the ancient Greeks certainly knew about angles. However the most famous Greek treatise on mathematics, Euclid's *Elements* only describes one angle (a right-angle) and describes all other angles as parts or multiples of a right angle. When astronomy became a more formalised pursuit, leading to the advent of trigonometry, angles naturally played an important role. Through Hipparchus and eventually Ptolemy, triangles could be solved (both plane and spherical) and the angle was firmly placed at the heart of geometry. In your studies you will definitely come across angles in trigonometry and they are essential to understanding the trigonometric ratios (see study guides: *Trigonometric Ratios: Sine, Cosine and Tangent*, *Solving Right-Angled Triangles*, *Further Trigonometry*, *The Sine Rule* and *The Cosine Rule*). You will also find they play a role in calculus and vectors and so in science in general.

An angle is made when two straight lines cross. The lines AB and BC are called the **sides of the angle** and the point where they meet, B, is called **the vertex of the angle**. It is usual to draw an arc joining the two lines to depict the angle. In the diagram below you can see a drawing of an angle:



Here A, B and C refer to the three points that define the angle which is labelled by the symbol θ. You will find that is very common to name angles using italic Greek letters such as θ, ϕ, γ and so on. There are many other different ways to denote an angle, you might commonly encounter the angle θ being described by ABC, $A\hat{B}C$, $\hat{B}$ or $\angle ABC$.

Units used to measure angles

As in any physical quantity, there are different unit systems to measure angles. In mathematics and science **the degree** and **the radian** are the most widely used. Many mistakes in the calculation of angles occur because your calculator may be set in the wrong mode. For example, you may have your calculator in degree mode when you need an answer in radians. You should always check if your calculator is in the appropriate mode for a calculation. You can consult the manual for your calculator to help you understand how to change the mode on your calculator. Most calculators have three modes to perform calculations involving angles degrees, radians and gradians:

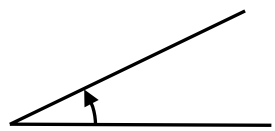
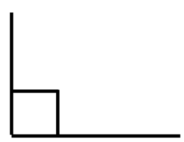
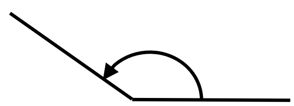
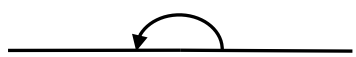
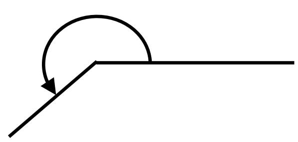
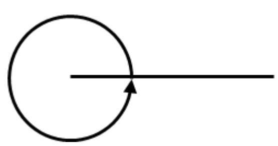
a) **Degrees**. Symbol $^\circ$, $1^\circ = 0.017\,\text{rad} = 1.11$ grad. In this unit system the full rotation around a circle is divided into 360 equal parts. Each part called a **degree** (from the Latin *de gradus*) and would be written 1°. The origin of this system is not clear with some assigning a link to Babylonian culture whose year had 360 days (plus five bad luck days). Others suggest it is based on the base 60 numeric system (sexagesimal) as 60 is a number which can be divided by many smaller whole numbers. One degree can be subdivided into 60 equal parts, each named a **minute**, which is written $1'$. In turn, each minute can be subdivided in 60 equal parts, each one called a **second**, which is written $1''$. Most angles are written in decimal form such as 47.5°. However in navigation bearings, angles are written in terms of degrees, minutes and seconds, for example $47^\circ 31' 13''$ and are said 47 degrees 31 minutes and 13 seconds.

b) **Radians**. Symbol **rad** or no symbol, $1\text{ rad} = 57.3^\circ = 63.66\,\text{grad}$. One **radian** (or **radius angle**) is the angle centred on a circle whose arc is the same length as the radius of that circle. Therefore the length of the arc of a full rotation in a circle is 2π radians (see table below), you may know that the circumference of a circle is 2π multiplied by the radius. It is very common to express radians as multiples of π. Although the concept of measuring angles in terms of arc length has been around for a long time, the term ‘radian’ was only first used by James Thomson in 1871. The use of radians as a way of measuring angles is very important when working with calculus. All calculus calculations involving a trigonometrical element are performed in radians. There are also some formulas which only work when the angle concerned is expressed in radians. For example equations for finding the arc length and area of a sector in a circle. In fact the radian is the SI unit for angle.

c) **Gradian**. Symbol **grad**, $1\text{ grad} = 0.9^\circ = 0.016\,\text{rad}$. A gradian is defined so that a full rotation is 400 grad and so a right angle is 100 grad. They were introduced to try and decimalise the measurement of angles. This system is rarely used today but you will find it as an option on most calculators.

Classification of angles

There are different types of angles, and they can be classified according to size. These are: **acute**, **right**, **obtuse**, **straight** and **reflex** angles as well as a **full rotation** or **turn**. In the table on the next page you can find an example for each case and how they are defined in terms of both degrees and radians.

Type of Angle	Figure	Range (degrees)	Range (radians)
Acute		$0° \le \theta < 90°$	$0 \le \theta < \frac{\pi}{2}$
Right		$\theta = 90°$	$\theta = \frac{\pi}{2}$
Obtuse		$90° < \theta < 180°$	$\frac{\pi}{2} < \theta < \pi$
Straight		$\theta = 180°$	$\theta = \pi$
Reflex		$180° < \theta < 360°$	$\pi < \theta < 2\pi$
Full rotation or Turn		$\theta = 360°$	$\theta = 2\pi$

You are often required to convert an angle from degrees to radians or from radians to degrees. You can try to remember the formulas but it is better to understand their origin, this will allow you to derive the relationships quickly. You start by recognising that all the way around a circle can be described by either 360° or 2π radians. This implies that the two are equal.

$$360° = 2\pi \text{ radians}$$

If you divide both sides of this relationship by 360 then you get a relationship for 1°:

$$1^\circ = \frac{\pi}{180}\text{ radians}$$

You can then work out a relationship for any number of degrees (say *n*°) by multiplying both sides by *n*:

$$n^\circ = \frac{n\pi}{180}\text{ radians}$$

Similarly, if you divide both sides of $360^\circ = 2\pi$ radians by 2π you get a definition for 1 radian:

$$\frac{180^\circ}{\pi} = 1\text{ radian}$$

You can then work out a relationship for any number of radians (say *n* radians) by multiplying both sides by *n*:

$$\frac{n \times 180^\circ}{\pi} = n\text{ radians}$$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

	Call:	01603 592761
	Ask:	ask.let@uea.ac.uk
	Click:	https://portal.uea.ac.uk/student-support-service/learning-enhancement

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