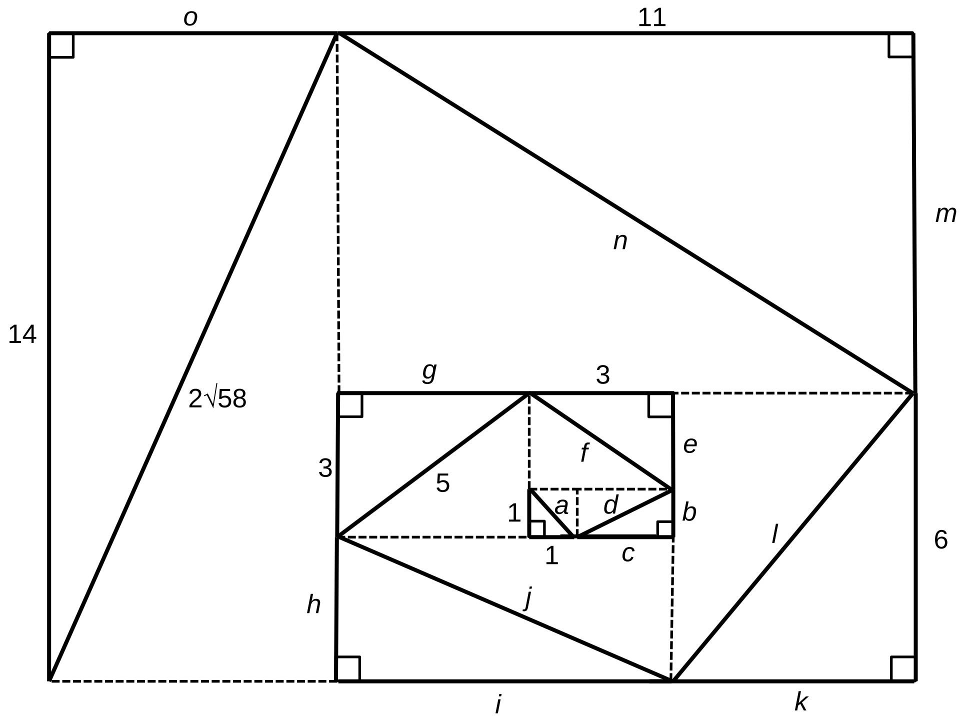
# Learning Enhancement Team

## *Model answers:* Solving Right-Angled Triangles

Solving Right-Angled Triangles study guide

Let's works out the lengths of the sides first. Pythagoras' Theorem is used throughout the explanation, if you are unsure about using it, the study guide: *Pythagoras' Theorem* should help you.

Begin by labelling all the sides you do not know from *a* to *o*:




Commentary: You can use Pythagoras' Theorem to calculate *a* as you know the length of the other two sides of this triangle. As *a* is the hypotenuse:

$$a^2 = 1^2 + 1^2 = 2$$
$$a = \sqrt{2}$$

Looking at the shape, you can see that $b = 1$.

Looking at the shape, $1+c=3$ and so $c=2$.

Now that you know *b* and *c*, you can use Pythagoras' Theorem to calculate *d*:

$$d^2 = 1^2 + 2^2 = 5$$
$$d = \sqrt{5}$$

Looking at the shape $b+e=3$, as $b=1$ then $e=2$.

Now that you know *e* (and the other side is 3), you can use Pythagoras' Theorem to calculate *f*:

$$f^2 = e^2 + 3^2 = 2^2 + 9 = 13$$
$$f = \sqrt{13}$$

You can find *g* either by using Pythagoras' Theorem to give:

$$5^2 = g^2 + 3^2$$
$$g^2 = 25 - 9 = 16$$
$$g = \sqrt{16} = 4$$

or by recognising that you have a Pythagorean Triple, the famous 345 triangle.

Looking at the shape $h+3=6$ so $h=3$.

Looking at the shape $g+3=i$, as you know that $g=4$, it follows that $i=7$.

Now that you know *h* and *i* you can use Pythagoras' Theorem to calculate *j*:

$$j^2 = h^2 + i^2 = 3^2 + 7^2 = 58$$
$$j = \sqrt{58}$$

Looking at the shape $i+k=11$, as you know that $i=7$, it follows that $k=4$.

Now that you know *k* (and the other side is 6), you can use Pythagoras' Theorem to calculate *l*:

$$l^2 = k^2 + 6^2 = 4^2 + 36 = 52$$
$$l = \sqrt{52}$$

Looking at the shape $m+6=14$, it follows that $m=8$.

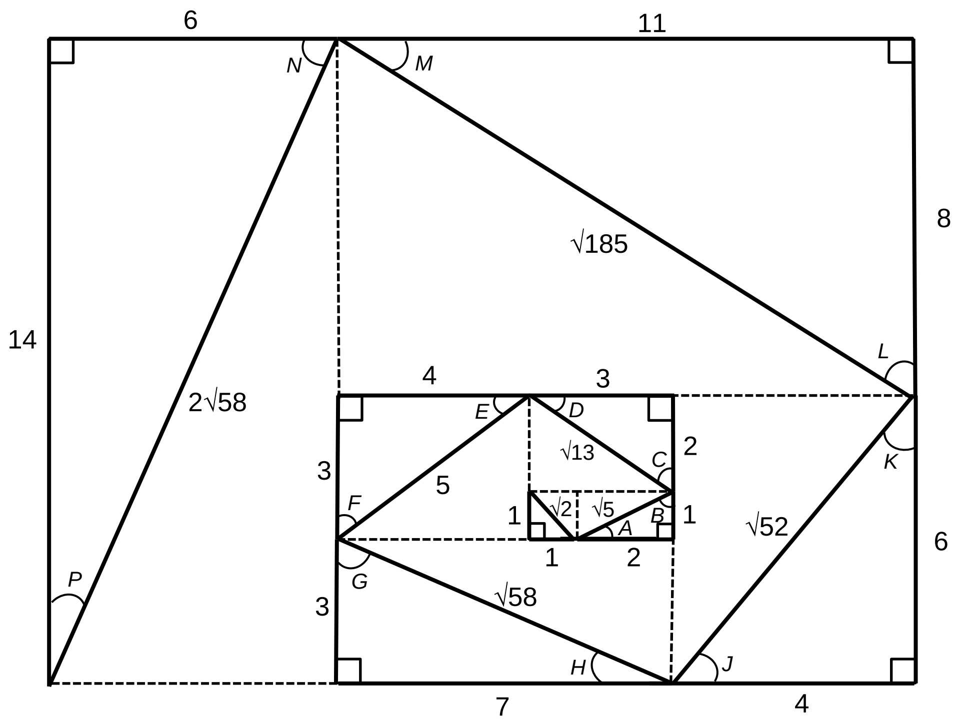
Now that you know *m* (and the other side is 11), you can use Pythagoras' Theorem to calculate *n*:

$$n^2 = m^2 + 11^2 = 8^2 + 121 = 185$$
$$n = \sqrt{185}$$

Finally, you can use Pythagoras' Theorem to calculate *o*:

$$\left(2\sqrt{58}\right)^2 = 14^2 + o^2$$
$$o^2 = \left(2\sqrt{58}\right)^2 - 14^2 = 232 - 196 = 36$$
$$o = 6$$

So the shape looks like this:




Now you know the lengths of the sides, you can use the trigonometric ratios to find the angles, see study guide: *Trigonometric Ratios: Sine, Cosine and Tangent*. To help you understand the methods used below. You can use the ratios for sine, cosine or tangent to solve the questions because you have all the sides of each triangle. However, the solutions below try to use the lengths given in the original question where possible.

The smallest triangle, the one which contains *a*, is isosceles and so the two unknown angles are identical and must be 45°.

For the triangle containing *A* and *B*:

$$\tan A = \frac{1}{2} \qquad \text{so} \qquad A = \tan^{-1}\left(\frac{1}{2}\right) = 26.57^\circ \text{ to 2 d.p.}$$

$$\tan B = \frac{2}{1} = 2 \quad \text{so} \qquad A = \tan^{-1}(2) = 63.43^\circ \text{ to 2 d.p.}$$

You can check this result by adding the angles together, they should give 90°.

For the triangle containing *C* and *D*:

$$\tan C = \frac{3}{2} \quad \text{so} \quad C = \tan^{-1}\left(\frac{3}{2}\right) = 56.31° \text{ to 2 d.p.}$$

$$\tan D = \frac{2}{3} \quad \text{so} \quad D = \tan^{-1}\left(\frac{2}{3}\right) = 33.69° \text{ to 2 d.p.}$$

You can check this result by adding the angles together, they should give 90°.

For the triangle containing *E* and *F*:

$$\sin E = \frac{3}{5} \quad \text{so} \quad E = \sin^{-1}\left(\frac{3}{5}\right) = 36.87° \text{ to 2 d.p.}$$

$$\cos F = \frac{3}{5} \quad \text{so} \quad F = \cos^{-1}\left(\frac{3}{5}\right) = 53.13° \text{ to 2 d.p.}$$

You can check this result by adding the angles together, they should give 90°.

For the triangle containing *G* and *H*:

$$\tan G = \frac{7}{3} \quad \text{so} \quad G = \tan^{-1}\left(\frac{7}{3}\right) = 66.80° \text{ to 2 d.p.}$$

$$\tan H = \frac{3}{7} \quad \text{so} \quad H = \tan^{-1}\left(\frac{3}{7}\right) = 23.20° \text{ to 2 d.p.}$$

You can check this result by adding the angles together, they should give 90°.

For the triangle containing *J* and *K*:

$$\tan J = \frac{6}{4} = \frac{3}{2} \quad \text{so} \quad J = \tan^{-1}\left(\frac{3}{2}\right) = 56.31° \text{ to 2 d.p.}$$

$$\tan K = \frac{4}{6} = \frac{2}{3} \quad \text{so} \quad K = \tan^{-1}\left(\frac{2}{3}\right) = 33.69° \text{ to 2 d.p.}$$

You can check this result by adding the angles together, they should give 90°.

It is interesting to note that this triangle and the triangle containing *C* and *D* are **similar**. That means that they contain the same angles but have different lengths of sides. You should also note that Pythagoras' Theorem still holds and each of the sides in the larger triangles are obtained by doubling those in the smaller triangle:

$$2^2 + 3^2 = \left(\sqrt{13}\right)^2$$

Multiplying each side length by 2 gives:

$$4^2 + 6^2 = \left(2\sqrt{13}\right)^2$$

which is true as $2\sqrt{13} = \sqrt{4 \times 13} = \sqrt{52}$ .

For the triangle containing $L$ and $M$:

$$\tan L = \frac{11}{8} \quad \text{so} \quad L = \tan^{-1}\left(\frac{11}{8}\right) = 53.97° \text{ to 2 d.p.}$$

$$\tan M = \frac{8}{11} \quad \text{so} \quad M = \tan^{-1}\left(\frac{8}{11}\right) = 36.03° \text{ to 2 d.p.}$$

You can check this result by adding the angles together, they should give 90°.

For the triangle containing $N$ and $P$:

$$\sin N = \frac{14}{2\sqrt{58}} = \frac{7}{\sqrt{58}} \quad \text{so} \quad N = \sin^{-1}\left(\frac{7}{\sqrt{58}}\right) = 66.80° \text{ to 2 d.p.}$$

$$\cos P = \frac{14}{2\sqrt{58}} = \frac{7}{\sqrt{58}} \quad \text{so} \quad P = \cos^{-1}\left(\frac{7}{\sqrt{58}}\right) = 23.20° \text{ to 2 d.p.}$$

It is interesting to note that this triangle and the triangle containing $G$ and $H$ are **similar**. That means that they contain the same angles but have different lengths of sides. You should also note that Pythagoras' Theorem still holds and each of the sides in the larger triangles are obtained by doubling those in the smaller triangle:

$$3^2 + 7^2 = \left(\sqrt{58}\right)^2$$

Multiplying each side length by two gives:

$$6^2 + 14^2 = \left(2\sqrt{58}\right)^2$$

You can check this result by adding the angles together, they should give 90°.