

Model answers: Trigonometric Ratios: Sine, Cosine and Tangent

Trigonometric Ratios
study guide



The following definitions are used in this question:

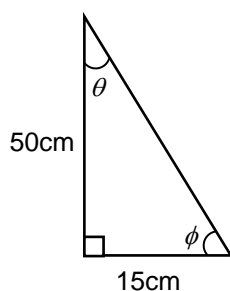
$$\text{sine of an angle} = \frac{\text{length of side **opposite** to the angle}}{\text{length of the **hypotenuse**}}$$

$$\text{cosine of an angle} = \frac{\text{length of side **adjacent** to the angle}}{\text{length of the **hypotenuse**}}$$

$$\text{tangent of an angle} = \frac{\text{length of side **opposite** to the angle}}{\text{length of side **adjacent** to the angle}}$$

Remember that the ratio that you can calculate depends on the sides that you are given and their relationship to the angle you are considering.

(i)



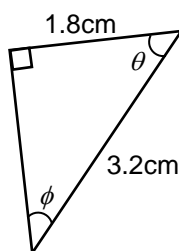
Firstly take the angle θ , you know the sides that are opposite and adjacent to it and so you can calculate the tangent of θ .

$$\tan \theta = \frac{15}{50} = 0.3$$

Now consider ϕ , you also know the opposite and adjacent sides but they are the other way around from θ and so:

$$\tan \phi = \frac{50}{15} = 3.33$$

(ii)



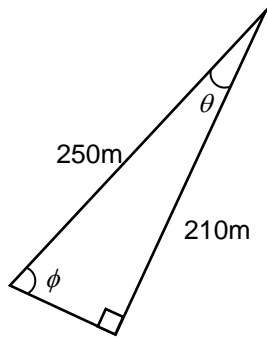
Firstly take the angle θ , you know the hypotenuse and the side adjacent to it and so you can calculate the cosine of θ

$$\cos \theta = \frac{1.8}{3.2} = 0.56$$

Now consider ϕ , you know the hypotenuse and the opposite side and so you can calculate the sine of this angle:

$$\sin \phi = \frac{1.8}{3.2} = 0.56$$

(iii)



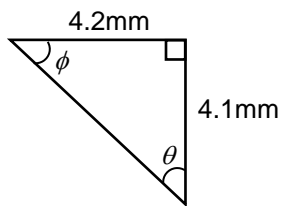
Firstly take the angle θ , you know the hypotenuse and the adjacent side so you can calculate the cosine of θ :

$$\cos \theta = \frac{210}{250} = 0.84$$

Now consider ϕ , you know the hypotenuse and the opposite side and so you can calculate the sine of ϕ :

$$\sin \phi = \frac{210}{250} = 0.56$$

(iv)



Firstly take the angle θ , you know the opposite and adjacent sides so you can calculate the tangent of θ :

$$\tan \theta = \frac{4.2}{4.1} = 1.02$$

Now consider ϕ , again you know the opposite and adjacent sides but this time the other way around:

$$\tan \phi = \frac{4.1}{4.2} = 0.98$$

2. The inverse trigonometric functions \sin^{-1} , \cos^{-1} and \tan^{-1} are used to calculate the angles in this question.

(i) $\theta = \tan^{-1}\left(\frac{15}{50}\right) = 16.70^\circ$

$$\phi = \tan^{-1}\left(\frac{50}{15}\right) = 73.30^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.

(ii) $\theta = \cos^{-1}\left(\frac{1.8}{3.2}\right) = 55.77^\circ$

$$\phi = \sin^{-1}\left(\frac{1.8}{3.2}\right) = 34.23^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.

(iii) $\theta = \cos^{-1}\left(\frac{210}{250}\right) = 32.86^\circ$

$$\phi = \sin^{-1}\left(\frac{210}{250}\right) = 57.14^\circ$$

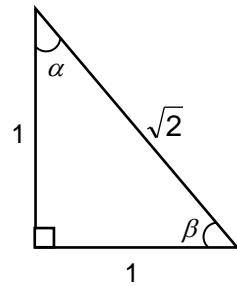
You can see that $\theta + \phi = 90^\circ$ as required.

$$(iv) \quad \theta = \tan^{-1}\left(\frac{4.2}{4.1}\right) = 45.69^\circ$$

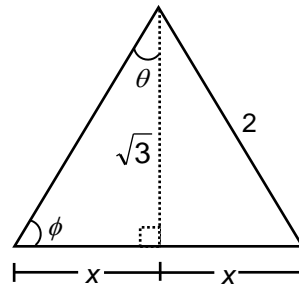
$$\phi = \tan^{-1}\left(\frac{4.1}{4.2}\right) = 44.31^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.

3. This question relates to the two triangles below.



(i)



(ii)

(a) As triangle (i) is isosceles, the two angles must have the same value. As $\alpha + \beta = 90^\circ$ then both $\alpha = 45^\circ$ and $\beta = 45^\circ$.

(b) As triangle (ii) is an equilateral triangle, each of its internal angles are 60° and so, as ϕ is an internal angle of the triangle, $\phi = 60^\circ$. The angle θ is half of ϕ as it is the result of bisecting the angle ϕ and so $\theta = 30^\circ$. Also x is half of 2 and so $x = 1$.

$$(c) \quad \sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

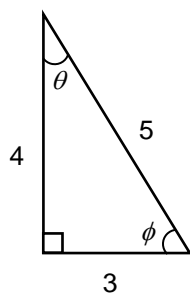
$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

4.

(i)



$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

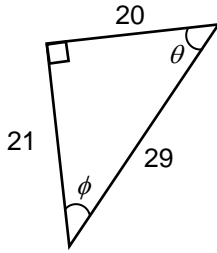
$$\tan \theta = \frac{3}{4}$$

$$\sin \phi = \frac{4}{5}$$

$$\cos \phi = \frac{3}{5}$$

$$\tan \phi = \frac{4}{3}$$

(ii)



$$\sin \theta = \frac{21}{29}$$

$$\cos \theta = \frac{20}{29}$$

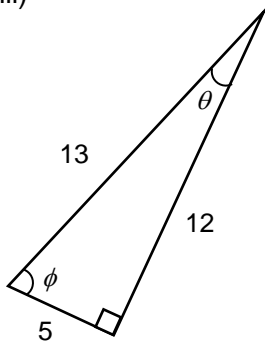
$$\tan \theta = \frac{21}{20}$$

$$\sin \phi = \frac{20}{29}$$

$$\cos \phi = \frac{21}{29}$$

$$\tan \phi = \frac{20}{21}$$

(iii)



$$\sin \theta = \frac{5}{13}$$

$$\cos \theta = \frac{12}{13}$$

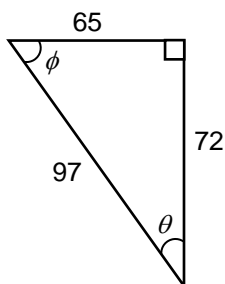
$$\tan \theta = \frac{5}{12}$$

$$\sin \phi = \frac{12}{13}$$

$$\cos \phi = \frac{5}{13}$$

$$\tan \phi = \frac{12}{5}$$

(iv)



$$\sin \theta = \frac{65}{97}$$

$$\cos \theta = \frac{72}{97}$$

$$\tan \theta = \frac{65}{72}$$

$$\sin \phi = \frac{72}{97}$$

$$\cos \phi = \frac{65}{97}$$

$$\tan \phi = \frac{72}{65}$$

5.

$$(i) \quad \theta = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ \quad \theta = \cos^{-1}\left(\frac{4}{5}\right) = 36.87^\circ \quad \theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.87^\circ$$

$$\phi = \sin^{-1}\left(\frac{4}{5}\right) = 53.13^\circ \quad \phi = \cos^{-1}\left(\frac{3}{5}\right) = 53.13^\circ \quad \phi = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.

$$(ii) \quad \theta = \sin^{-1}\left(\frac{21}{29}\right) = 46.40^\circ \quad \theta = \cos^{-1}\left(\frac{20}{29}\right) = 46.40^\circ \quad \theta = \tan^{-1}\left(\frac{21}{20}\right) = 46.40^\circ$$

$$\phi = \sin^{-1}\left(\frac{20}{29}\right) = 43.60^\circ \quad \phi = \cos^{-1}\left(\frac{21}{29}\right) = 43.60^\circ \quad \phi = \tan^{-1}\left(\frac{20}{21}\right) = 43.60^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.

$$(iii) \quad \theta = \sin^{-1}\left(\frac{5}{13}\right) = 22.62^\circ \quad \theta = \cos^{-1}\left(\frac{12}{13}\right) = 22.62^\circ \quad \theta = \tan^{-1}\left(\frac{5}{12}\right) = 22.62^\circ$$

$$\phi = \sin^{-1}\left(\frac{12}{13}\right) = 67.38^\circ \quad \phi = \cos^{-1}\left(\frac{5}{13}\right) = 67.38^\circ \quad \phi = \tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.

$$(iv) \quad \theta = \sin^{-1}\left(\frac{65}{97}\right) = 42.08^\circ \quad \theta = \cos^{-1}\left(\frac{72}{97}\right) = 42.08^\circ \quad \theta = \tan^{-1}\left(\frac{65}{72}\right) = 42.08^\circ$$

$$\phi = \sin^{-1}\left(\frac{72}{97}\right) = 47.92^\circ \quad \phi = \cos^{-1}\left(\frac{65}{97}\right) = 47.92^\circ \quad \phi = \tan^{-1}\left(\frac{72}{65}\right) = 47.92^\circ$$

You can see that $\theta + \phi = 90^\circ$ as required.



These model answers are one of a series on mathematics produced by the Learning Enhancement Team.

Scan the QR-code with a smartphone app for [more resources](#).



University of East Anglia

STUDENT SUPPORT SERVICE