Learning Enhancement Team

STUDENT SUPPORT SERVICE

Steps into Trigonometry

Trigonometric Ratios: Sine, Cosine and Tangent

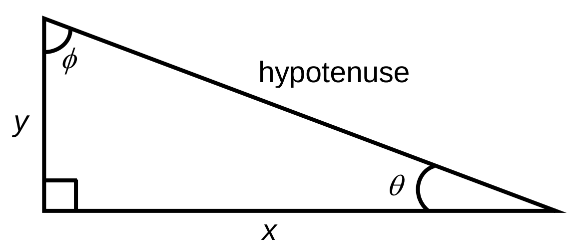
This guide introduces the trigonometric ratios sine, cosine and tangent.

Introduction

A right-angled triangle is a triangle which contains an angle equal to 90° and is a very important shape in mathematics. Many problems in science which involve 2- or 3-dimensional objects require some use of right-angled triangles. A fundamental property of right-angled triangles is that **Pythagoras' Theorem** holds (see study guide: *Pythagoras' Theorem*). Just as importantly they are used to define the **trigonometric ratios sine, cosine** and **tangent** – three of the most important concepts in mathematics.

Labelling a triangle

In order to help define the trigonometric ratios it is important that you understand how to correctly identify the sides of a triangle *in relation to* its angles. Let's start by drawing a right-angled triangle.



This triangle contains three sides; a longest side which is always called the **hypotenuse**, a side of length x and one of length y. You also have three angles; the right-angle itself (depicted by the small square) and two others given by θ (theta) and ϕ (phi). If you look carefully you can see that the hypotenuse is the side that is **opposite** the right angle – in a way the hypotenuse plays no part in the right-angle which is defined by the join between sides x and y. **The hypotenuse is always opposite the right-angle in a right-angled triangle.**

Now if you look at the angle θ, can you see that side *y* is opposite this angle? So *y* is said to be **opposite** to θ. The side *x* (along with the hypotenuse) plays a part in defining θ and is said to be **adjacent** to θ. Definitions of adjacent and opposite sides are different for the angle ϕ. Can you see that the side *x* is opposite the angle ϕ and that side *y* is adjacent to ϕ? This is the crucial point here, **when you are defining the opposite and adjacent sides of an angle, the sides you choose depend on the angle you are interested in.** Moreover the hypotenuse always remains the longest side, the one opposite the right-angle. This idea of sides which are opposite and adjacent with respect to certain angles is essential in defining the trigonometric ratios. The longer of the sides (*x* or *y*) is always opposite the larger of the angles (θ or ϕ).

Trigonometric ratios

You can use the idea of opposite and adjacent sides, along with the hypotenuse, to define the **trigonometric ratios**. The triangle at the beginning of this guide is used as the basis for the following definitions but they can be applied to any right-angled triangle.

Sine

In a right-angled triangle the sine of an angle is defined as:

$$\text{sine of an angle} = \frac{\text{length of side opposite to the angle}}{\text{length of the hypotenuse}}$$

As the hypotenuse in a triangle is always the longest side, the value for the sine of an angle is never bigger than 1. In mathematical writing, the word “sine” is shortened to “sin” and the angle of interest is written after for example $\sin\theta$. There is no multiplication here and the text “$\sin\theta$” implies the sine of the angle θ. Applied to the triangle above:

$$\sin\theta = \frac{y}{\text{hypotenuse}} \qquad \sin\phi = \frac{x}{\text{hypotenuse}}$$

Cosine

In a right-angled triangle the cosine of an angle is defined as:

$$\text{cosine of an angle} = \frac{\text{length of side adjacent to the angle}}{\text{length of the hypotenuse}}$$

The value of the cosine of an angle is never bigger than 1, for the same reason as the sine. When writing mathematically the word “cosine” is shortened to “cos” and, as with the sine, the angle of interest is written after, for example $\cos\theta$. Again there is no multiplication here and the text “$\cos\theta$” implies the cosine of the angle θ. Applied to the triangle above:

$$\cos\theta = \frac{x}{\text{hypotenuse}} \qquad \cos\phi = \frac{y}{\text{hypotenuse}}$$

Tangent

In a right-angled triangle the tangent of an angle is defined as:

$$\text{tangent of an angle} = \frac{\text{length of side opposite to the angle}}{\text{length of side adjacent to the angle}}$$

You may notice that the tangent represents the gradient of a line (usually defined as the change in *y* divided by the change in *x*, see study guide: *Finding Equations of Straight Lines*). The word “tangent” is shortened to “tan” with $\tan\theta$ denoting the tangent of the angle θ, again there is no multiplication involved. Applied to the triangle above:

$$\tan\theta = \frac{y}{x} \qquad \tan\phi = \frac{x}{y}$$

Interestingly you can use the definitions sine and cosine above to show that:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \tan\phi = \frac{\sin\phi}{\cos\phi}$$

In other words:

The tangent of an angle is the sine of that angle divided by its cosine.

Methods to remember trigonometric ratios

Over the years many mnemonics and memory devices have been created to help the student remember the trigonometric ratios. The most famous is probably the word **SOHCAHTOA** (pronounced "so – ka – toe – ah") which helps you remember that:

SOH: **S**ine of an angle is **O**pposite side divided by **H**ypotenuse.

CAH: **C**osine of an angle is **A**djacent side divided by **H**ypotenuse.

TOA: **T**angent of an angle is **O**pposite side divided by **A**djacent side.

Others include "**S**ome **O**ld **H**orses **C**hew **A**pples **H**appily **T**hroughout **O**ld **A**ge",

"**S**illy **O**ld **H**enry **C**aught **A**lbert **H**ugging **T**wo **O**ld **A**unts" and

"**S**ome **O**f **H**er **C**hildren **A**re **H**aving **T**rouble **O**ver **A**lgebra".

Inverse trigonometric functions

With the definitions of the trigonometric ratios you calculate a property of an angle (the sine, cosine or tangent) via the ratio of the relevant sides of a right-angled triangle. You can also find the size of an angle from a given trigonometric ratio value by using the **inverse trigonometric functions**. The inverses of sine, cosine and tangent are written $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$ respectively. Here the "index" does not represent a reciprocal but an inverse, so $\sin^{-1}$ is said "inverse sine" not "sine to the minus 1", and so on. You can find an angle (say θ from the above triangle) from the equations:

$$\theta = \sin^{-1}\left(\frac{y}{\text{hypotenuse}}\right)$$

$$\theta = \cos^{-1}\left(\frac{x}{\text{hypotenuse}}\right)$$

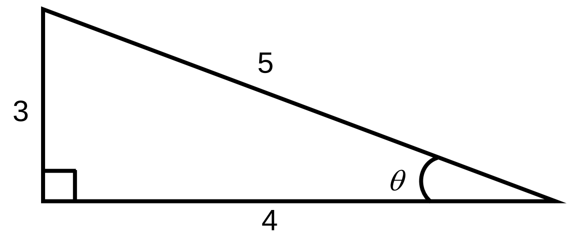
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Here the equation you choose depends on the sides that you know. These are difficult to calculate by hand and so the use of a calculator is strongly recommended. This means you should familiarise yourself with where the inverse trigonometric function buttons are on your particular calculator. Note that the inverse trigonometric functions are also commonly called arc sin, arc cos and arc tan.

Examples

1. Finding the value of a trigonometric ratio.

Given the triangle below, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.



The 3, 4, 5 right-angled triangle is a common one; it is one of the **Pythagorean Triples** (see study guide: *Pythagoras' Theorem*). With respect to the angle θ can you see that 3 is the opposite side and 4 the adjacent side? The hypotenuse is always the longest side, in this case 5. So, from SOHCAHTOA and after substitution of the relevant values:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{3}{5} = 0.6$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{4}{5} = 0.8$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{3}{4} = 0.75$$

Notice that you do not know the size of the angle θ.

2. Finding the size of an angle.

You can find the value of an angle from a known trigonometric ratio using the inverse trigonometric functions. From example 1 you know that:

$\sin\theta = 0.6$

$\cos\theta = 0.8$

$\tan\theta = 0.75$

You can use either $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$ to find the angle θ. It is important that you use the correct inverse, $\sin^{-1}$ for $\sin\theta$, $\cos^{-1}$ for $\cos\theta$ or $\tan^{-1}$ for $\tan\theta$. Remember that the angle does not change, θ is still θ, regardless of the inverse you pick, so you should get the same answer:

$$\theta = \sin^{-1}(0.6) = 36.87^\circ$$

$$\theta = \cos^{-1}(0.8) = 36.87^\circ$$

$$\theta = \tan^{-1}(0.75) = 36.87^\circ$$

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

Call: 01603 592761

Ask: ask.let@uea.ac.uk

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