Learning Enhancement Team

University of East Anglia
STUDENT SUPPORT SERVICE

Model Answers: Pythagoras' Theorem

Pythagoras' Theorem study guide

1.

(i) $a = \sqrt{c^2 - b^2}$

$a^2 + b^2 = c^2$ subtract b^2 from each side.
$a^2 = c^2 - b^2$ take the square root of each side
$a = \sqrt{c^2 - b^2}$

It is common to forget to take the square root of your answer. Remember that you are after *a* and not its square. It is also common to take the square root of c^2 and b^2 to leave $c - b$, this is also not correct, you must take the square root of the whole expression $c^2 - b^2$.

(ii) $b = \sqrt{c^2 - a^2}$

$a^2 + b^2 = c^2$ subtract a^2 from each side.
$b^2 = c^2 - a^2$ take the square root of each side
$b = \sqrt{c^2 - a^2}$

It is common to forget to take the square root of your answer. Remember that you are after *b* and not its square. It is also common to take the square root of c^2 and a^2 to leave $c - a$, this is also not correct, you must take the square root of the whole expression $c^2 - b^2$.

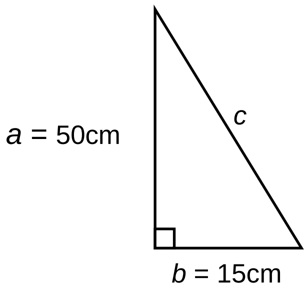
(iii) $c = \sqrt{a^2 + b^2}$

$a^2 + b^2 = c^2$ take the square root of each side
$\sqrt{a^2 + b^2} = c$

It is common to forget to take the square root of your answer. Remember that you are after *c* and not its square. It is also common to take the square root of a^2 and b^2 to leave $a + b$, this is also not correct, you must take the square root of the whole expression $a^2 + b^2$.

2.

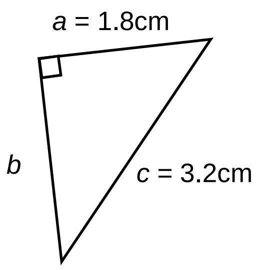
(i)



Firstly label the triangle, making sure that the hypotenuse is *c*. It does not matter which of the other sides are *a* and *b*. Here $a = 50$ cm and $b = 15$ cm with *c* unknown. Substitute these values into Pythagoras' Theorem from 1(iii):

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{50^2 + 15^2} = \sqrt{2500 + 225} = \sqrt{2725} = 52.20 \text{ cm}$$

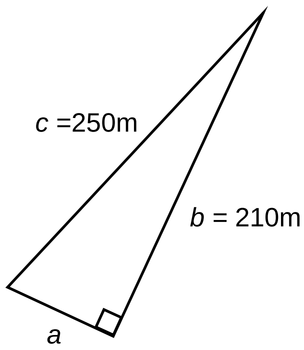
(ii)



Firstly label the triangle, making sure that the hypotenuse is *c*. It does not matter which of the other sides are *a* and *b*. Here $a = 1.8$ cm and $c = 3.2$ cm with *b* unknown. Substitute these values into Pythagoras' Theorem from 1(ii):

$$b = \sqrt{c^2 - a^2}$$
$$b = \sqrt{3.2^2 - 1.8^2} = \sqrt{10.24 - 3.24} = \sqrt{7} = 2.66 \text{ cm}$$

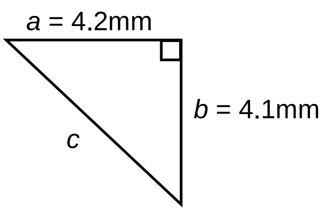
(iii)



Firstly label the triangle, making sure that the hypotenuse is *c*. It does not matter which of the other sides are *a* and *b*. Here $b = 210$ m and $c = 250$ m with *a* unknown. Substitute these values into Pythagoras' Theorem from 1(i):

$$a = \sqrt{c^2 - b^2}$$
$$a = \sqrt{250^2 - 210^2} = \sqrt{62500 - 44100}$$
$$= \sqrt{18400} = 135.65 \text{ m}$$

(iv)

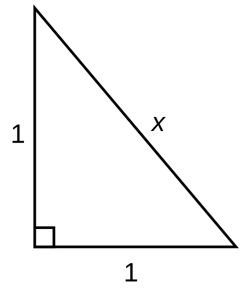


Firstly label the triangle, making sure that the hypotenuse is *c*. It does not matter which of the other sides are *a* and *b*. Here $a = 4.2$ mm and $b = 4.1$mm with *c* unknown. Substitute these values into Pythagoras' Theorem from 1(iii):

$$c = \sqrt{a^2 + b^2}$$
$$c = \sqrt{4.2^2 + 4.1^2} = \sqrt{17.64 + 16.81} = \sqrt{34.45} = 5.87 \text{ mm}$$

3.

(i)



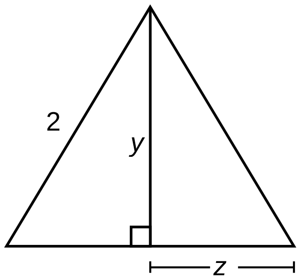
Here the unknown side, x, is the hypotenuse so you know that its square is equal to the sum of the squares of the other two sides. In other words:

$$x^2 = 1^2 + 1^2 = 2$$

and so taking the square root gives $x = \sqrt{2}$.

This triangle is an extremely useful triangle to learn about as it gives information about the angle 45°. This is explored in more detail in the worksheets: *Trigonometric Identities: Sine Cosine and Tangent* and *Solving Right-Angled Triangles*.

(ii)



Firstly, you should realise that z is half the length of the base. As the triangle is equilateral, the base has a length of 2 and so $z = 1$.

The base of the triangle which contains y is also 1. So:

$1^2 + y^2 = 2^2$ subtract 1 from each side

$y^2 = 4 - 1 = 3$ take the square root

$y = \sqrt{3}$

This triangle is an extremely useful triangle to learn about as it gives information about the angles 30° and 60°. This is explored in more detail in the worksheets: *Trigonometric Identities: Sine Cosine and Tangent* and *Solving Right-Angled Triangles*.

4.

(i) If you double all the numbers in $[3,4,5]$ you get $[6,8,10]$ and so $6^2 + 8^2$ should equal 10^2.

Check: $6^2 + 8^2 = 36 + 64 = 100$ which is 10^2.

If you double all the numbers in $[5,12,13]$ you get $[10,24,26]$ and so $10^2 + 24^2$ should equal 26^2.

Check: $10^2 + 24^2 = 100 + 576 = 676$ which is 26^2.

So it seems that, if you double the numbers in a Pythagorean Triple you get a new Pythagorean Triple.

(ii) If you treble all the numbers in $[3,4,5]$ you get $[9,12,15]$ and so $9^2 + 12^2$ should equal 15^2.

Check: $9^2 + 12^2 = 81 + 144 = 225$ which is 15^2.

If you treble all the numbers in $[5,12,13]$ you get $[15,36,39]$ and so $15^2 + 36^2$ should equal 39^2.

Check: $15^2 + 36^2 = 225 + 1296 = 1521$ which is 39^2.

So it seems that, if you treble the numbers in a Pythagorean Triple you get a new Pythagorean Triple.

(iii) If you multiply all the numbers by 10 in $[3,4,5]$ you get $[30,40,50]$ and so $30^2 + 40^2$ should equal 50^2.

Check: $30^2 + 40^2 = 900 + 1600 = 2500$ which is 50^2.

So it seems that, if you multiply the numbers in a Pythagorean Triple by 10 you get a new Pythagorean Triple.

If you halve all the numbers in $[5,12,13]$ you get $\left[\frac{5}{2}, 6, \frac{13}{2}\right]$ and so $\left(\frac{5}{2}\right)^2 + 6^2$ should equal $\left(\frac{13}{2}\right)^2$.

Check: $\left(\frac{5}{2}\right)^2 + 6^2 = \frac{25}{4} + 36 = \frac{169}{4}$ which is $\left(\frac{13}{2}\right)^2$.

So it seems that, if you halve the numbers in a Pythagorean Triple you get a new Pythagorean Triple.

If you multiply all the numbers in $[3,4,5]$ by 2.4 you get $[7.2, 9.6, 12]$ and so $7.2^2 + 9.6^2$ should equal 12^2.

Check: $7.2^2 + 9.6^2 = 51.84 + 92.16 = 144$ which is 12^2.

So it seems that, if you double the numbers in a Pythagorean Triple you get a new Pythagorean Triple.

(iv) Start with Pythagoras' Theorem $a^2 + b^2 = c^2$ which you know to be true. Looking at the left hand side of this, if you multiply *a* and *b* by some number *n* you get $(na)^2 + (nb)^2$, your aim is to show that this is equal to n^2c^2. If you can do this then you have shown that multiplying *a*, *b* and *c* by *n* you produce some mathematics that also fits Pythagoras' Theorem. If you open the brackets $(na)^2 + (nb)^2 = n^2a^2 + n^2b^2$. You can now factorise as there is a common factor of n^2 which gives $n^2(a^2 + b^2)$. You can see that n^2 is multiplied by $a^2 + b^2$ which is c^2 from Pythagoras' Theorem. So $n^2(a^2 + b^2) = n^2c^2$ and you are done.