Learning Enhancement Team

University of East Anglia

STUDENT SUPPORT SERVICE

Steps into Algebra

Solving Quadratic Equations using the Quadratic Formula

This guide looks at quadratic equations and gives a method for their solution using the quadratic formula. The solutions are described graphically as particular points on the corresponding quadratic function.

Introduction

Before reading this guide it is recommended that you read the study guide: *Quadratic Functions* to familiarise yourself with some of the terms used here.

Every quadratic function takes the form:

$$y = ax^2 + bx + c$$

This equation is known as the general form of the quadratic function. When you solve a quadratic equation, what you are doing is finding the points where the quadratic function crosses the x-axis. **At every point on the *x*-axis, *y* takes a value of 0**, this is a crucial fact. By setting y equal to zero in the general quadratic function above you get the **general quadratic equation**:

$$ax^2 + bx + c = 0$$

The solutions of this equation give the points at which the graph of the corresponding quadratic function crosses the x-axis. In mathematics the solutions have a special name and are known as the **roots** of the equation. There are a variety of methods for solving quadratic equations (see the study guides: *Solving Quadratic Equations by Factorisation* and *Completing the Square*). This guide concerns the most common technique for solving quadratic equations, using the **quadratic formula**.

The quadratic formula

If you have to solve a quadratic equation you can use the quadratic formula to find the values of x:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You must identify the values of a, b, and c in your particular quadratic equation, substitute them into the quadratic formula and perform the resulting calculation.

You can solve *every* quadratic equation, and hence find the roots, directly by using the formula above. However the calculation can be complicated. If you *can* factorise the quadratic expression then solving by factorisation is quicker and easier than using the formula – however some algebraic manipulation is involved. You should certainly use the quadratic formula when you cannot work out the factorised form of the quadratic expression and it is especially useful when the value of a is not 1.

Example: Find the roots of $-x^2 + 3x + 2 = 0$.

To solve this question using the formula you must determine the values of a, b and c and substitute them into the quadratic formula. To do this try writing the quadratic equation you are interested in under the general quadratic equation and then draw boxes around the corresponding values:

$$\begin{array}{l} \boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0 \\ \boxed{-}x^2 \boxed{+3}x \boxed{+2} = 0 \end{array}$$

Can you see that $a = -1$, $b = +3$ and $c = +2$? It is important to include the signs to help you see whether a, b, or c are negative. When you have found a, b, and c you substitute them into the quadratic formula, using brackets to help you. Do not try to do the calculation all at once as you could make a mistake. For this example:

$x = \frac{-(+3) \pm \sqrt{(+3)^2 - 4 \cdot (-1) \cdot (+2)}}{2 \cdot (-1)}$	substitute in $a = -1$, $b = +3$ and $c = +2$.
$= \frac{-3 \pm \sqrt{9 + 8}}{-2}$	use BODMAS to remove the brackets.
$= \frac{-3 \pm \sqrt{17}}{-2}$	add 9 and 8.

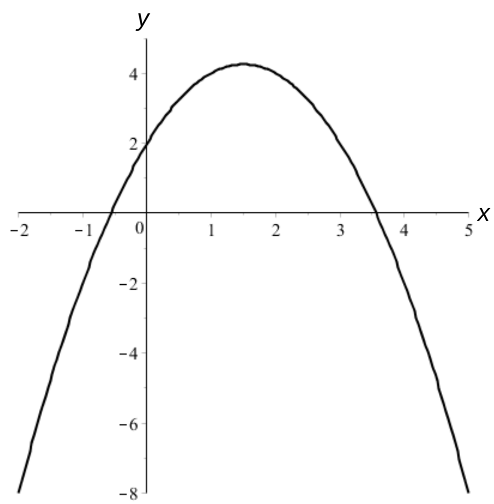
When you get to this stage the ± symbol plays an important role; you have to choose either + or –. Each choice gives a separate answer, which are your two roots:

$$x = \frac{-3 \pm \sqrt{17}}{-2}$$

choose + ↙ ↘ choose –

$$x = \frac{-3 + \sqrt{17}}{-2} = -0.561 \qquad x = \frac{-3 - \sqrt{17}}{-2} = 3.561$$

So your two roots are 3.561 and -0.561. The roots also tell you that the corresponding quadratic function $y = -x^2 + 3x + 2$ crosses the *x*-axis at approximately $x = -0.561$ and $x = 3.561$ by looking at the graph below:



$y = -x^2 + 3x + 2$

The discriminant

Part of the quadratic formula is an interesting piece of mathematics called the **discriminant**. Specifically the discriminant is the piece of mathematics underneath the square root symbol: $b^2 - 4ac$. The value of the discriminant for a given quadratic equation gives you information about the form of the corresponding quadratic function. If you need to **sketch** a quadratic function, calculating the discriminant can save you time and effort. There are three specific types of discriminant:

(i) $b^2 > 4ac$. If the value of b^2 is larger than that of $4ac$ **then the value of the discriminant is positive**. This means you have positive number under the square root sign and, due to the ± symbol in the quadratic formula, you have to choose either + or –. This gives **two (real) roots** indicating that the corresponding quadratic function intersects the *x*-axis in **two places**.

(ii) $b^2 = 4ac$. If the value of b^2 is equal to that of $4ac$ **then the value of the discriminant is zero**. This gives you zero under the square root sign in the quadratic formula. As $\sqrt{0} = 0$ the $\pm$ symbol in the quadratic formula plays no further part in the calculation as you are adding or subtracting zero. This means you have **one (real) root** at $-b/2a$. This means that the corresponding quadratic function just touches the *x*-axis in **a single place**.

(iii) $b^2 < 4ac$. If the value of b^2 is smaller than that of $4ac$ **then the value of the discriminant is negative**. This gives you a negative number under the square root sign. The square root of a negative number is not a real number (it is called a **complex number**). The $\pm$ symbol in the quadratic formula means you still have **two roots** but they are complex numbers. Complex numbers cannot be shown on a conventional graph and this means that corresponding quadratic functions **never cut the *x*-axis**.

When you are beginning to study mathematics you will almost certainly be given examples of quadratic equations with discriminants that are either positive or zero. A quick calculation of the discriminant can help you determine the shape of quadratic function you are investigating. This can help you to decide whether the answers you get are reasonable or not.

Want to know more?

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