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Model answers: Solving Quadratic Equations using the Quadratic Formula

Solving Quadratic Equations using the Quadratic Formula study guide

In order to solve a quadratic equation using the quadratic formula you will often need to manipulate the equation to take the form:

$$ax^2 + bx + c = 0$$

Only then can you work out *a*, *b* and *c* and then use them in the formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

to find the solutions for *x*, known as roots. In these model answers the roots are given to two decimal places unless otherwise stated.

1.

(a) $x^2 + 7x + 12 = 0$ $\quad (x = -3 \text{ and } x = -4)$

This question is already in the form $ax^2 + bx + c = 0$ and so you can write the two equations one above another and use boxes to help you identify *a*, *b* and *c*. You should include the sign of the number as this is important.

$$\boxed{1}x^2 \boxed{+7}x \boxed{+12} = 0$$
$$\boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0$$

Here $a = +1$, $b = +7$ and $c = +12$, so you can substitute these values into the quadratic formula to give:

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} = \frac{-7 \pm \sqrt{49 - 48}}{2} = \frac{-7 \pm \sqrt{1}}{2} = \frac{-7 \pm 1}{2}$$

Taking the positive sign of the ± gives $x_1 = \frac{-7+1}{2} = \frac{-6}{2} = -3$

Taking the negative sign of the ± gives $x_2 = \frac{-7-1}{2} = \frac{-8}{2} = -4$

You can factorise this quadratic equation by inspection and find the roots that way, see study guide: *Solving Quadratic Equations by Factorisation*. You should always ask yourself if the problem can be solved by factorisation (give yourself a time limit and if you cannot work them out within it then use the quadratic formula instead). Here:

$$x^2 + 7x + 12 = (x+3)(x+4)$$

And so:

$$(x+3)(x+4) = 0$$

Which means that the roots are $x = -3$ and $x = -4$ as shown by the formula.

(b) $x^2 + 7x + 11 = 0$ ($x = -2.38$ and $x = -4.62$)

This question is already in the form $ax^2 + bx + c = 0$ and so you can write the two equations one above another and use boxes to help you identify *a*, *b* and *c*. You should include the sign of the number as this is important.

$$\boxed{1}x^2\boxed{+7}x\boxed{+11} = 0$$
$$\boxed{a}x^2\boxed{+b}x\boxed{+c} = 0$$

Here $a = +1$, $b = +7$ and $c = +11$, so you can substitute these values into the quadratic formula to give:

$$x = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 1 \cdot 11}}{2 \cdot 1} = \frac{-7 \pm \sqrt{49-44}}{2} = \frac{-7 \pm \sqrt{5}}{2}$$

Taking the positive sign of the ± gives $x_1 = \frac{-7+\sqrt{5}}{2} = -2.38$

Taking the negative sign of the ± gives $x_2 = \frac{-7-\sqrt{5}}{2} = -4.62$

(c) $x^2 - 13x + 11 = 0$ ($x = 12.09$ and $x = 0.91$)

This question is already in the form $ax^2 + bx + c = 0$ and so you can write the two equations one above another and use boxes to help you identify *a*, *b* and *c*. You should include the sign of the number as this is important.

$$\boxed{1}x^2 \boxed{-13}x \boxed{+11} = 0$$
$$\boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0$$

Here $a = +1$, $b = -13$ and $c = +11$. As the value of b is negative, it is good practice to use brackets to make sure you do not make any mistakes when you substitute negative values into the quadratic formula:

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4 \cdot 1 \cdot 11}}{2 \cdot 1} = \frac{13 \pm \sqrt{169 - 44}}{2} = \frac{13 \pm \sqrt{125}}{2}$$

Taking the positive sign of the ± gives $x_1 = \dfrac{13 + \sqrt{125}}{2} = 12.09$

Taking the negative sign of the ± gives $x_2 = \dfrac{13 - \sqrt{125}}{2} = 0.91$

Often you will see roots of quadratic equations given as **surds** which retain the square roots in the answer. For more information about surds see the study guide: *Different Kinds of Numbers*. Here the answers in surd form are $x_1 = \dfrac{13 + 5\sqrt{5}}{2}$ and $x_2 = \dfrac{13 - 5\sqrt{5}}{2}$.

Expressing the roots in surd form can help you see the symmetry they possess. Here the roots are equally spaced around $13/2$ which is 6.5.

(d) $10 - 3x - x^2 = 0$ ($x = -5$ and $x = 2$)

Be careful here as you need to reorder the equation into the form $ax^2 + bx + c = 0$ before deciding on the values of *a*, *b* and *c*. To do this you write the x^2 term first, then the *x* term and finally the constant so you get:

$$\boxed{-1}x^2 \boxed{-3}x \boxed{+10} = 0$$
$$\boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0$$

So in this case $a = -1$, $b = -3$ and $c = +10$. As you have negative values, use brackets to make sure you do not make any mistakes when substituting into the quadratic formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot (-1) \cdot 10}}{2 \cdot (-1)} = \frac{3 \pm \sqrt{9 + 40}}{-2} = \frac{3 \pm \sqrt{49}}{-2} = \frac{3 \pm 7}{-2}$$

Taking the positive sign of the ± gives $x_1 = -\dfrac{3 - 7}{-2} = 2$

Taking the negative sign of the ± gives $x_2 = \dfrac{3 + 7}{-2} = -5$

You can factorise this quadratic equation by inspection and find the roots that way, see study guide: *Solving Quadratic Equations by Factorisation*. Here:

$$x^2 + 3x - 10 = (x-2)(x+5)$$

And so:

$$(x-2)(x+5) = 0$$

Which means that the roots are $x = 2$ and $x = -5$ as shown by the formula.

(e) $4x^2 = 4x + 1$ $(x = -0.21$ and $x = 1.21)$

To solve this equation you first need to rearrange it to fit the form $ax^2 + bx + c = 0$. You do this by subtracting $4x$ and 1 from each side to give $4x^2 - 4x - 1 = 0$. If you find this difficult you can read the study guide: *Rearranging Equations* or talk to a **Learning Enhancement Tutor**. Using the result you get:

$$\boxed{4}\,x^2 \boxed{-4}\,x \boxed{-1} = 0$$
$$\boxed{a}\,x^2 \boxed{-b}\,x \boxed{+c} = 0$$

So you use $a = +4$, $b = -4$ and $c = -1$ in the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 4 \cdot (-1)}}{2 \cdot 4} = \frac{4 \pm \sqrt{16+16}}{8} = \frac{4 \pm \sqrt{32}}{8} = \frac{1 \pm \sqrt{2}}{2}$$

Using the fact that $\sqrt{32} = 4\sqrt{2}$ (see study guide: *Laws of Indices*).

Taking the positive sign of the ± gives $x_1 = \dfrac{1+\sqrt{2}}{2} = 1.21$

Taking the negative sign of the ± gives $x_2 = \dfrac{1-\sqrt{2}}{2} = -0.21$

The roots could be given in surd form as $x_1 = \dfrac{1+\sqrt{2}}{2}$ and $x_2 = \dfrac{1-\sqrt{2}}{2}$.

(f) $-4x^2 - 5x = 1$ $(x_1 = -0.25$ and $x_2 = -1)$

Firstly you need to rearrange the equation to fit the form $ax^2 + bx + c = 0$. You do this by adding $4x^2$ and $5x$ to each side. If you find this difficult you can read the study guide: *Rearranging Equations* or talk to a **Learning Enhancement Tutor**. Adding to each has the added bonus of removing negative signs from your coefficients to give you $4x^2 + 5x + 1 = 0$ and so:

$$\boxed{4}x^2\boxed{+\ 5}x\boxed{+\ 1}=0$$
$$\boxed{a}x^2\boxed{+\ b}x\boxed{+\ c}=0$$

So now you can see that the values $a=+4$, $b=+5$ and $c=+1$, are used in the quadratic formula to give:

$$x=\frac{-5\pm\sqrt{5^2-4\cdot 4\cdot 1}}{2\cdot 4}=\frac{-5\pm\sqrt{25-16}}{8}=\frac{-5\pm\sqrt{9}}{8}=\frac{-5\pm 3}{8}$$

Taking the positive sign of the ± gives $x_1=\frac{-5+3}{8}=\frac{-2}{8}=-0.25$

Taking the negative sign of the ± gives $x_2=\frac{-5-3}{8}=-1$

(g) $x+2=3x^2$ ($x_1=1$ and $x_2=-0.67$)

Firstly you need to rearrange the equation to fit the form $ax^2+bx+c=0$. You do this by subtracting x and 2 from each side to give $3x^2-x-2=0$. If you find this difficult you can read the study guide: *Rearranging Equations* or talk to a **Learning Enhancement Tutor**. So:

$$\boxed{3}x^2\boxed{-\ 1}x\boxed{-\ 2}=0$$
$$\boxed{a}x^2\boxed{+\ b}x\boxed{+\ c}=0$$

You can see that $a=+3$, $b=-1$ and $c=-2$, and you substitute these values into the quadratic formula to give:

$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4\cdot 3\cdot(-2)}}{2\cdot 3}=\frac{1\pm\sqrt{1+24}}{6}=\frac{1\pm\sqrt{25}}{6}=\frac{1\pm 5}{6}$$

Taking the positive sign of the ± gives $x_1=\frac{1+5}{6}=\frac{6}{6}=1$

Taking the negative sign of the ± gives $x_1=\frac{1-5}{6}=\frac{-4}{6}=\frac{-2}{3}=-0.67$

(h) $2x^2=18$ ($x_1=3$ and $x_2=-3$)

Firstly you need to rearrange the equation to fit the form $ax^2+bx+c=0$ by subtracting 18 from each side. This gives $2x^2-18=0$ and so:

$$2\ x^2\qquad -18=0$$
$$\boxed{2}x^2\boxed{+\ 0}x\boxed{-18}=0$$
$$\boxed{a}x^2\boxed{+\ b}x\boxed{+\ c}=0$$

You can see that $a = +2$, $b = 0$ and $c = -18$, so you can substitute these values into the quadratic formula to give:

$$x = \frac{-0 \pm \sqrt{0^2 - 4 \cdot 2 \cdot (-18)}}{2 \cdot 2} = \frac{0 \pm \sqrt{0 + 144}}{4} = \frac{\pm\sqrt{144}}{4} = \frac{\pm 12}{4} = \pm 3$$

Taking the positive sign of the ± gives $x_1 = 3$

Taking the negative sign of the ± gives $x_2 = -3$

When $b = 0$, you can directly rearrange the quadratic equation to find *x.* Here you divide both sides by 2 and then take the square root so:

$$\begin{aligned} 2x^2 &= 18 \\ x^2 &= 9 \\ x &= \pm\sqrt{9} \end{aligned}$$

Which gives the same results as the quadratic formula. Any quadratic equation where $b = 0$ can be solved by rearranging and will result in roots $x_1 = -x_2$.

(i) $6x^2 - 3x = 0$ ($x_1 = 0.5$ and $x_2 = 0$)

This question is already in the form $ax^2 + bx + c = 0$ and so:

$$\begin{array}{l} \boxed{6}x^2 \boxed{-3}x \boxed{+0} = 0 \\ \boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0 \end{array}$$

Which shows that $a = +6$, $b = -3$ and $c = 0$, so you can substitute these values into the quadratic formula to give:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4 \cdot 6 \cdot 0}}{2 \cdot 6} = \frac{3 \pm \sqrt{9 - 0}}{12} = \frac{3 \pm \sqrt{9}}{12} = \frac{3 \pm 3}{12}$$

Taking the positive sign of the ± gives $x_1 = \frac{3+3}{12} = \frac{1}{2} = 0.5$

Taking the negative sign of the ± gives $x_2 = \frac{3-3}{12} = 0$

When $c = 0$ you can solve a quadratic equation by simple factorisation as there is always a common factor of at least *x* in each term (see study guides: *Solving Quadratic Equations by Factorisation* and *Simple Factorisation*). Here the common factor is $3x$ so:

$6x^2 - 3x = 0$ becomes $3x(2x - 1) = 0$

So either:

$3x = 0$ which implies that $x_1 = 0$

Or $2x - 1 = 0$ which implies that $x_2 = \frac{1}{2}$

Any quadratic equation where $c = 0$ will have a one root of $x_1 = 0$ and another of $x_2 = -\frac{b}{a}$.

(j) $4\left(x^2 + 4x - \frac{1}{4}\right) = -3$ ($x = -3.87$ and $x = -0.13$)

We first need to re-express the equation to take it to the form $ax^2 + bx + c = 0$ by opening the bracket and the adding 3 to each side. This gives $4x^2 + 16x + 2 = 0$ and so:

$$\begin{array}{|c|c|c|c|c|c}
4 & x^2 & +16 & x & +2 & = 0 \\
a & x^2 & + \ b & x & +c & = 0
\end{array}$$

Which shows that $a = +4$, $b = +16$ and $c = +2$. You can use these numbers in the quadratic formula to give:

$$x = \frac{-16 \pm \sqrt{16^2 - 4 \cdot 4 \cdot 2}}{2 \cdot 4} = \frac{-16 \pm \sqrt{256 - 32}}{8} = \frac{-16 \pm \sqrt{224}}{8} = \frac{-16 \pm 4\sqrt{14}}{8} = \frac{-4 \pm \sqrt{14}}{2}$$

Which uses the fact that $\sqrt{224} = 4\sqrt{14}$.

Taking the positive sign of the ± gives $x_1 \frac{-4 + \sqrt{14}}{2} = -0.13$

Taking the negative sign of the ± gives $x_2 = \frac{-4 - \sqrt{14}}{2} = -3.87$

(k) $x^2 = 104$ ($x = -10.20$ and $x = 10.20$)

By subtracting 104 from each side of the equation you get $x^2 - 104 = 0$ which fits the pattern $ax^2 + bx + c = 0$:

$$\begin{array}{|c|c|c|c|c|c}
1 & x^2 & +0 & x & -\ 104 & = 0 \\
a & x^2 & +\ b & x & +\ c & = 0
\end{array}$$

Then $a = +1$, $b = 0$ and $c = -104$ which you can substitute into the quadratic formula to find:

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4 \cdot 1 \cdot (-104)}}{2 \cdot 1} = \frac{0 \pm \sqrt{0 + 416}}{2} = \frac{\pm \sqrt{416}}{2}$$

Taking the positive sign of the ± gives $x_1 = \frac{\sqrt{416}}{2} = 10.20$

Taking the negative sign of the ± gives $x_2 = \frac{-\sqrt{416}}{2} = -10.20$

Question 1(h) explains that, when $b = 0$, you can directly rearrange the quadratic equation to find *x.* Here you take the square root of both side to obtain:

$$\begin{aligned} x^2 &= 104 \\ x &= \pm\sqrt{104} \end{aligned}$$

Which gives the roots you calculated from the quadratic formula.

(l) $2\left(2x - \tfrac{1}{3}x^2\right) + x - 6 = \tfrac{1}{3}x^2$ ($x = 2$ and $x = 3$)

You need to rearrange this equation carefully to express is as $ax^2 + bx + c = 0$. Take your time. Firstly you should open the brackets to get:

$$4x - \tfrac{2}{3}x^2 + x - 6 = \tfrac{1}{3}x^2$$

Then subtract $\tfrac{1}{3}x^2$ from each side to get:

$$4x - x^2 + x - 6 = 0$$

Then finally collect the like terms and write them as an x^2 term then an *x* term and finally the constant. This (finally) gives:

$$\begin{array}{c} \boxed{-1}x^2 \boxed{+5}x \boxed{-6} = 0 \\ a\,x^2 + b\,x + c = 0 \end{array}$$

Which shows that $a = -1$, $b = +5$ and $c = -6$. You can use these values in the quadratic formula to get:

$$x = \frac{-5 \pm \sqrt{5^2 - 4\cdot(-1)\cdot(-6)}}{2\cdot(-1)} = \frac{-5 \pm \sqrt{25 - 24}}{-2} = \frac{-5 \pm \sqrt{1}}{-2} = \frac{-5 \pm 1}{-2}$$

Taking the positive sign of the ± gives $x_1 = \frac{-5+1}{-2} = \frac{-4}{-2} = 2$

Taking the negative sign of the ± gives $x_2 = -\frac{-5-1}{-2} = \frac{-6}{-2} = 3$

You can factorise this quadratic equation by inspection and find the roots that way, see study guide: *Solving Quadratic Equations by Factorisation*. Here:

$$-x^2+5x-6=(2-x)(x-3)$$

And so:

$$(2-x)(x-3)=0$$

Which means that the roots are $x=2$ and $x=3$ as shown by the quadratic formula.

2. (a) Discriminant is -71 and so no real roots.

For a quadratic equation in the form $ax^2+bx+c=0$, the discriminant is given by b^2-4ac. So for $3x^2-x+6=0$:

$$\boxed{3}x^2\boxed{-\ 1}x\boxed{+\ 6}=0$$
$$\boxed{a}x^2\boxed{+b}x\boxed{+\ c}=0$$

Which shows that $a=3$, $b=-1$ and $c=6$ which gives a discriminant of:

$$(-1)^2-4\cdot 3\cdot 6=1-72=-71$$

Since $\sqrt{-71}$ is not a real number, there are no real values of *x* which solve this quadratic equation and so it has no real roots. The solutions to this quadratic equation are **complex numbers** (see study guide: *Different Kinds of Numbers*). From this you can conclude that the quadratic function $y=3x^2-x+6$ does not cut the *x*-axis.

(b) Discriminant is 0 and so one root at $x=-1$.

For a quadratic equation in the form $ax^2+bx+c=0$, the discriminant is given by b^2-4ac. So for $x^2+2x+1=0$:

$$\boxed{1}x^2\boxed{+\ 2}x\boxed{+\ 1}=0$$
$$\boxed{a}x^2\boxed{+b}x\boxed{+\ c}=0$$

Which shows that $a=1$, $b=2$ and $c=1$ which gives a discriminant of:

$$2^2-4\cdot 1\cdot 1=4-4=0$$

A quadratic equation with a discriminant of 0 has one real root at $-b/2a$ and the function touches the *x*-axis at this value only. For $x^2+2x+1=0$ the root is:

$$x=\frac{-2}{2\cdot 1}=-1$$

(c) Discriminant is -64 and so no real roots.

For a quadratic equation in the form $ax^2+bx+c=0$, the discriminant is given by b^2-4ac. So for $-x^2+6x-25=0$:

$$\begin{array}{rlrlrl} \boxed{-1} & x^2 & \boxed{+\ 6} & x & \boxed{-\ 25} & =0 \\ \boxed{a} & x^2 & \boxed{+b} & x & \boxed{+\ c} & =0 \end{array}$$

Which shows that $a=-1$, $b=+6$ and $c=-25$ which gives a discriminant of:

$$6^2-4\cdot(-1)\cdot(-25)=36-100=-64$$

Since $\sqrt{-64}$ is not a real number, there are no real values of x which solve this quadratic equation and so it has no real roots. The solutions to this quadratic equation are **complex numbers** (see study guide: *Different Kinds of Numbers*). From this you can conclude that the quadratic function $y=-x^2+6x-25$ does not cut the x-axis.

(d) Discriminant is -231 and so no real roots.

For a quadratic equation in the form $ax^2+bx+c=0$, the discriminant is given by b^2-4ac. So for $4x^2-3x+15=0$:

$$\begin{array}{rlrlrl} \boxed{4} & x^2 & \boxed{-\ 3} & x & \boxed{+\ 15} & =0 \\ \boxed{a} & x^2 & \boxed{+b} & x & \boxed{+\ c} & =0 \end{array}$$

Which shows that $a=+4$, $b=-3$ and $c=+15$ which gives a discriminant of:

$$(-3)^2-4\cdot4\cdot15=9-240=-231$$

Since $\sqrt{-231}$ is not a real number, there are no real values of x which solve this quadratic equation and so it has no real roots. The solutions to this quadratic equation are **complex numbers** (see study guide: *Different Kinds of Numbers*). From this you can conclude that the quadratic function $y=15-3x+4x^2$ does not cut the x-axis.

(e) Discriminant is 25 and so two real roots at $x=0.67$ and $x=1$.

For a quadratic equation in the form $ax^2+bx+c=0$, the discriminant is given by b^2-4ac. You need to rearrange the equation to the form $ax^2+bx+c=0$ before finding the discriminant. You can do this by adding $3x$ and subtracting 3 from each side of the equation $9-9x^2=3-3x$, this gives:

$$\begin{array}{rlrlrl} \boxed{-3} & x^2 & \boxed{+\ 1} & x & \boxed{+2} & =0 \\ \boxed{a} & x^2 & \boxed{+b} & x & \boxed{+c} & =0 \end{array}$$

Which shows that $a = -3$, $b = +1$ and $c = +2$ which gives a discriminant of:

$$1^2 - 4 \cdot (-3) \cdot 2 = 1 + 24 = 25$$

You can use this result in quadratic formula to find the roots:

$$x = \frac{-1 \pm \sqrt{25}}{2 \cdot (-3)} = \frac{-1 \pm 5}{-6}$$

Taking the positive sign of the ± gives $x_1 = \frac{-1+5}{-6} = \frac{4}{-6} = -\frac{2}{3} = 0.67$

Taking the negative sign of the ± gives $x_2 = -\frac{-1-5}{-6} = \frac{-6}{-6} = 1$

(f) Discriminant is 49 and so two real roots at $x = -5$ and $x = 2$.

For a quadratic equation in the form $ax^2 + bx + c = 0$, the discriminant is given by $b^2 - 4ac$. You need to rearrange the equation to the form $ax^2 + bx + c = 0$ before finding the discriminant. You can do this by adding $3x$ and subtracting 10 from each side of the equation $x^2 = -3x + 10$, this gives:

$$\begin{array}{l} \boxed{1}x^2 \boxed{+3}x \boxed{-10} = 0 \\ \boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0 \end{array}$$

Which shows that $a = +1$, $b = +3$ and $c = -10$ which gives a discriminant of:

$$3^2 - 4 \cdot 1 \cdot (-10) = 9 + 40 = 49$$

You can use this result in quadratic formula to find the roots:

$$x = \frac{-3 \pm \sqrt{49}}{2 \cdot 1} = \frac{-1 \pm 7}{2}$$

Taking the positive sign of the ± gives $x_1 = \frac{-3+7}{2} = \frac{4}{2} = 2$

Taking the negative sign of the ± gives $x_2 = -\frac{-3-7}{2} = \frac{-10}{2} = 5$

(g) Discriminant is -11 and so no real roots.

For a quadratic equation in the form $ax^2 + bx + c = 0$, the discriminant is given by $b^2 - 4ac$. You need to rearrange the equation to the form $ax^2 + bx + c = 0$ before finding the discriminant. You can do this by multiplying each side by x, then subtracting x from each side and finally adding 3 to each side of $\frac{x-3}{x} = x$ this gives:

$$\boxed{1}x^2 \boxed{-1}x \boxed{+3} = 0$$
$$\boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0$$

Which shows that $a = +1$, $b = -1$ and $c = +3$ which gives a discriminant of:

$$(-1)^2 - 4 \cdot 1 \cdot 3 = 1 - 12 = -11$$

Since $\sqrt{-11}$ is not a real number, there are no real values of x which solve this quadratic equation and so it has no real roots. The solutions to this quadratic equation are **complex numbers** (see study guide: *Different Kinds of Numbers*). From this you can conclude that the quadratic function $y = x^2 - x + 3$ does not cut the x-axis.

(h) Discriminant is 25 and so two real roots at $x = 4$ and $x = -1$.

For a quadratic equation in the form $ax^2 + bx + c = 0$, the discriminant is given by $b^2 - 4ac$. You need to rearrange the equation to the form $ax^2 + bx + c = 0$ before finding the discriminant. For $\frac{1}{x-4} = \frac{x}{4-x}$ you multiply each side by each denominator in turn, open the resulting brackets, subtract 4 and add x to each side, this gives:

$$\boxed{1}x^2 \boxed{-3}x \boxed{-4} = 0$$
$$\boxed{a}x^2 \boxed{+b}x \boxed{+c} = 0$$

Which shows that $a = +1$, $b = -3$ and $c = -4$ which gives a discriminant of:

$$(-3)^2 - 4 \cdot 1 \cdot (-4) = 9 + 16 = 25$$

You can use this result in quadratic formula to find the roots:

$$x = \frac{3 \pm \sqrt{25}}{2 \cdot 1} = \frac{3 \pm 5}{2}$$

Taking the positive sign of the ± gives $x_1 = \frac{3+5}{2} = \frac{8}{2} = 4$

Taking the negative sign of the ± gives $x_2 = -\frac{3-5}{2} = \frac{-2}{2} = -1$