Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Trigonometry

Introduction to Triangles

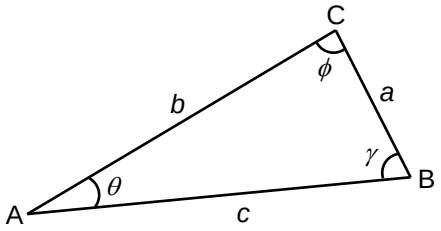
This guide introduces some of the terminology associated with triangles and some of their basic properties. This guide also lists the different types of triangle.

Introduction

A triangle is a shape which you should be familiar with as they are one of the most important shapes in mathematics. A triangle has three sides and three corners giving three internal angles and can be completely described by the lengths of the three sides and size of the three angles. The mathematical study of triangles is known as **trigonometry** (often abbreviated simply to 'trig'). The word is derived from the Greek words *trigonon* meaning "triangle" and *metron* meaning "measure". The rigorous investigation of triangles has resulted in some of the most fundamental and far-reaching theorems and rules in the history of mathematics. It is important that you are familiar with some of the basic words and properties of triangles and trigonometry if you are to become successful in any study of mathematics at University. This guide gives an overview of the terminology involved in describing a triangle and a description of the different types of triangles you may come across in your studies.

Describing triangles

There are many (sometimes confusing) ways of writing down the sides and angles of a triangle. Let's use the triangle below to explore the way that sides, angles and triangles themselves are denoted in mathematics to try and clarify why mathematicians write what they write.



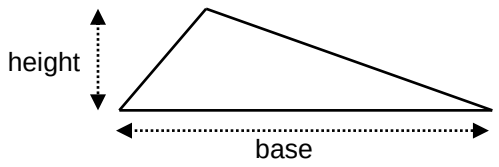
You can think of the triangle above as the shape formed when three points (which are not in a straight line) are connected by straight lines. The points in triangles are conventionally labelled using three consecutive capital letters, in this case A, B, C. You may see the triangle itself represented by the symbol "ΔABC". Sides of the triangle link the points and are written in a variety of ways. For example you may see the line linking points A and B denoted by "AB" or equally "BA", alternatively the line can be represented by a single character which is connected to the point which is *opposite* the side in question. Look carefully at the triangle above; can you see that the point C is opposite the side AB? Similarly A is opposite BC and B is opposite AC. The relevant side is conventionally labelled as the lower case version of the opposite point, so AB is opposite C and can be labelled *c*. Similarly BC can be labelled *a* and AC is labelled *b*.

The angles in the triangle can also be denoted in a variety of ways. Usually lower case Greek letters are used. Specifically θ (theta), then ϕ (phi) and then γ (gamma) are used if one, two or three angles are unknown. An alternative (and quite common) way of denoting an angle is describing the path to take to make the angle. For example to make the angle θ in the triangle above you could trace the path from B to A to C (or equally C to A to B) and this is written as "$\angle$BAC" (or "$\angle$CAB").

Properties of triangles

1. **The interior angles of any triangle sum to 180°**. This means that you can calculate the size of the third angle of a triangle when you know the size of the other two.

2. The **area of a triangle** can be calculated if you know the length of the **base** of the triangle and its **height**, it is given by half the base multiplied by the height or $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$. You can use any side as the base; the height is the distance to the peak of the triangle when the triangle is laid on the side you use as the base.



If you do not know the height of the triangle you can use **Heron's formula** to calculate the area of the triangle. For a triangle with sides of length *a*, *b* and *c*:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is the **semiperimeter** of the triangle and is given by:

$$s = \frac{a+b+c}{2} \quad .$$

3. The sum of the lengths of any two sides of a triangle is always longer that the length of the other side. This is an important property of triangles known as the **triangle inequality**.

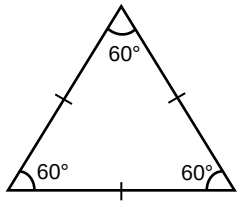
4. Triangles which have the same lengths of sides and sizes of angles are said to be **congruent**.

5. Triangles which have the same three internal angles (in the same order) are said to be **similar**.

Types of triangles

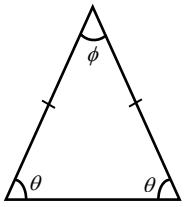
1. Equilateral Triangle

In an equilateral triangle all the sides are of equal length, this also means that all the angles are of equal size (60°). To indicate that sides are of equal length a small line is put through their centre (see right). An equilateral triangle is a **regular polygon**.



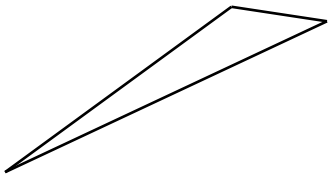
2. Isosceles Triangle

An isosceles triangle has two equal sides (with the line through) and two equal angles (here shown by θ). It is useful to note that $2\theta + \phi = 180$.



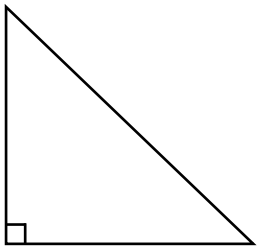
3. Scalene Triangle

A scalene triangle has three unequal sides and three unequal angles.



4. Right-Angled Triangle

A right-angled triangle contains an angle of 90° (the right-angle) and implies that the sum of the other two angles is 90°. A right-angle is depicted by a small square within the triangle (see example to the right). Right-angled triangles can be either isosceles (with other two angles equal to 45°) or scalene. Right-angled triangles are very important in mathematics; they led Pythagoras to formulate his famous theorem and are the basis for the definition of the trigonometric ratios sine, cosine and tangent (see study guides: *Pythagoras' Theorem* and *Trigonometric Ratios: Sine, Cosine and Tangent*).



5. Other Types of Triangles

There are other descriptions for triangles which are not as common as the previous four. A triangle which does not have a right-angle in it is known as an **oblique triangle**. A triangle in which all interior angles are less than 90° is called an **acute triangle**. An equilateral triangle is both oblique and acute. Finally a triangle which contains an interior angle larger than 90° is called an **obtuse triangle**.

Want to know more?

If you have any further questions about this topic you can make an appointment to see a **Learning Enhancement Tutor** in the **Student Support Service**, as well as speaking to your lecturer or adviser.

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