Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Steps into Trigonometry

Solving Right-Angled Triangles

This guide discusses methods and strategies for solving right-angled triangles. This entails finding the lengths of sides and sizes of angles.

Introduction

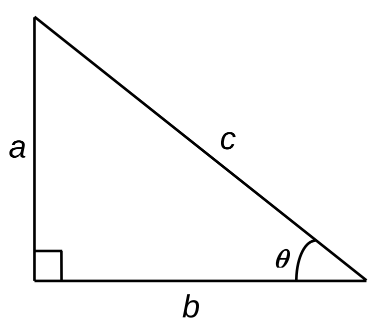
A triangle is described by six quantities, its three sides and three angles. Many mathematical problems involve triangles and it is beneficial to know the values of all the sides and angles. However you will rarely know them all. In mathematics the process of finding all the sides and angles in a triangle is called **solving the triangle**. Importantly:

In order to solve a triangle you need to know three of the six quantities and at least one of these must be a side.

This study guide concentrates on methods and strategies for solving a specific type of triangle – one which contains an angle of 90° which is commonly called a **right-angle**. These types of triangles are called **right-angled triangles**.

If you are trying to solve a right-angled triangle you can use some very useful relationships from trigonometry to help you. These are **Pythagoras' theorem** and the **trigonometric ratios sine, cosine and tangent**. If you need to refresh your knowledge of these pieces of mathematics, reading the study guides*: Pythagoras' Theorem* and *Trigonometric Ratios: Sine, Cosine and Tangent* can help. The information which this guide uses is summarised in the box below.

For right-angled triangles



$$\sin\theta = \frac{a}{c} \qquad \cos\theta = \frac{b}{c}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{a}{b}$$

$$a^2 + b^2 = c^2 \qquad \text{(Pythagoras' theorem)}$$

All the angles add to 180°

Methods for solving right angled triangles

There are some simple techniques that you can employ in order to help you to solve right angled triangles:

(i) **Draw a triangle.** This seems obvious, but it really helps to visualise the problem and will help you to make good decisions as you work through the problem.

(ii) **Label the sides and angles consistently.** Decide on the way you label your sides and angles and do this consistently every time you solve a triangle. For example in the box on page 1 the sides are labelled *a*, *b*, *c* and *c* is always the hypotenuse. Angles of interest are labelled by the Greek letters θ and ϕ.

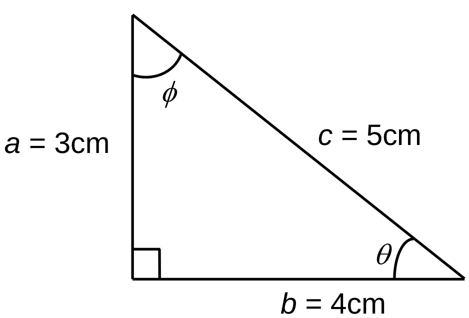
(iii) **Write the trigonometric ratios and Pythagoras' theorem and be consistent with the triangle you have drawn.** By doing this you can make a good decision about which equations can help you solve the triangle in question.

(iv) **Make sure you can rearrange an equation properly.** Most mistakes when solving triangles come from incorrectly rearranging the equation you choose. Reading the study guide: *Rearranging Equations* can help you with this skill.

If you follow these simple rules you should make improvements when you need to solve right-angled triangles.

Example: Solve the right-angled triangle with sides of length 3cm, 4cm and 5cm.

This is the famous 345 triangle (a Pythagorean triple) and the question is really asking for the two other angles. So you should begin by drawing and labelling a triangle.



You know that the longest side is 5cm and so that is labelled *c*. The other two sides are labelled as *a* and *b*, it does not matter which.

The two unknown angles are labelled θ and ϕ.

Next you write the equations which will help you solve the problem:

$$\sin\theta = \frac{a}{c} \qquad \cos\theta = \frac{b}{c} \qquad \tan\theta = \frac{a}{b} \qquad a^2 + b^2 = c^2 \qquad \theta + \phi = 90°$$

You can see from the triangle that you have drawn that you know *a*, *b* and *c* and you can use this knowledge to help you decide what to do next. **In mathematics you can solve an equation to find an unknown if the equation contains only one unknown**. If you substitute *a*, *b* and *c* into the equations you get:

$\sin\theta = \frac{3}{5}$ $\cos\theta = \frac{4}{5}$ $\tan\theta = \frac{3}{4}$ $5^2 = 3^2 + 4^2$ $\theta + \phi = 90°$

The three circled equations all have one unknown and you can use any of them to calculate the angle θ. Remember that the quantity you are trying to find is the angle θ and that $\sin\theta$, $\cos\theta$ and $\tan\theta$ are properties of that angle NOT the angle itself. You will need to use the inverse trigonometric function to find the angle (see study guide: *Trigonometric Ratios: Sine, Cosine and Tangent*) so:

From $\sin\theta = \frac{3}{5}$ it follows that $\theta = \sin^{-1}\left(\frac{3}{5}\right) = 36.87°$ to 2 d.p.

From $\cos\theta = \frac{4}{5}$ it follows that $\theta = \cos^{-1}\left(\frac{4}{5}\right) = 36.87°$ to 2 d.p.

From $\tan\theta = \frac{3}{4}$ it follows that $\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$ to 2 d.p.

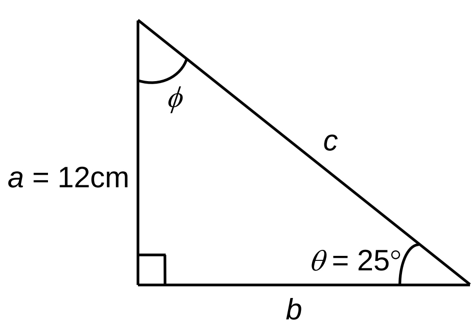
You can see that, regardless of the equation you choose, $\theta = 36.87°$ to 2 decimal places. If it is not specified in the question, you should choose an appropriate number of decimal places (2 is usually fine) but make sure you write this down as "to 2 d.p.".

You can now use $\theta + \phi = 90°$ to find the remaining angle, as $\theta = 36.87°$:

$$\begin{aligned} 36.87° + \phi &= 90° \\ \phi &= 90° - 36.87° \\ &= 53.13° \end{aligned}$$

In a right-angled triangle you can always use $\theta + \phi = 90°$ to find one angle if you know the other.

Example: A right-angled triangle contains an angle of $\theta = 25°$ and the side opposite this angle is 12 cm long. Solve this triangle.



You can use the information in the question to draw the triangle to the left.

Drawing the basic triangle and labeling $\theta = 25°$ gives $a = 12$ cm.

Now you can substitute the known values into the equations as follows:

$$\sin 25 = \frac{12}{c} \qquad \cos 25 = \frac{b}{c} \qquad \tan 25 = \frac{12}{b} \qquad c^2 = 12^2 + b^2 \qquad 25° + \phi = 90°$$

You can use the circled equations to find the missing values as there is only one unknown in each:

From $\sin 25 = \frac{12}{c}$ it follows that $c = \frac{12}{\sin 25} = 28.39$ cm to 2 d.p.

From $\tan 25 = \frac{12}{b}$ it follows that $b = \frac{12}{\tan 25} = 25.73$ cm to 2 d.p.

From $25° + \phi = 90°$ it follows that $\phi = 90° - 25° = 65°$

Important: You **cannot** use Pythagoras' Theorem and the definitions for sine, cosine and tangent to solve a triangle **without a right-angle**. To solve a triangle which does not have right angle you have to use the sine rule and the cosine rule (see study guides: *The Sine Rule* and *The Cosine Rule*).

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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Ask: ask.let@uea.ac.uk
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