Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Trigonometry

Pythagoras' Theorem

This guide introduces a fundamental part of trigonometry, Pythagoras' Theorem. It illustrates how Pythagoras' Theorem is used to solve problems involving right-angled triangles.

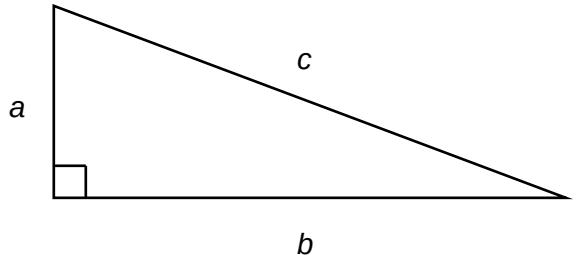
Introduction and history

Pythagoras of Samos was thought to be an Ionian Greek mathematician and philosopher who lived around 500 BC. His work and teachings gained him a group of disciples who, together with Pythagoras himself, developed and formalised a range of ideas which are still studied today. The most famous idea attributed to Pythagoras is a geometrical theorem which relates the lengths of the sides of right-angled triangles. A right-angle is an angle with a size of precisely 90°.

Right-angled triangles are extremely common in mathematics. They appear not only in trigonometry but also in vectors, complex numbers, geometry and many other mathematical disciplines. There are also many problems in science that can be modelled using right-angled triangles. Pythagoras' Theorem plays an important role in being able to solve these problems.

Pythagoras' theorem

Let's start by drawing a right-angled triangle. It is customary to assign lower case letters *a*, *b* and *c* to the side lengths. The longest side is usually labelled *c* and is called the **hypotenuse**; it is always opposite the right-angle which is indicated by a small square.



For any such right-angled triangle the following theorem always applies:

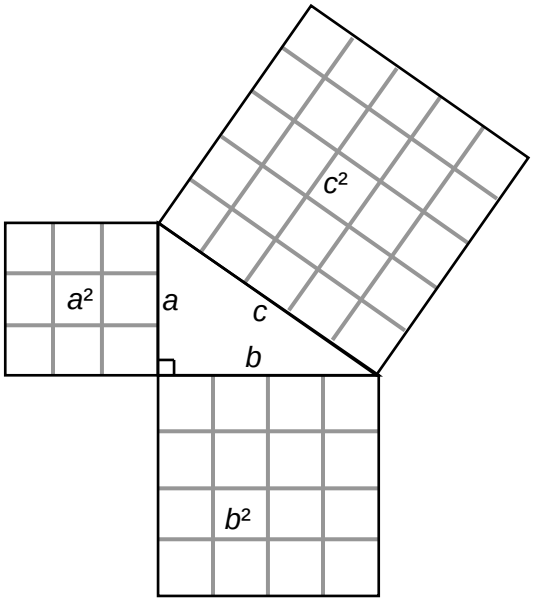
$$a^2 + b^2 = c^2$$

This is known as **Pythagoras' Theorem**. Pythagoras' Theorem is useful as you can calculate the length of one side of a right-angled triangle if you know the lengths of the other two. A good tip is to identify *c* for your triangle first. This enables you to substitute the two known lengths into Pythagoras' Theorem correctly. Remember that *c* is always the longest side (the side opposite the right angle) and the two shorter sides are *a* and *b* (it does not matter which is which). There are certain triangles in which *a*, *b* and *c* take the value of whole numbers; these are special cases of Pythagoras' Theorem and are called **Pythagorean Triples**. More commonly your triangle will not have all three sides with lengths which are whole numbers. When calculating lengths you should give your answers to a small number of decimal places (one or two) and state this as part of your answer.

Pythagoras' Theorem is often stated in the following way, which might make it easier for you to remember:

In a right-angled triangle, the square of the hypotenuse equals the sum of the squares of the other two sides.

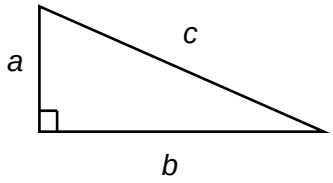
This concept is represented by the following diagram:



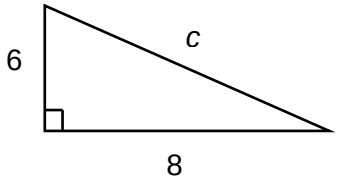
The area $a^2 = 3 \times 3 = 9$ and $b^2 = 4 \times 4 = 16$; if you add these together you get 25 which is indeed equal to c^2. So 3, 4 and 5 form a Pythagorean Triple.

Example: Find the length of the hypotenuse of a right-angled triangle with side lengths of 6 and 8.

Firstly the question states that you are interested in a right-angled triangle and so Pythagoras' Theorem can be applied. The question also states that you are trying to find the hypotenuse which implies that 6 and 8 must be the lengths of the two shortest sides. As with all trigonometry problems, it is useful to represent the problem as a diagram:



becomes:



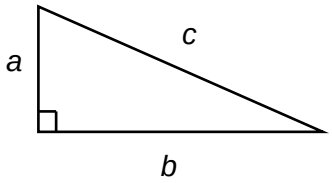
So you substitute into Pythagoras' Theorem and proceed as follows:

$a^2 + b^2 = c^2$	Pythagoras' Theorem
$6^2 + 8^2 = c^2$	substitute $a = 6$ and $b = 8$
$36 + 64 = c^2$	squaring
$100 = c^2$	addition
$\sqrt{100} = c$	square rooting
$c = 10$	answer

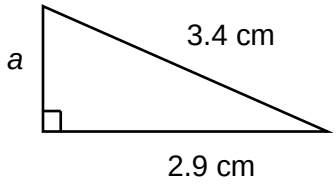
Note that $\sqrt{100}$ is ±10. However the minus option is nonsensical as you cannot have a negative length and so you take the positive value.

Example: The longest two sides of a right-angled triangle are 2.9 cm and 3.4 cm. Calculate the shortest side.

Again, you can use Pythagoras' Theorem because you are dealing with a right-angled triangle. In this case however you are given the hypotenuse and the second longest side meaning you have to work out the shortest side (*a*).



becomes:



You can now follow the same technique as above, substitute the known lengths into the theorem and then simplify, rearrange and solve in order to calculate the length *a*.

$a^2 + b^2 = c^2$	Pythagoras' Theorem
$a^2 + 2.9^2 = 3.4^2$	substitute $b = 2.9$ and $c = 3.4$
$a^2 + 8.41 = 11.56$	squaring
$a^2 = 11.56 - 8.41 = 3.15$	subtract 8.41 from each side
$a = \sqrt{3.15}$	square root
$a = 1.77$ cm to 2 d.p.	answer (to 2 decimal places)

Want to know more?

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